Asset Redeployability, Liquidation Value, and Endogenous Capital Structure Heterogeneity

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Abstract

Firms with lower leverage are not only less likely to experience financial distress but are also better positioned to acquire assets from other distressed firms. With endogenous asset sales and values, each firm's debt choice then depends on the choices of its industry peers. With indivisible assets, otherwise identical firms may adopt different debt policies—some choosing highly levered operations (to take advantage of ongoing debt benefits), others choosing more conservative policies to wait for acquisition opportunities. Our key empirical implication is that the acquisition channel can induce firms to *reduce* debt when assets become more redeployable.

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Firms with more leverage are more likely to experience future financial distress. Importantly, their expected costs of bankruptcy are likely to be higher not only when they themselves, but also when their industry peers have taken on more debt. More firms will then want to sell the same types of assets at the same time, and their peer firms—who would otherwise have been the natural asset buyers—become themselves more limited in their capacity to absorb these assets (Shleifer and Vishny (1992)).[1] As a result, the fire-sale discounts relative to fundamental asset values will become steeper. And, therefore, the debt choices of individual firms today, aggregated into industry debt, can themselves influence the asset liquidation values and have anticipative feedback into firms' debt choices in the first place.

Like most earlier literature, in our model, firms choose their capital structures before they learn their profitabilities. Leverage confers direct value benefits, such as signaling benefits, incentive enhancements, or tax shields. However, leverage can lead to distress costs for firms that later experience negative shocks. In the event of default, the creditors must decide whether to liquidate on the one hand, or to reorganize and continue operations on the other. If they liquidate, firms receive the prevailing market price for their assets. The assets will then be in the hands of buyers who can presumably put them to better use. If they reorganize, firms keep the assets but may still suffer some impairments, such as direct costs and strained relationships with key stakeholders. A distressed firm is not worth as much as it would have been in the absence of default.

Unlike in most earlier literature, in our model,[2] debt-laden capital-constrained firms are not only more likely to sell but also less likely to buy assets. We assume that all firms are competitive and can anticipate but not internalize the effects of their peers. The mechanism in our model that coordinates their debt choices is the endogenous asset price. For example, suppose that some firms adopt more aggressive debt policies. In the future, this will increase the supply and reduce the demand for liquidated assets, resulting in a lower equilibrium price. In turn, the anticipated lower price creates two motivations for the remaining firms today: (1) they will fear running into financial distress more; and (2), if they reduce their own debt, they will be more likely to enjoy future vulture buying opportunities. Thus, their best response to higher debt by their peers is lower debt for themselves.

[1] Bolton, Santos, and Scheinkman (2011) motivate preferred [industry] purchasers able and willing to pay more than outsiders with adverse selection.

[2] We will discuss the literature in great detail in Section III.A. Moreover, Table 3 shows succinctly how our model's key implications relate to and differ from this earlier literature.

The “opportunistic-acquisition” channel can reverse an important implication of models with only the “financial distress” channel. In Williamson (1988) and Harris and Raviv (1990), when assets are more redeployable, firms take on *more* debt because their distress costs will be lower (Benmelech, Garmaise, and Moskowitz (2005)). By contrast, in our model, greater redeployability creates more favorable future buying opportunities and firms may take on *less* debt to take advantage of them. The need of peers to liquidate can create a real growth option in the sense of Myers (1977) or McDonald and Siegel (1986) that can then itself feed back into debt choices and asset prices. No previous model has shown a negative comparative static with respect to redeployability.

Interestingly, when assets are indivisible, a-priori homogeneous firms sometimes split endogenously into two coexisting types who specialize in leverage and role. Some firms lever up to take advantage of the direct ongoing value benefits of debt—even anticipating distress and having to fire-sell—while other firms maintain conservative capital structures (“dry powder”) to take advantage of these anticipated future fire sales (as in the acquisition model of Morellec and Zhdanov (2008)).

Some publicly-traded corporations and industries seem to fit the assumptions of our model. For example, in the shipping industry, where assets are costly and indivisible, Diana Shipping (ticker: DSX) strategically chooses a low-debt conservative capital structure (unlike most other shipping firms) to expand its fleet when ships are liquidated at fire-sale prices.[3] Another natural domain of our model are private companies operating in more local markets. Anecdotal evidence suggests that some local real-estate developers are aggressive, while others wait more patiently for the future fire-sale opportunities in the next downturn. In the context of our model, such heterogeneity can arise more naturally or be amplified for projects such as large local developments (like shopping malls) that are difficult to parcel up.

Our model can also offer further insights. For example, there may be too many or too few asset transfers relative to first-best in our model. And, relevant to the literature on M&A activity, we show that transfer efficiency can be either procyclical or countercyclical, depending on parameters. Thus, for example, any tax policy designed to improve allocational efficiency must be context sensitive. Moreover, our model can also offer predictions

[3]For example, in its 2011 annual report, Diana Shipping stated that its strategy of maintaining a conservative balance sheet allowed it to "seize upon opportunities to deploy our strong cash position to acquire vessels at attractive valuations (p.4)." That year, it used its excess cash to purchase two Panamax dry bulk carriers from distressed sellers at deep fire-sale prices.

on other observables, such as asset transfer quantities and prices, recovery rates and credit spreads, default and liquidation probabilities, and so on.

Our paper also makes a more general point. Most theories of capital structure are about how parameters influence the optimal debt choice. Most empirical tests use normalized leverage, typically dividing it by firm value. When the market value is used, the problem is that not only debt but also firm-value should change with parameters. This matters less when debt and value respond in opposite directions, although what is interpreted as a test on debt could merely be a test on value. It matters more when debt and value respond in the same direction. The empirical metric, debt-to-market-value, then measures merely whether debt or value changes faster. In our specific model, we illustrate this general point by showing how an increase in the direct benefits of debt always increases debt but not always debt-to-value ratios.

Our paper now proceeds as follows: Section I lays out a basic no-distress model, in which firms with low leverage can later purchase assets from other firms that will turn out to have low productivity. Redeployability favors less leverage, as buyers want the opportunity to purchase poorly performing assets down the line. Debt has an effect only through its influence on this "opportunistic-acquisition" channel. Section II adds the more recognized "financial-distress" channel. Without the acquisition channel, redeployability always favors more leverage, because sellers can rely on the lesser downside. With both the purchase channel and the distress channel, more asset redeployability at first favors higher leverage (lesser distress costs dominate) and then decreases in leverage (greater acquisition opportunities dominate). Section III puts the model in perspective and describes its relation to prior research. In particular, it explains why our paper offers the very first model for many of the conjectures in Shleifer and Vishny (1992), and the relation of our model to Gale and Gottardi (2011) and Acharya and Viswanathan (2011). Section IV concludes.

I The Opportunistic-Acquisition Channel

[Insert Table 1 here: **Variables**]

In this section, we introduce a model in which lower leverage allows firms to undertake more acquisitions in the future. In the next section, lower leverage will also reduce expected financial-distress reorganization costs. Table 1 summarizes the key variables in our model.

A Model Setup and Assumptions

We consider an industry with risk-neutral competitive firms. Each firm has a manager who maximizes the value of the firm. This is not to discount the real-world importance of intra-firm agency conflicts, but to show that our results can obtain even when they are not present.[4] All information is public upon realization, again to show that asymmetric information concerns are not required for our results, not to discount their real-world importance.

Assets and Types: At time 0, each firm owns one indivisible unit of a productive asset.[5] The productivity of this asset is a random variable, denoted $\tilde{v}_i$, whose realization will be publicly observed at time 1. All firms are ex-ante identical and it is common knowledge that their firm type is distributed uniformly on the interval $\tilde{v}_i \in [0, 1]$. After firm productivity is realized at time 1, firms with enough capital (low leverage and high productivity) can acquire assets offered by other firms in the industry at the prevailing endogenous price P. We always assume free disposal, so $P \geq 0$. All assets generate a payoff at time 2, which depends on the holder's realized productivity v_i. $\tilde{v}_i$ P

Financing: At time 0, each firm can finance its asset purchase (but not slack excess cash) with equity or debt. The face value of debt is constrained to be $F_i \in [0, 1]$.[6] F_i

[4] We provide an online appendix in which we model managers that maximize equity and not debt values. Maximizing firm value is the same as maximizing equity value out of default and debt value in default.

[5] At time 0, all firms are identical and we can define their preferred investment amount to be one unit. As we describe below, some firms may wish to purchase liquidated assets at time 1, but these buying firms are then aware of their higher productivity.

All agents are risk-neutral and there is no time discounting, so the expected rate of return on debt is zero. We assume that debt F_i offers immediate net benefits that confer a proportional value $\tau \cdot F_i$. This τ can include the tax benefit of debt (which may or may not be socially valuable), but we have a much broader concept in mind.[7] The parameter τ can reflect the ability of debt to allow financially-constrained firms to take on more productive projects, any positive incentive effects from debt, lower fund-raising costs, and so on; all net of debts' unmodelled costs. This benefit is not dissipated by subsequent events and accrues to the original owners. We show in Appendix C that all our main results hold when the debt benefit is available to pay creditors and fund acquisitions. τ

Liquidation: At time 1, after each firm has learned its productivity realization v_i, it can decide whether to sell its asset at the prevailing price P or to continue operations. Because managers' objectives are aligned with their firms', their decision to liquidate or continue is efficient, given their earlier time 0 debt choice. The value from continuing operations is v_i. The liquidation price of the asset is determined by perfectly competitive buyers and sellers. Thus, firm i sells iff $v_i < P$. Although the firm's own debt choice has no influence on the asset's price, each firm knows that the asset price is determined by the collective choices of all firms in the industry.

Acquisition: Although firms can acquire liquidated assets, we assume there is some cost associated with redeployment. This could be because assets need to be customized. Repurposing can require, e.g., moving costs, reprogramming, retraining of workers, and coordination with other complementary assets. In our model, we assume that an asset with productivity v_i to its current owner (firm i) has productivity of $\eta \cdot v_j$ to a potential acquirer (firm j), where $\eta < 1$. Higher values of η imply that assets can be redeployed more easily (at lower cost). In this specification, an asset that transfers from a low-productivity seller i to a high-productivity buyer j enjoys upgraded productivity ($v_j > v_i$), but not to the same η

[6]An upper limit on F_i ensures that the promised debt payment is never greater than the firm's highest possible cash flow (sans direct benefits). A higher value of F_i would not result in higher proceeds from the debt issuance, because the increment would not be paid. A better assumption would be to impose the upper limit and assess the (tax and other debt) benefits not on the promised but on the expected debt payoff. Unfortunately, this specification forces the model into numerical rather than algebraic solutions.

[7]The model in the text interprets τ broadly as the direct benefits of debt. However, we have solved the model in which τ can represent the tax shield (where taxes also negatively affect firm value), or any combination of tax and non-tax benefits. This requires multiplying our objective functions (except the additive $\tau \cdot F_i$ term) by $1-\tau$. The solutions are appropriately proportional, except that the τ parameter becomes its monotonic transformation $\tau/(1-\tau)$. And with the exception of $\partial V^*/\partial \tau$, which is specifically marked in Table 2, all comparative statics remain the same. This is covered in Appendix E.

extent that it would have had if buyer j had owned it all along. Thus, holding productivity fixed across firms, the asset is also worth more to the current owner than to a potential buyer if both have equal productivity. Taking both firm-specificity and own productivity into account, firms find it worthwhile to buy liquidated assets only if they are sufficiently more productive—acquiring liquidated assets at price P is positive NPV for all firms with $v_j > P/\eta$.

As in Shleifer and Vishny (1992), the natural buyers of liquidated assets are other firms in the industry with appropriate expertise. These firms have limited capital, and they are constrained in their ability to acquire the asset at time 1 if they took on too much debt at time 0. Similar limits are also central in other papers, most prominently Duffie (2010). It can be justified, e.g., by *cash-in-the-market* financing (Gale and Gottardi (2015)), where firms are assumed to be unable to raise outside funding on short notice. Our model can go a step further, because it includes one parameter that can help capture at least some cross-sectional or time-series variation in the cash-in-the-market immediacy constraint. Long-term demand curves are more elastic than short-term demand curves. Our parameter η would be lower when assets are shorter-lived, when they require more informed buyers or due diligence (the crisis time relative to the life time of the cash flows), and when they are more difficult to put to use by outsiders. Of course, η also has to encapsulate further real-life aspects, such as how quickly transfer activity would have to take place when aggregate economic and financial conditions are worse. In the extreme allowed in our model, η can approach 1 if industry firms can simply wait out any crisis and search until they can find the nearly perfect buyer.

In our specific model, the only financing available to a firm at time 1 is its internal equity, which is the maximum of zero and $(v_i - F)$. We assume that each firm can only acquire one unit of the liquidated asset at time 1.[8] This reflects limited organizational capacity to take on too many new assets at one time.

Timing: Figure 1 illustrates the timing of decisions more precisely.

[8]The indivisibility assumption is important for the existence of a mixed equilibrium. However, our other qualitative results hold if the asset is divisible and firms can acquire as much of the asset as they can afford with their residual equity $(v_i - F_i)$. See Section I.D for more detail.

[Insert Figure 1 here: **Game Tree for the Acquisition Model**]

Objective: At time 0, each firm chooses a debt level F_i to maximize its ex-ante value,

$$(1) \qquad V(P, F_i) \;=\; \int_0^P P\,dv + \int_P^1 v\,dv + \int_{\min\{P+F_i,1\}}^1 \max\{0, \eta \cdot v - P\}\,dv + \tau \cdot F_i \;.$$

The first term represents the payoff P if the asset is eventually liquidated ($v_i \leq P$). The second term represents the payoff if the firm chooses to continue operations ($P < v_i \leq 1$). The third term represents the expected surplus if the firm chooses to acquire liquidated assets. The limits of integration recognize that the firm only has sufficient capital to acquire the asset if $v_i \geq P + F_i$ and the integrand ($\max\{0, \eta \cdot v_i - P\}$) recognizes that the firm only acquires the asset if it is positive NPV ($v_i > P/\eta$). The final term represents the immediate benefit of debt.

If the firm's financing constraint is binding (i.e., $1 \geq P + F_i \geq P/\eta$), the expected surplus associated with acquiring assets at time 1 is

$$\int_{P+F_i}^1 (\eta \cdot v - P)\,dv \;=\; \frac{\eta \cdot [1-(P+F_i)^2]}{2} \;-\; P \cdot (1 - P - F_i)\;,$$

which is decreasing in the own debt choice F_i. Thus, debt is costly because it reduces future profitable buying opportunities. Furthermore, the surplus is (negative) quadratic in F_i. As the debt level increases, the marginal cost of debt also increases as firms are forced to forgo more and more profitable acquisition opportunities. This is in contrast to the marginal benefits of debt which we have assumed to be linear, leading to the possibility of internal optimal debt levels. Moreover, this cost of debt is increasing when future buying opportunities are of higher quality (i.e., assets are more easily redeployed or the price is lower). When the price of the asset is determined endogenously, as in our model, it will depend partly on how easily the asset can be redeployed. Therefore, the net effect of asset redeployability on equilibrium debt choice is not yet clear.

Because there is no aggregate uncertainty in our model, and we have infinitely many industry participants,[9] firms can anticipate the equilibrium price P at time 0. Therefore,

[9] Appendix D considers an extension with aggregate uncertainty.

each firm can consider its debt choice in one of three regions, outlined by a marginal cost defined by the right-most integral in (1):

1. For low debt, $F_i \leq P/\eta - P$, the marginal cost of debt is zero: increasing debt is not costly because the firm's financing constraint is not binding. Thus, because the marginal benefit of debt is positive (τ), it is always optimal for the firm to increase debt beyond this region.

2. For medium debt, $P/\eta - P < F_i < 1 - P$, the marginal cost is $\eta \cdot (P + F_i) - P$: increasing debt is costly because the firm's financing constraint is now binding, i.e., it may have to forego acquiring positive NPV assets that will be liquidated.

3. For high debt, $F_i \geq 1 - P$, the marginal cost is again zero: the debt is so high that the firm would not be able to finance the acquisition of the asset even if it were to turn out to be the highest productivity, v_i=1. The discontinuous drop in the marginal cost is the result of our indivisibility assumption. At this point the firms cannot afford to purchase an entire asset. If they were allowed to purchase fractional assets, then further debt would still lead to foregone purchases. Increasing debt has no additional costs but additional benefits. Therefore, if the optimal debt is at least $1 - P$, given the τ benefit of debt, it is optimal for such a firm to push its debt to the permitted maximum, here F_i=1.[10]

Together, this means that there are two potential optimal debt levels. One is in the interior region where the marginal benefit is equal to the marginal cost, and one is at the upper boundary where the marginal benefit exceeds the marginal cost but firms have hit the debt constraint. In equilibrium, we will find that firms are sometimes indifferent between these two choices. This means that firms may make different debt choices even if they are identical ex-ante. In particular, firms adopting high-debt strategies (to take advantage of the ongoing debt benefits) will be able to coexist with firms adopting low-debt strategies (to take advantage of future asset buying opportunities at fire-sale prices).

Market Clearing: The equilibrium price for liquidated assets is determined by supply and demand. Because firms may choose different debt strategies, the market clearing price has to be a function of the frequency distribution of firm debt choices. Let $\mathscr{F}(F)$ represent

[10]All our results hold in a modified model in which the benefits of debt also become available for collateral.

the cumulative distribution function of firms over admissible debt choices $F \in [0,1]$, i.e., the proportion of firms choosing $F_i \le F$ is given by $\mathscr{F}(F)$.

Supply: All firms with values $v_i \le P$ will liquidate their asset regardless of their debt choice. Therefore, the aggregate supply of the liquidated assets is

$$\int_0^1 \int_0^P 1 \ dv \, d\mathscr{F}(F) \ \ (= P) \ . \tag{2}$$

Demand: Acquiring one unit of the liquidated asset is positive NPV iff $v_i > P/\eta$. Firms will have sufficient funding to do so iff $v_i \ge P + F_i$, and they will have no demand if they have more debt than $1 - P$. Therefore, the aggregate demand for liquidated assets is

$$\int_0^{1-P} \int_{\max\{P+F, P/\eta\}}^1 1 \ dv \, d\mathscr{F}(F) \ . \tag{3}$$

B Equilibrium

Definition 1 *An equilibrium is a distribution $\mathscr{F}(F)$ over admissible debt choices $F \in [0,1]$ at time 0 and a price $P \in [0,1]$ for the liquidated asset at time 1, such that*

- *firms act optimally at time 1; and*
- *given a market clearing price P (and their optimal decisions at time 1), firms choose debt F_i to maximize firm value at time 0, according to the distribution $\mathscr{F}(F)$; and*
- *given the distribution of firm debt choices $\mathscr{F}(F)$, the price P clears the market for liquidated assets at time 1.*

C Solution

Firms are competitive so they take the price of liquidated assets, P, as given. Maximizing ex-ante firm value in equation (1) yields the optimal (interior) debt face value, F^*

$$F^*(P) = \frac{\tau + (1-\eta)\cdot P}{\eta} ,$$

and the maximized firm value of

$$V(P, F^*(P)) = \frac{1+P^2}{2} + \frac{\eta^2 + (P+\tau)^2 - 2\cdot\eta\cdot(1+\tau)\cdot P}{2\cdot\eta} .$$

The optimal debt choice is higher when the benefits of debt (τ) are greater and when future acquisition opportunities are worse—when assets are more expensive (P) and more difficult to redeploy (η). However, as we explained above, the equilibrium asset price P^* also depends on the exogenous parameters, so the parameter net effects are yet to be determined.

Together, the equilibrium asset price equates supply, as in (2), with demand, as in (3); and each firm, given the asset price and its optimal decisions at time 1, chooses debt at time 0 to maximize its value, as in (1).

We must also consider the debt choice at the upper boundary, F_i=1, and compare the firm values between the two debt choices. For a given price, the high debt strategy may appear more attractive. However, if all firms choose the maximum leverage, there is no one left to purchase the liquidated assets and the price falls to zero. This makes the interior debt choice more attractive. Mixed strategies may be the only way to balance these forces.

Theorem 1 *In the absence of financial-distress reorganization costs, there exists a unique equilibrium for all parameter values:*

- ▹ *If $\tau \le \eta^2/(3\cdot\eta+2)$, there is a pure-strategy equilibrium with price $P^* = (\eta-\tau)/(1+\eta)$, in which all firms choose $F_L^* = (2\cdot\tau+1-\eta)/(1+\eta)$.*
- ▹ *If $\eta^2/(3\cdot\eta+2) < \tau \le \eta$, there is a mixed-strategy equilibrium with price*

$$P^* = \eta - \tau + \eta\cdot\tau - \sqrt{\eta^2\cdot\tau^2 + 2\cdot\eta\cdot\tau\cdot(\eta-\tau)} ,$$

in which proportion h^* of firms choose F_H^*=1, and proportion $1-h^*$ of firms choose F_L^*, where

$$F_L^* = \frac{\tau + (1-\eta)\cdot P^*}{\eta},$$
$$h^* = \frac{1-2\cdot P^* - F_L^*(P^*)}{1-P^*-F_L^*(P^*)}.$$

▷ *If $\eta < \tau \leq 1$, there is a pure-strategy equilibrium with price P^*=0, in which all firms choose F_L^*=1.*

(All proofs are in the appendix.)

If the benefits of debt (τ) are low, all firms choose a low-debt strategy, so that they can maintain financial flexibility to acquire the asset at time 1. For intermediate values of τ, some firms choose a high-debt strategy to take advantage of the immediate benefits of debt, while other firms choose a low-debt strategy to take advantage of future investment opportunities.[11] For high values of τ, the immediate debt benefits outweigh any potential benefit from asset acquisitions, so all firms choose the high-debt strategy. In this case, with no buyers, all assets will end up being discarded rather than being transferred from low-productivity to high-productivity firms.

D Implications

[Insert Figure 2 here: **Comparative Statics for Heterogeneity in the Acquisition-Only Model** ($\phi = 0$)]

A visual perspective can help the intuition. Figure 2 plots the comparative statics for heterogeneity h^*.

Type Heterogeneity: This plot shows how heterogeneity in *ex-ante* debt strategies (h^*) arises endogenously. For high redeployability (η) and low debt benefits (τ), all firms choose to operate with very little debt (eager for the opportunity to buy assets from

[11]The mixing need not be the same for every firm. The same results obtain in a non-symmetric equilibrium in which h^* firms follow the F_H=1 with certainty, and $1-h^*$ firms never follow it; or, similarly, any combination of probabilities that lead to an aggregate h^* fraction of firms pursuing F_H=1.

lower productivity firms in the future). For low redeployability and high debt benefits, all firms choose to operate with very high debt (in order to obtain the debt benefits). For intermediate redeployability and debt benefits, ex-ante homogeneous firms naturally divide into two kinds of firms—some pursuing the high-debt operating strategy, others pursuing the lower-debt opportunistic waiting strategy.

This heterogeneity is caused by the indivisibility of the asset.[12] Once a firm has taken on so much debt that it will not be able to purchase the asset, it faces no further marginal cost to taking on more debt. If assets were divisible, our comparative statics below would continue to hold, but all firms would act alike. We can thus speculate that heterogeneity in ex-ante strategies increases in asset indivisibility—for example, in real-world situations in which purchases require assuming large pieces (like entire divisions or factories), and not just diversifiable and spreadable small bits and pieces (like retail product inventories).

Implication 1 *When assets are indivisible, ex-ante identical firms may specialize: Low-debt firms coexist with high-debt firms. The region with endogenous heterogeneity is characterized by intermediate levels of redeployability and debt benefits.*

Some of this intuition for leverage and role specialization has also appeared in Morellec and Zhdanov (2008). In their model, there are two potential and strategic acquirers and one target. One potential acquirer decides to specialize in obtaining the tax benefits (with high leverage), while the other specializes in becoming the real acquirer (with low leverage). This is because the equity of a low-debt acquirer does not need to share as much surplus with its own creditors (due to the fact that the debt becomes safer after the acquisition, because the firm becomes larger). The target itself is a third firm, whose value is determined by the competition of the two acquirers. In contrast, in our model, all firms can be acquirers and targets. The leverage of the non-acquiring firms becomes a price-setting component. Despite the obvious similarity, the models also have their differences with respect to heterogeneity. For many parameters, no heterogeneity can emerge in our model. And with many atomistic firms rather than just two, and with firms themselves becoming potential targets, our model can analyze the link between indivisibility and heterogeneity: If there are many firm types and distressed assets are divisible, then all firms would act alike and there would never be heterogeneity (see Section I.D). Moreover,

[12] Allen and Gale (1994) discuss divisibility, but their heterogeneity arises from heterogeneity in funding needs.

our model's main concern is endogenous distress, itself caused by the very same leverage. We are not aware of any models in the financial distress literature (described further in Table 3) that have featured similar endogenous heterogeneity.

[Insert Figure 3 here: **Comparative Statics for the Acquisition-Only Model Leverage (ϕ=0)**]

Figure 3 plots leverage-related comparative statics in this acquisition-channel-only model.

Leverage: The left and middle plots shows that the face value of debt and the value of debt today *decrease* in asset redeployability.[13] This is because, in equilibrium, future buying opportunities are more attractive when assets are more easily redeployed. Hence, firms choose lower debt upfront to enable more opportunistic purchasing in the future. It is this opportunistic-acquisition channel that pushes against the more common intuition that firms take on more debt when their assets are more redeployable because distress costs are lower. Naturally, this implication is robust only to the extent that it characterizes an acquisition constraint. If firms in the industry—broadly defined as firms that are suitable buyers—can purchase liquidating assets regardless of their own leverage (e.g., perhaps because they can raise infinite financing instantly), then this implication is unlikely to hold.

Leverage Ratios: Although debt is unambiguously increasing in τ, the right plots in Figure 3 show that this is not true for debt-to-value ratios. The implication of this simple point—that value is also endogenous—is more wide-reaching than just our model. Almost every capital-structure theory has been formulated in terms of debt, while almost every reduced-form empirical capital-structure test has been operationalized in terms of debt-to-value ratios. But with endogenous values, debt-to-value ratios measure primarily the relative speed of the change of debt vis-a-vis the speed of change of value. Thus, empirical test coefficients in naive leverage-ratio regressions may not be translatable into support or rejection of underlying theories.

Implication 2 *Because firm value is also endogenous, comparative statics on debt levels need not be the same as comparative statics on debt-value ratios.*

[13] With one tiny region exception, which can be seen at the bottom left figure, the discussion applies to both the debt of the low firm (F_L^*) and the debt of the industry ($h^* \cdot 1 + (1-h^*) \cdot F_L^*$). Our focus is on industry debt, so the discussion omits some trivial tiny-region caveats.

[Insert Figure 4 here: **Peer Effects on Debt Choice**]

Peer Effects on Debt Choice: An important aspect of our model is that each firm's debt choice is influenced by its peers via the endogenously determined price of liquidated assets. Recall that the optimal (interior) debt choice is

$$F_L^*(P) = \frac{\tau + (1-\eta)\cdot\eta\cdot P}{\eta} ,$$

which is increasing in the price P. The intuition is that future vulture buying opportunities are more attractive when the anticipated asset price is low, so firms have more incentives to reduce debt in order to be more likely to have the financing available to make asset acquisitions. When they conjecture that their peer firms take on more debt, the aggregate demand for the asset declines. The resulting lower equilibrium asset price gives other firms the incentives to reduce debt. This is illustrated in Figure 4, which plots the equilibrium price and debt choices as a function of the benefits of debt, τ. (In this example, $\eta=1/2$.) For high values of τ, a fraction of firms choose a high-debt strategy ($F_H=1$), resulting in higher industry debt and a lower asset price than would have obtained if all firms had chosen the low-debt strategy (represented by the dashed-lines). Consequently, firms choosing the low-debt strategy—recognizing that more valuable future buying opportunities will become available—shade their leverage below what would have been optimal if industry debt had been lower.

Implication 3 *Holding parameters constant, with endogenous liquidation values, firms' equilibrium debt choices are negatively influenced by those they conjecture for their peers.*

In real life, peers are likely to have similar parameters for ϕ, η, τ, which would lead them to choose similar capital structures. However, conditional on parameters, higher peer debt gives firms a (marginal) incentive to take less debt, because equilibrium liquidation prices turn lower. However, Leary and Roberts (2014) find evidence even of conditional peer effects, suggesting forces beyond those in our model (such as learning of unknown parameters in industries in which correlated distress is of lesser concern).

II The Distress-Reorganization Channel

The main cost of debt in standard trade-off models like Williamson (1988) and Harris and Raviv (1990) is not debt's constraint on future asset purchases, but its financial-distress cost. Firms that have taken on too much debt will suffer not because they can no longer *buy* when there are fire sales, but because they will have to *sell* when they are in trouble. We now extend our model to show how the two channels work in tandem: the opportunistic-acquisition channel means that debt reduces the demand for liquidated assets, while the financial-distress channel means that debt increases the supply of liquidated assets. Each channel plays the dominant role in some parameter region. Moreover, adding the financial-distress channel makes the model more realistic and adds a wealth of implications.

A Setup and Assumptions

The model is similar to the one from the previous section with the following changes:

Impairment: We now assume that there is a dissipative cost when reorganizing-and-continuing in the event of default ($v_i < F_i$). Reorganization here is not necessarily Chapter 11, with its large fixed-cost component, but can also be informal. It seems realistic that the reorganization costs are smaller when the firm is closer to being able to meet its debt obligations.

We specify the distressed reorganization cost to be linear in the shortfall, i.e., $\phi \cdot (F_i - v_i)$. The parameter ϕ represents the losses to a firm's value that are due to being ϕ
unable to meet pre-agreed debt.[14] The costs could be due to, e.g., direct distractions; damaged relationships with key stakeholders (suppliers, employees, and customers) when the firm is reorganized (Titman (1984));[15] or the residual effects of creditor-manager conflicts (after mitigation by negotiations and side-payments). Moreover, it is cheap to

[14]We focus on the natural case in which $\phi \in [0,1]$. If $\phi \to \infty$, then firms never reorganize. In this region, there is no heterogeneity, but changes in D and D/V are still ambiguous in redeployability η and debt benefits τ. It is still the case that the comparative statics for debt levels can differ from those of debt ratios. The only new comparative static is that Q^* may decrease in η.

[15]For example, Opler and Titman (1994) shows that distressed firms lose market share relative to their conservatively financed peers in industry downturns.

"buy" bridge funds or leniency by creditors when it concerns one dollar rather than one million dollars.

We always assume limited liability, so firm value under continuation is $\max\{0, v_i - \phi \cdot (F_i - v_i)\}$.

Liquidation: At time 1, the manager must decide not only whether to purchase liquidated assets at price P, as in the previous section, but also whether to reorganize in the event of financial distress or liquidate. Financial distress arises when the firm value is below the face value of debt. The firm value is P if it liquidates, and $\max\{0, v_i - \phi \cdot (F_i - v_i)\}$ if it continues. For lower firm values v_i, liquidation is better; for higher values, impaired operations is better. Since we assume managers maximize firm value it is straightforward to show that the firm optimally liquidates for all values v_i below a critical value Λ, Λ

(4) $$\Lambda(F_i) \equiv \frac{P + \phi \cdot F_i}{1 + \phi} .$$

A priori, firms expect to liquidate assets more often when the expected liquidation price (P) is higher and when the relative value from reorganization and continuing operations in distress is lower (i.e., when debt, F_i, is higher or when the reorganization impairment, ϕ, is worse). However, ϕ also has an influence on the equilibrium price, so its net effect is yet to be determined.

Acquisition: As in Section I, the decision whether or not to buy the liquidated asset depends on the transferability of the asset η, the price P, and the firm's capital availability. The asset value to firm i is $\eta \cdot v_i$, so it is positive NPV to acquire the asset iff $v_i > P/\eta$. However, firm i only has sufficient capital to acquire the asset iff $v_i - F_i \geq P$.

[Insert Figure 5 here: **Game Tree for the Full Model ($F_i > P$)**]

Timing: Figure 5 illustrates the revised model.

Objective: At time 0, the firm chooses its debt, again taking the expected (and fully anticipated) time 1 price P of liquidating assets as given; and anticipating its own optimal time 1 decisions (a) whether to liquidate or continue operating, and (b) whether to purchase or not purchase other firms' liquidating assets. Therefore, the ex-ante (time 0) value of each firm is

(5)

$$V(P,F_i) = \tau \cdot F_i + \begin{cases} \displaystyle\int_0^P P\,dv \;+\; \int_P^1 v\,dv + \int_{\min\{P+F_i,1\}}^1 \max\{0, \eta\cdot v - P\}\,dv & \text{if } F_i < P \\ \displaystyle\int_0^{\Lambda(F_i)} P\,dv \;+\; \int_{\Lambda(F_i)}^{F_i} [v - \phi\cdot(F_i - v)]\,dv \;+\; \int_{F_i}^1 v\,dv \;+\; \int_{\min\{P+F_i,1\}}^1 \max\{0,\eta\cdot v - P\}\,dv & \text{if } F_i \geq P \end{cases}$$

If the firm takes on less debt than what the asset will be worth, the first row applies and we are back to the case of the previous model. Each firm would know it would operate without possible impairment by distress. If the firm takes on more debt, the second row applies and there are now five terms in the (always-continuous) value objective. The first term is the τ benefit of debt, which accrues immediately.[16] The second term reflects the payoff, P, if the firm is eventually liquidated ($v_i \leq \Lambda(F_i)$), where $\Lambda(F_i)$ is given by equation (4). The third term represents the payoff to the firm if it is distressed but chooses to reorganize and continue ($v_i \in [\Lambda(F_i), F_i]$), in which case it receives v_i less the dissipative costs of reorganization $\phi \cdot (F_i - v_i)$. The fourth term is the value of the firm if it is not distressed ($v_i \in [F_i, 1]$) and continues unimpaired. The fifth term represents the expected surplus if the firm acquires liquidated assets. The limits of integration recognize that the firm only has sufficient capital to acquire the asset if $v_i \geq P + F_i$, and the integrand ($\max\{0, \eta \cdot v_i - P\}$) recognizes that the firm only acquires the assets if its NPV is positive given its own type ($v_i > P/\eta$).

Market Clearing: The equilibrium price for liquidated assets is determined by supply and demand:

Supply: As explained above, firms choosing $F_i \leq P$ will liquidate when their realized productivity $v_i \leq P$. Firms choosing $F_i > P$ will liquidate when their realized productivity $v_i \leq \Lambda(F_i)$, as described in equation (4). Therefore, the aggregate supply of liquidated assets is

$$\int_0^P \int_0^P 1\;dv\,d\mathscr{F}(F) \;+\; \int_P^1 \int_0^{\Lambda(F)} 1\;dv\,d\mathscr{F}(F)\,. \tag{6}$$

[16] In this formulation, the debt benefits cannot be used to stave off liquidation or impairment or to finance the purchase of the asset. However, as already noted above, Appendix C shows that a model in which firms can do so is isomorphic to the current one. All our principal conclusions continue to hold.

Demand: Acquiring one unit of the liquidated asset is positive NPV iff $v_i > P/\eta$. Moreover, firms will have sufficient funding to do so iff $v_i \geq P + F_i$. Therefore, the aggregate demand is

$$\int_0^{1-P} \int_{\max\{P+F,P/\eta\}}^{1} 1 \; dv \, d\mathscr{F}(F) \; . \tag{7}$$

B Access to Infinite Financing / Eliminating The Acquisition Channel

Before solving the model, it is useful to consider a benchmark in which firms in the industry have infinite access to capital. In this case, the acquisition channel is no longer a constraint. Competition among firms results in an equilibrium with $P^*=\eta$, in which (only) the highest-productivity firms ($v_i=1$) can purchase *all* assets available for sale. At this high a price, purchasing assets is zero NPV even for the highest-productivity firms and negative NPV for all other firms. Therefore, the acquisition profit terms in both rows in (16) drop out. In the first row ($F_i < P$), there is also no disadvantage to raising debt, so firms would always be better off increasing debt and leaving this region. This leaves only the second row for consideration.

Substituting $\Lambda(F_i) = (P+\phi \cdot F_i)/(1+\phi)$ into the objective and taking the derivative with respect to F_i yields the first-order condition for the (interior) optimal debt choice. The symmetric pure-strategy equilibrium debt choice is

$$F^* = P^* + (1+1/\phi)\cdot\tau = \eta + (1+1/\phi)\cdot\tau \; .$$

The optimal debt choice is increasing in the benefits of debt (τ) and asset redeployability (η), and decreasing in the costs of reorganization (ϕ). This is the standard result in earlier literature. In particular, debt increases in asset redeployability, because more redeployable assets have higher liquidation values ($P^*=\eta$), thereby reducing distress costs. There is no countervailing cost of debt with unlimited capital—increasing debt never precludes firms from acquiring valuable (high η) assets.[17]

[17] Substituting F^* into Λ yields the equilibrium liquidation threshold $\Lambda(F^*) = P + \tau = \eta + \tau$. The optimized firm value is

$$V^* \equiv V(F^*) = \frac{1+(\eta+\tau)^2+\tau^2/\phi}{2} \; .$$

C Equilibrium With Acquisitions and Financial Distress Channels

The trade-offs associated with the firm's debt choice in our general model with both the opportunistic-acquisition channel and the distress-reorganization channel depend again on the level of debt F_i vis-a-vis the predictable asset price P. For example, consider the case where $\eta \geq 1/2$. (All cases are derived in the Appendix.) Each firm takes P as given and considers its possible debt choice in one of four distinct regions:

1. In the first region, $F_i \leq P/\eta - P$, the marginal cost of debt is zero. Firm value is described by the first row of equation (16): increasing debt does not increase reorganization costs (because the firm will always liquidate in distress) and the firm does not forego asset-acquisition opportunities (because the financing constraint is not binding).

2. In the second region, $P/\eta - P < F_i \leq P$, the marginal cost of debt is $\eta \cdot F_i - P \cdot (1-\eta)$. Firm value is still described by the first row of equation (16): increasing debt still does not increase reorganization costs, but the firm now may forego some positive NPV asset-acquisition opportunities.

3. In the third region, $P < F_i < 1-P$, the marginal cost of debt is $\eta \cdot F_i - P \cdot (1-\eta) + (F_i - P) \cdot \phi/(1+\phi)$. Firm value is now described by the second row of equation (16): increasing debt raises the expected reorganization costs *and* results in the firm foregoing some positive NPV buying opportunities.

4. In the fourth region, $1-P \leq F_i \leq 1$, the marginal cost of debt is $(F_i - P) \cdot \phi/(1+\phi)$. Firm value is again described by the second row of equation (16): the firm's debt is now so high that it would *never* be able to buy assets even if it turned out to be the highest productivity type, $v_i{=}1$. Therefore, the only remaining marginal cost of debt is the increase in expected reorganization costs.

Importantly, the marginal cost of debt is weakly increasing in F_i over the first three regions, but then jumps down at $F_i{=}1-P$ (because the firm can now never afford to purchase the asset), after which it increases again. Consequently, as in our model without reorganization costs, there is again a region with a mixed equilibrium, in which some firms choose low debt and others choose high debt.

Equilibrium requires again that firms make optimal decisions at time 1 (both continuation and asset acquisition); their debt choices at time 0 maximize firm value (16), given the anticipated asset price and optimal decisions at time 1; and the market for liquidated assets clears, i.e., supply in equation (6) is equal to demand in equation (7).

The description of the equilibrium solution for all parameters is very detailed and depends on different parameter regions for the reasons just described. Therefore, for the sake of the exposition, in the following theorem we describe equilibria for a particularly relevant parameter region—when ϕ is modest and η is large, for all values of τ—and leave the full description and proof of the theorem for all parameters for the appendix.

Theorem 2 *Assume (i)* $\eta \geq 2/3$ *and (ii)* $\phi < (3\eta-2)/(6-3\eta)$ *and let*

$$\tau_1 = \frac{2(\phi+1)\eta^2 - \phi + \eta\cdot(\phi+1) - \sqrt{(\eta+1)^2\cdot(\phi+1)\cdot(\eta^2\phi+\eta^2-2\phi-\eta\phi)}}{3\eta(\phi+1)+3\phi+2},$$

$$\tau_2 = \frac{(2\eta-1)\cdot(2\eta+\phi+2\eta\phi) - \sqrt{(2\eta-1)^2\cdot(1+\phi)\cdot[4\eta^2(1+\phi)+\eta\phi-2(\eta+\phi)]}}{2+3\phi},$$

$$\begin{aligned}\tau_3 = {} & \frac{2\eta^2(1-\phi^2)+\eta(1+9\phi+\phi^2-5\phi^3)+2\phi+12\phi^2+7\phi^3}{2+14\phi+20\phi^2+9\phi^3+3\eta\cdot(1-\phi)\cdot(1+\phi)^2} \\ & + \frac{\sqrt{(1+\phi)\cdot(1+\eta+5\phi-\eta\phi)^2\cdot[\eta^2(1+\phi)(1+3\phi^2)-3\eta\phi^2(2+\phi)-2\phi^3]}}{2+14\phi+20\phi^2+9\phi^3+3\eta\cdot(1-\phi)\cdot(1+\phi)^2},\end{aligned}$$

$$\tau_4 = \frac{\eta+\phi+\eta\phi}{1+2\phi}.$$

For any set of parameter values that satisfy the above restrictions, there exists a unique equilibrium. The following is a complete characterization of the equilibrium:

▹ *If* $0 \leq \tau \leq \tau_1$, *there is a pure-strategy equilibrium with price*

$$P^* = \frac{\eta-\tau}{1+\eta},$$

in which all firms choose

$$F_L^* = \frac{1-\eta+2\tau}{1+\eta}.$$

▹ *If $\tau_1 < \tau \le \tau_2$, there is a mixed-strategy equilibrium with price*

$$\begin{aligned} P^* &= \frac{\phi\eta - (1+\phi)\cdot[\tau - \eta(1+\tau)]}{1+\phi(1+\eta)} \\ &\quad - \frac{\sqrt{\eta(\phi+1)\cdot\{2\tau\cdot[\eta\phi + (\eta-\tau)\cdot(1+\phi)] + \eta\tau^2(\phi+1) - \phi\cdot(1+\tau-\eta)^2\}}}{1+\phi(1+\eta)} , \end{aligned}$$

in which fraction h^* of firms choose F_H^*=1, and fraction $1-h^*$ choose F_L^*, where

$$\begin{aligned} F_L^* &= \frac{\tau}{\eta} + \frac{(1-\eta)}{\eta}\cdot P^* , \\ h^* &= \frac{(1+\phi)\cdot[\eta - \tau - (1+\eta)\cdot P^*]}{\eta\phi + (1+\phi)\cdot(\eta-\tau) - [1+\phi(1+\eta)]\cdot P^*} . \end{aligned}$$

▹ *If $\tau_2 < \tau \le \tau_3$, there is a mixed-strategy equilibrium with price*

$$\begin{aligned} P^* &= \frac{\phi\cdot[1+2\phi(1-\tau) - 3\tau] + \eta(1+\phi)\cdot[1+\tau+(2+\tau)\phi] - \tau}{1+(6-3\eta)\cdot(1+\phi)\cdot\phi} \\ &\quad - \frac{\sqrt{(1+\phi)\cdot(\eta+\phi+\eta\phi)\cdot\left\{\begin{array}{l} 3\eta^2\phi(1+\phi) - 2[\phi(\tau-1)+\tau]^2 \\ \quad + \eta[\phi(\tau-1)+\tau]\cdot[2+(\tau-1)\phi+\tau] \end{array}\right\}}}{1+(6-3\eta)\cdot(1+\phi)\phi} , \end{aligned}$$

in which h^* firms choose F_H^*=1, and $1-h^*$ choose F_L^*, where

$$\begin{aligned} F_L^* &= \frac{(1+\phi)\cdot\tau}{\eta+\phi+\eta\phi} + \frac{(1-\eta)\cdot(1+\phi)+\phi}{\eta+\phi+\eta\phi}\cdot P^* \\ h^* &= \frac{(1+\phi)\cdot[\eta+\phi+\eta\phi - (1+2\phi)\tau - (1+\eta+5\phi-\eta\phi)\cdot P^*]}{(1+2\phi)\cdot[\eta+\phi+\eta\phi-(1+\phi)\tau] - (1+5\phi-\eta\phi+5\phi^2-\eta\phi^2)\cdot P^*} . \end{aligned}$$

▹ *If $\tau_3 < \tau \le \tau_4$, there is a pure-strategy equilibrium with price*

$$P^* = \frac{\eta+\phi+\eta\phi - \tau\cdot(1+2\phi)}{1+\eta+5\phi-\eta\phi} ,$$

in which all firms choose

$$F_L^* = \frac{1+2\phi+\tau+(\tau-\eta)\cdot(1+\phi)}{1+\eta+5\phi-\eta\phi} .$$

▹ *If* $\tau_4 < \tau \leq 1$*, there is a pure-strategy equilibrium with price* P^*=0*, in which all firms choose*

$$F_L^* = \min\left\{ 1 \ , \ \frac{\tau(1+\phi)}{\eta(1+\phi)+\phi} \right\} .$$

D Implications

Unlike our model from the previous section, debt is now costly for two reasons: first, it reduces future purchasing opportunities; *and* second, it increases the expected costs of financial distress. The model still has only three parameters—the redeployability of assets (η), which is central to our acquisition channel; the reorganization impairment parameter (ϕ), which is central to our financial distress channel; and a compensating direct benefit of debt (τ). Yet, the model can offer many implications. Of course, it remains too stylized to consider its implications to be either quantitative or universal. Instead, our model should be viewed as suggestive of economic forces in contexts in which both the financial-distress and the opportunistic-acquisition channels are important for firms that can become either sellers or buyers of distressed assets in the future.

[Insert Table 2 here: **Summary of Comparative Statics**]

This subsection discusses the model's comparative statics. They are summarized in Table 2 and illustrated in the graphs that follow. The graphical approach is more intuitive, although the model's implications are also algebraically demonstrable using the closed-form solutions in Theorem 2.

[Insert Figure 6 here: **Comparative Statics for Heterogeneity when** ϕ=0.25]

Figure 6 shows the proportion of firms choosing maximum debt, firm values, and leverage in the case in which ϕ=0.25. This parameter means that reorganization would consume one quarter of each dollar's *shortfall*. This seems high for large firms, although it is not unreasonable for midsize and smaller firms (Bris, Welch, and Zhu (2006)).

1 Heterogeneity (h^*)

As in the model of the previous section, heterogeneity in ex-ante leverage strategies can arise endogenously because our assets are indivisible. Figure 6 shows the now parabolic convex region that separate the set of homogeneous (pure) from the set of heterogeneous (mixed) equilibria. These mixed equilibria occur again when otherwise identical firms infer that the corner solution, with maximum permitted debt of F_H=1, is as good for them as the best interior debt choice (F_L^*). Not surprisingly, mixed equilibria can only occur in regions in which firms want to choose fairly high debt to begin with.

Comparing the heterogeneity when ϕ=0 in Figure 2 with its equivalent when ϕ=0.25 in Figure 6 shows that reorganization costs ϕ shrink the heterogeneous region. For sufficiently low values of either debt benefits τ or redeployability η, there are now only homogeneous equilibria. Nevertheless, the set of mixed equilibria remains non-trivially large. In detail:

- When the debt benefits τ are low, all firms choose low debt because the benefits of a high-debt strategy are too small to compensate for the foregone investment opportunities. Similarly, when the redeployability η is low, liquidation values are low and again all firms choose low debt because a high-debt strategy results in excessive distress costs
- At some point, with high enough redeployability and debt benefits, some firms can begin to specialize in waiting for acquisition opportunities. Heterogeneous equilibria appear only for intermediate values of τ and high values of η. Thus, the heterogeneous region becomes smaller than it was in Figure 2.
- Finally, when the debt benefits become overwhelming, all firms end up choosing high debt and no firm finds it worth waiting for opportunities, even though such firms expect large distress costs.

2 Firm Value

Firm value always decreases monotonically in reorganization costs ϕ.

Firm value also usually increases in redeployability η. However, it can occur in a tiny parameter region (with high redeployability, low reorganization costs, and high benefits)—too small to be even visible in this graph—that firm value can decrease.

The effect of direct debt benefits (τ) on firm value is our *only* comparative-static implication that depends on the source of the direct benefits of debt:

- If the debt advantages are not from taxes but from incentive or information causes (and purely additive), as in the model presented here in our main text, then firm value always increases in τ.
- If the debt gains are from taxes, firm value can still increase or decrease in τ. This is somewhat surprising. As expected, for small tax-rates, taxes reduce the firm value directly (through their multiplicative $1-\tau$ factor on the value part of the objective function). The levered firm merely is less negatively effected by the required tax payments. However, for higher tax rates (about halfway up in the feasible region), equilibrium firm value also increase again in the tax rate. This is partly due to the ability of firms with very low expected values to resell the still-valuable tax credits on the market, and partly due to an equilibrium effect that is determined by the interplay of leverage and redeployment. Higher tax rates can therefore raise firm value![18]

Appendix E derives and illustrates value and leverage ratios in the two extreme cases. (The leverage ratios comparative statics do not change with the source of the debt benefits τ.)

3 Leverage

We are now ready to proceed to the focus of our paper, corporate leverage, when there are both the traditional financial-distress channel and the novel opportunistic acquisition channel. For what follows, we continue to assume that the benefits of debt (τ) accrue to shareholders. The debt F_i in our model corresponds to the face value at time 1. Because the expected return on debt is zero in our model, the market value of debt at time 0 is

$$D(F_i) \equiv \begin{cases} F_i & \text{if } F_i < P^* \\ F_i - (1+\phi)\cdot(F_i^2 - \Lambda(F_i)^2)/2 & \text{otherwise} \end{cases} .$$

[18] Note that firms with high debt can pass on the tax shield even when expected earnings are low and they are likely to go bankrupt. However, tax revenue can also improve when reallocational efficiency improves.

For a low face value of debt, there is no possibility of default. For a high face value of debt, the expected payout to creditors is equal to the promised payoff, F_i, less the expected loss to creditors.

Below, we will be considering the market value of the low type debt at time 0 in equilibrium, which is denoted $D_L^* \equiv D(F_L^*)$. We will also be considering the industry average market value of the debt at time 0 in equilibrium, which we will call $D_{Ind}^* \equiv h^* \cdot D(F_H^*) + (1-h^*) \cdot D(F_L^*)$.

[Insert Figure 7 here: **Comparative Statics for Industry Leverage when** ϕ=0.25]

Absolute Leverage: The left plot in Figure 7 shows that industry debt can first increase and then decrease in redeployability η (for low debt benefits τ). For these very low debt benefit values, the financial-distress channel dominates when redeployability is low. At first, when redeployability increases, firms take on more debt. It makes little sense for such firms to speculate on purchasing assets—the assets are simply not valuable enough. Eventually, when redeployability increases further, the potential to buy assets becomes more lucrative, the asset-acquisition channel begins to dominate, and firms again take on less debt. Finally, for higher debt benefits τ, only the asset-acquisition channel matters again. It dominates for all redeployability parameters η. Firms always find it more important to keep leverage low because of the opportunity to pounce on future opportunities.[19]

Implication 4 *For low debt benefits τ and low asset redeployability η, the financial-distress channel dominates. Firms take on more debt when assets become more redeployable. For higher debt benefits τ and higher asset redeployability η, the opportunistic-acquisition channel dominates. Firms take on less debt when assets become more redeployable.*

Leverage-Value Ratios: There are now two reasons why empirical debt-ratios (the plot on the right in Figure 7) may not increase in redeployability. The first effect is the aforementioned endogenous-value effect. Both debt and firm value increase with the direct ongoing debt benefit, and thus the leverage ratio can even decrease in τ. The second effect is the acquisition channel.

[19]The plots for the low-debt firm are identical when there is no mixing, and very similar when there is mixing. There is a tiny obscure region in which the low-debt firm may reduce its debt face value when the benefits increase.

Together, a simple linear regression explaining leverage ratios with redeployability proxies is not a powerful test. Instead, a better test would posit a U-shape—first an increasing and then a decreasing effect. When redeployability is low, a small increase in redeployability induces firms to fear distress less and they increase leverage. This is the case regardless of the source of debt gains. When redeployability is high, a small increase in redeployability induces firms to hold out for better acquisition opportunities and they decrease leverage.

A glance at the left and the right plot makes it obvious that the face value of debt and the resulting leverage ratio show completely different behavior. There are wide regions in which the face value of debt increases and the the leverage ratio decreases, and vice-versa.

Implication 5 *Debt face values and leverage ratios can have different comparative statics. One may go up when the other goes down, and vice-versa. This is because parameters effect not only the debt but also the firm value.*

4 Ancillary Implications

[Insert Figure 8 here: **Ancillary Comparative Statics for** ϕ=0.25]

Our model can also offer implications on other measures that were not its primary focus. This section provides a sampling.[20]

Credit Spreads: Creditors are indifferent between providing funding and not providing funding to the low type if the credit spread is

$$r(F_L^*) \equiv \frac{F_L^*}{D_L^*} - 1 \ . \tag{8}$$

The top left plot in Figure 8 shows that credit spreads increase when debt benefits are higher. Higher τ encourages firms to take on more debt, which increases the expected

[20]We could also offer further implications on other outcomes (such as on the average values and discounts of assets in production and transfer) that would be more difficult to measure empirically.

loss to creditors. Higher η (redeployability) leads to higher recovery rates and (all else equal) lower credit spreads, but firms may optimally choose higher debt levels which increases the likelihood of default. Although the former effect almost always dominates, resulting in lower spreads when redeployability is greater, there is a very small parameter region in which the credit spread increases when redeployability is greater. Finally (not plotted), just as in Leland (1994), credit spreads may increase or decrease in reorganization costs. Higher ϕ lead to lower recovery rates in the event of default, but also cause firms to choose lower levels of debt, which reduces the likelihood of default.[21]

Asset Liquidation Price (P): All three price-related comparative statics are unambiguous (though they can be quite flat): asset prices increase in redeployability and reorganization costs, and decrease in debt benefits. We already discussed earlier in the context of our model without reorganization costs why the asset price increases with redeployability and decreases with debt benefits. Higher reorganization costs have two competing effects on price: on one hand, they result in greater supply of the asset, because liquidation becomes relatively more desirable than continuing operations in financial distress. On the other hand, they result in greater demand for the asset, because firms take on less debt and therefore have more access to financing. Though not necessarily universal, in our specific model, the latter effect always dominates.

Asset Sales (Q): Asset sales always increase in redeployability and decrease in reorganization costs, but are ambiguous in debt benefits. The dominant effect of greater redeployability is to make the asset more valuable to a potential buyer, resulting in greater demand and higher asset sales. Higher reorganization costs make asset sales more appealing relative to the direct alternative of reorganization. Higher debt benefits increase firm debt. This results both in less demand for the asset (because of tighter financing constraints), and in greater asset supply (because of more firms in trouble). The net effect is ambiguous. Appendix Section D discusses the cyclicality of asset sales when there is uncertainty in the industry or economy.

[21] Our qualitative comparative statics results for credit spreads hold even when we allow creditors to have access to the immediate debt benefits (see Appendix C). Of course, quantitative predictions about credit spreads will depend on whether creditors have access to the immediate debt benefits—which they may in the real world. It is possible to change the model to entertain different assumptions on the disposition of these benefits.

Distressed Reorganization Observables: The model also offers secondary predictions for two quantities related to distressed reorganization:

- ▹ The liquidation frequency for the low type conditional on being in financial distress is $\Lambda(F_L^*)/F_L^*$ if $F_L^* \geq P$. If $F_L^* < P$, firms will always liquidate and never continue. The plot shows that firms liquidate more often in distress when assets are more redeployable. The dominant effect here is that more redeployable assets have higher liquidation values which makes liquidation more desirable. Higher distress costs reduce continuation values *holding debt fixed*, but higher distress costs also result in lower optimal debt which increases continuation values. The first effect dominates and the conditional liquidation frequency increases in reorganization cost ϕ. Increasing the benefits of debt τ leads to higher debt levels and declining liquidation values. This makes liquidation less attractive and therefore less frequent.
- ▹ The expected losses associated with reorganizing the firm, $E_\nu[\phi \cdot (F_L^* - \nu) \cdot \mathbb{1}_{\Lambda(F_L^*) \leq \nu \leq F_L^*}]$—possibly at least a partial transfer to and thus a partial proxy of the size for the legal reorganization industry—increase in τ, decrease in η, and are ambiguous in ϕ. The dominant effect of increasing τ is to increase debt which increases the likelihood of distress and the dissipative cost of reorganizing and continuing in distress. The dominant effect of increasing η is to increase liquidation values which makes impaired continuation less likely and reduces expected reorganization costs. The effect of ϕ is ambiguous because it increases reorganization costs *holding debt fixed* but reduces optimal debt.

III Discussion and Literature Context

A Welfare

Our paper has largely deemphasized welfare implications and government policy prescriptions, because we view the model as too stylized to offer policy prescriptions. Our model assumes production, reallocation, incentive and tax[22] effects as parameters; and we are simply not confident enough to take a stance to what extent these aspects are dissipative or redistributive to some parties elsewhere in the economy.

[Insert Figure 9 here: **Allocational Efficiency**]

We are however comfortable to discuss briefly one part of the overall social welfare within the context of our model. This can help to clarify one conceptual aspect of the trade-offs that government should be aware of. How do corporate income taxes—one component of τ—influence re-allocational efficiency?

Figure 9 shows that the answer is ambiguous:

Implication 6 *Increases in the benefits of debt—as can be effectuated by tax code changes—can result in socially less or more efficient redeployment activity.*

[22] To the extent that some of the firm's debt benefits come through the tax shelter (though there are others!), there is a related conceptual puzzle. If, as is widely acknowledged, debt has a potentially negative effect on the stability of firms individually and system-wide, why would the government want to impose them differentially on equity and not on debt? A government could impose taxes on projects instead—for example, in Germany, home ownership is subsidized not through interest deductibility, but through non-recaptured depreciation. We can speculate that default forces more reallocation of resources from less productive to more productive uses; and by increasing the value of debt, the government can calibrate both the equilibrium reallocation frequency and reallocation state dependence. However, debt is a fairly blunt instrument, used by governments that are themselves not great experts about when reallocation is better or worse. The mutual industry-peer externalities discussed in our paper further suggest that it could be a dangerous instrument—if it forces only a few firms to sell, prices are reasonably appropriate, but at some point, feedback effects can reallocate assets less towards the highest-value user of assets and more towards the least-levered users of assets.

This is because there is typically an intermediate level of debt, in which asset transfer activity is socially ideal.[23] Tax policy can then push firms toward or away from this ideal. This is easiest to understand in the context of the total direct debt benefits:

- ▷ For low τ, firms choose low leverage, resulting in high demand for liquidated assets. If reorganization costs are high—which makes liquidation more likely in financial distress—this can also result in high supply of assets, and the economy can have too many asset transfers relative to the efficient level. Increasing the tax advantage of debt then pushes firms towards more debt, which helps because it will reduce the expected transfer activity.
- ▷ For high τ, firms choose high leverage, resulting in low demand for liquidated assets. The economy has too few asset transfers relative to the efficient level. Increasing the tax advantage of debt further would only push firms towards even more debt and thereby worsen the reallocation.[24]

A reasonable interpretation is that government tax policy towards debt should moderate other debt benefits.

For comparison, in Gale and Gottardi (2015), in which asset sale prices are also endogenous, the thought experiment about the social cost of debt as a tax shelter is different. In their model, in the absence of a corporate debt response (to undo taxes), such taxes would *always* reduce socially beneficial productive operations. Debt, by undoing taxes, tends to increase productive activity and can thereby improve social welfare. Taking the leverage responses of firms into account, the net effect of an increase in taxes on production and thus welfare could be positive or negative. Interestingly, Gale-Gottardi consider a novel policy mechanism—forcing firms to take on more debt. This can in turn induce firms to increase investment voluntarily.

[23]Assets are identical and it is always the lowest-use owners who transfer assets to the highest-use owners. Thus, the total quantity transferred is the only metric of relevance.

[24]Similar to this point, when redeployability is low and reorganization costs are high, firms would choose high leverage. This is because maintaining financial flexibility is less valuable when it is unlikely that there will be good buying opportunities later. Again, transfer activity would be too low from a social perspective, and increasing the tax advantage of debt would only hurt more. (The opposite is the case when redeployability is high and reorganization costs are low.)

B Generalizations of the Model

The most important takeaways of our model are that

1. firms' leverage choices are affected by their peers through the equilibrium price of liquidated assets;
2. indivisibility of assets may result in heterogeneity in leverage strategies;
3. leverage level effects are not isomorphic to leverage-ratio effects;
4. the acquisition channel means that increased asset redeployability can also have a negative effect on leverage, especially when debt benefits and redeployability are high to begin with;
5. and tax policy and non-tax related debt benefits can have ambiguous effects on re-allocational efficiency, firm value, and tax receipts. For example, for some large tax rates, a further increase in tax rates can increase both firm value and tax revenues.

To illustrate them, our model had to employ a set of assumptions for tractability, such as the uniform distribution on values; linearity in η, ϕ, and τ; stark integration limits; limited liability and free disposal; limited capital; uncorrelated shocks; no further countervailing important omitted effects (e.g., due to agency or inside information), and so on. None of our takeaways lean especially heavy on specificity in these assumptions, and we would expect the key insights to survive in models in which they are reasonably relaxed. In particular:

- ▷ Outside Buyers: Our model is sensitive to the assumption that buying is limited to firms inside the industry. Our qualitative results would continue to hold if there is limited demand from outside the industry—this would increase liquidation values and mitigate, but not eliminate, the incentive to choose lower debt to take advantage of buying opportunities. It would also have a similar effect as an increase in redeployability, η. But if assets are just as valuable outside the industry and potential buyers have practically unlimited capital, then our acquisition channel vanishes, as discussed in Section II.B. More commonly, neither zero nor infinite capital availability inside the industry, and neither perfect nor useless redeployability outside the industry is likely to be a realistic description; and these unmodeled forces can help push eta towards lower or higher levels.

- ▹ Correlated shocks: It could be that all assets in an industry are simultaneously affected by a recession, or that (e.g., consumer taste) shocks help some firms at the same time they hurt others. For example, if shocks are positively correlated, fire sales will be deeper in bad times (more sellers and fewer buyers) and shallower in good times (fewer sellers and more buyers). This may create an incentive to take on less debt initially to take advantage of the great investment opportunities available in bad times, above and beyond the incentive to avoid financial distress oneself. Appendix Section D sketches an extension of our model to industry uncertainty. It shows that there are parameter regions where reallocation of assets is procyclical and regions where it is counter-cyclical.
- ▹ Agency Conflicts: When managers (and equity) have stronger incentives not to declare bankruptcy and even weaker incentives to liquidate (and if creditors cannot renegotiate managers out of collectively inefficient choices, as in Benmelech and Bergman (2008)), then firms would likely be less inclined to liquidate at the same time, given the same amount of debt. However, this would not necessarily be the outcome. In turn, this could have equilibrium repercussions for the optimal level of debt and/or various restrictions written into debt that can enhance the incentives of firms to liquidate. The outcome would likely depend on how extra debt calibrates the relative incentives.

Our paper has endogenous heterogeneity. More realistically, there would be both exogenous heterogeneity and endogenous heterogeneity. We have not modelled differences in behavior across types, however. Firms with higher ex-ante quality could have both more debt capacity and expect to be buyers. It is not clear whether this would lead them to behave differently from lower quality types.

C Related Literature

Our model was built around the fundamental tradeoff between taxes and financial-distress costs, first raised in Robichek and Myers (1966). As this encompasses most of the modern theory of corporate capital structure, we can only highlight some work especially close to the assumptions and results of our own paper. Harris and Raviv (1990), Leland (1994), Leland and Toft (1996), Gryglewicz (2011), and many others, have provided the theoretical formalizations to help understand firm tradeoffs and behavior. Industry debt choices have been proposed by Maksimovic and Zechner (1991), Fries, Miller, and Perraudin (1997), and others.[25]

The costs of financial distress were further dissected into components, such as debt overhang (Myers (1977)), the damaged relationships with key stakeholders (Titman (1984)), or reduced market share (Opler and Titman (1994)).

Allen and Gale (1994) and Acharya and Viswanathan (2011) develop models of asset sales in which potential buyers face entry costs or are financially constrained so that equilibrium prices depend on funding availability of industry peers. Unlike our model, these models have specific fixed funding needs, with an endogenous determination of whether they can raise them. (In this sense, they do not choose an optimal capital structure.) Furthermore, assets are divisible in these models; however, we show that when assets are indivisible there may be mixed equilibria in which some firms adopt high-debt strategies to take advantage of tax benefits and others adopt low-debt strategies to take advantage of asset fire sale opportunities.

Gale and Gottardi (2015) offer a theory in which debt is an optimal choice and fire-sale prices are also endogenous.[26] In their model, frictions and especially taxes lead firms to take on too few projects from a social point of view. Debt can reduce the tax burden and thereby enhance the desire of firms to take projects. An endogenous reduction in price upon resale[27] comes into play, because when many firms have taken on too much debt, the induced price reduction then works against this social advantage of debt. As remedy, they propose forcing firms to take on more debt. This induces them to undertake more projects, which in their model is socially valuable. As noted, our model has a different structure, parameters, and focus. It considers social welfare only in passing, because our own model assumes production, reallocation, and tax costs as parameters, and we are less confident about the dissipative/redistributive cost-benefit issues for them.

[25]Bolton, Santos, and Scheinkman (2011) motivate preferred purchasers through adverse selection. Asset specificity plays a role in Marquez and Yavuz (2013), though assets have exogenous prices. More specific assets can increase productivity (reducing debt) and increase continuation values (increasing debt).

[26]In Gale and Gottardi (2011), leverage is not a choice that firms consider. (Projects are 100% leverage by assumption.)

[27]Assets are as productive to buyers as they were to sellers. Sales are costly to the firm, but not to the economy.

A number of empirical papers have provided evidence about the existence and nature of these fire sales. Asquith, Gertner, and Scharfstein (1994) showed that financially-distressed firms often liquidate assets at discounts to fundamental value. Pulvino (1998) showed that there are periods in which many airlines were hit by negative shocks at the same time, how this depressed airplane prices, and how financially unconstrained airlines then increased their buying activity, while constrained airlines did not. Acharya, Bharath, and Srinivasan (2007) investigated this effect more generally. Taking this yet a step further, Benmelech, Garmaise, and Moskowitz (2005) showed that firms take on more debt when assets are easier to redeploy. Rajan and Ramcharan (2016) show how financial intermediation failures have reduced land sale values through fire sales.

They interpreted their findings as support for an optimal capital structure theory, in which assets that were more redeployable allowed industries to take on more debt.

D Relation to Shleifer-Vishny 1992

Like Jensen and Meckling (1976) did for agency issues, the seminal Shleifer and Vishny (1992) paper laid out a research agenda for corporate behavior when asset prices could be depressed by the need of many firms to sell simultaneously and the resulting fire-sale liquidations.[28] However, Shleifer-Vishny do not present an economic model in the traditional sense. Their paper model lays out a set of inequalities that an equilibrium should satisfy in which two firms' debt choices could influence one another. It offers neither a specific solution to these conditions nor any comparative statics. This model sketch is then followed by an insightful discussion of economic possibilities. However, much of this discussion does *not* follow from their model. It is therefore not clear from their paper whether it is possible to construct a model of firms that exhibit the kinds of behavior they conjecture. Our own paper has provided such a model.

Their Section II describes a three-period model with two industry firms plus one outsider. Both firms choose the minimum debt levels that eliminate overinvesting in prosperous times. The model describes a set of constraints[29] under which each firm is either

[28]Duffie (2010) went even further, attributing temporarily depressed prices not just to firms and industry assets, but even to financial claims in wide distribution.

[29]The model has 12 exogenous firm parameters (4 cash flow parameters and 2 investment flow parameters, per firm); 1 internal and 1 external asset value parameter; 1 probability that governs the state of the

a potential purchaser for the other firm or a bystander, suffering from its cash constrained inability to buy the selling firm's assets. Other model parameters can push this within-industry value above or below the external asset value. The price of the liquidating asset is the lower of the two. Both firms can calculate the other-firm price in advance, which they can take into consideration when they choose whether to have debt (and avoid the agency costs) or not to have debt (and avoid costly liquidation).

With only one outsider and one insider competing, the opportunistic buying decision is limited to whether each firm wants to have low enough debt to beat the external price (be the acquiror) if the other firm runs into difficulties, or not. Any buying firm always pays the outside value. The other firm's high or low debt choice thus has a (binary) influence on the value of the firm in distress. If there is any competition in the external market, the price of the liquidating firm would become completely independent of industry debt choices (i.e., the other firm). Instead, the price would always be determined by the external market value alone. To resurrect feedback from the industry price to debt choices back into the model would require the introduction of multiple insiders, who would compete against one another when setting the price. The dynamics of such a model would be more complex and are not at all obvious.

The S-V model does not derive comparative statics from model solutions. In contrast, our model derives specific optimal debt levels, equilibrium prices, and values for firms in industries that are subject to distress and tax costs. It thus offers comparative statics with respect to taxes and distress costs, and especially with respect to redeployability. This yielded our key result about where *debt can go up or down with redeployability*, in contrast to Williamson (1988) and Harris and Raviv (1990). The "important general principle: optimal leverage or debt capacity falls as liquidation value falls" (p.1354) in Shleifer-Vishny holds only for the seller's leverage when a threshold is crossed and he chooses discontinuously to take on no debt. Our model characterized how leverage choices may change continuously in reaction to liquidation prices due to both buying and selling concerns.

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economy; 4 endogenous debt parameters (short-term and long-term, per firm), resulting in the *key* resulting maximum value that a buyer can pay, given own debt overhang; and 14 equality and inequality constraints (guaranteeing such conditions as debt overhang being optimal, firms needing to raise capital, control of agency in good times being more important than liquidation in bad times, etc.). 5 conditions are redundant. There are also 7 other explicit assumptions and some implicit assumptions (like $d \geq 0$). It is not difficult to find 15 parameters that satisfy their 14 conditions.

In the region of the S-V model where two equilibria exist, either the buyer chooses a low (zero) debt level and the seller a high one, or vice versa. The heterogeneity is exogenous—the model cannot accommodate identical firms. In this region "it would not be an equilibrium either for both firms to have a lot of debt or for neither to have debt. This case of two equilibria suggests the notion of an industry debt capacity..." (p.1354), similar to Miller (1977). Of course, firms are not exactly homogeneous ex-ante, as characterized by our model. However, our model can explain (a) how the notion of an industry debt level matters even when all firms choose the same debt and (b) when otherwise similar firms may or may not end up choosing different debt strategies. The latter choice depends crucially on the indivisibility of the asset. With divisible assets, all identical firms would choose the same debt.

E Summary

[Insert Table 3 here: **Model Implication and Features Comparison**]

Table 3 highlights the key difference between our model and the most closely related papers in the literature. Primarily,

1. Our model has offered a negative comparative static with respect to redeployability.
2. Our model has generated endogenous heterogeneity among firms that are ex-ante homogeneous, with an important link to asset indivisibility.

Secondarily,

3. Our model has offered comparative statics on leverage (D/V), not just on debt levels.
4. It is among very few models in which prices are endogenous.

IV Conclusion

Our paper has sketched a model in which firms could anticipate and participate in industry asset sales, with more levered firms as sellers and less levered firms as buyers. This turns prices into mediators of industry leverage interactions, and ambiguates the role of asset redeployability. When redeployability is low, an increase therein induces firms to take on more debt in order to take advantage of higher fire sales prices as potential sellers—as in the earlier literature. However, when redeployability is high, an increase therein induces firms to take on less debt in order to take advantage of fire sales as potential buyers.

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V Tables and Figures

Table 1: Variables

v_i	$v_i \sim U[0,1]$	Unlevered firm/asset type
Exogenous Parameters		
ϕ	$0 \leq \phi \leq 1$	Reorganization impairment $\phi \cdot (F_i - v_i)$ for firms continuing in default.
η	$0 \leq \eta \leq 1$	Asset redeployability
τ	$0 \leq \tau \leq 1$	Other (net) benefits of debt
Endogenous Quantities		
F_i	$0 \leq F_i \leq 1$	Face Value of Debt for firm i, promised **for time 1**.
D_i	$0 \leq D_i \leq F_i$	Value of Debt **at time 0**, as in (8)
h	$0 \leq h \leq 1$	Proportion of F_H^*=1 types
$\Lambda(F_i, P)$	$0 \leq \Lambda \leq 1$	Liquidation/continuation threshold, $(P + \phi \cdot F_i)/(1+\phi)$
$V(F_i, P)$	$V \geq 0$	Firm value **at time 0**
P	$P \geq 0$	Price of liquidated assets **at time 1**
Q	$Q \geq 0$	Assets transferred **at time 1**
r	$r \geq 0$	Credit Spread, $F_i^*/D_i^* - 1$, as in (8)

Table 2: Summary of Comparative Statics

Panel A: Key Comparative Statics on Value and Leverage

		Redeploy-ability η	Reorganization Cost ϕ	Direct Debt Benefits τ
Optimized Firm Value	V^*	0.9,0.2,0.9† 0.9,0.0,0.1	$\downarrow$	*
Debt Face Value, Industry	F^*_{Ind}	0.6,0.0,0.1 0.1,0.2,0.1	$\downarrow$	$\uparrow$
Low-Debt Firm	F^*_L		0.1,0.2,0.1 0.6,0.0,0.1†	0.02,0.02,0.02†† 0.1,0.2,0.1
Debt , Industry	D^*_{Ind}	0.8,0.0,0.1 0.1,0.9,0.1	$\downarrow$	0.9,0.9,0.9 0.1,0.3,0.1
Low-Debt Firm	D^*_L		0.1,0.2,0.1 0.6,0.0,0.1	
Debt / Value, Industry	D^*_{Ind}/V^*	0.8,0.0,0.1 0.1,0.9,0.1	0.1,0.2,0.1 0.9,0.5,0.5	0.1,0.1,0.1 0.1,0.4,0.1
Low-Debt Firm	D^*_L/V^*			

Panel B: Ancillary Comparative Statics

Low Type Credit Spread	$r(F^*_L)$	0.4,0.1,0.4 0.1,0.2,0.1	0.1,0.2,0.1 0.08,0.0,0.02†	0.02,0.02,0.02†† 0.1,0.2,0.1
Asset Price	P^*	$\uparrow$	$\uparrow$	$\downarrow$
Asset Price/Max Value (NPV 0)	P^*/η	0.1,0.2,0.1 0.4,0.3,0.5	$\uparrow$	$\downarrow$
Asset Sales #	Q^*	$\uparrow$	$\uparrow$	0.6,0.0,0.1 0.1,0.2,0.1
Low Type Liquidation Freq.	$\Lambda(F^*_L)/F^*_L$	$\uparrow$	$\uparrow$	$\downarrow$
Low Type Reorganization Cost	$E_v[\phi \cdot (F^*_L - v) \cdot \mathbb{1}_{\Lambda(F^*_L)\le v \le F^*_L}]$	$\downarrow$	0.1,0.2,0.1 0.9,0.0,0.8	0.02,0.02,0.02†† 0.1,0.2,0.1

† Small region. Less than 2% of the parameter space.
†† Minuscule region. Less than 0.001% of the parameter space. Considered effectively unambiguous in the text.

Explanation: Ambiguous comparative statics are illustrated with two examples (order η, ϕ, τ), in which one derivative is negative (red) and another is positive (blue). $\partial V^*/\partial \tau$ is indicated by a "*", because it depends on the source of the debt benefits. It is positive if the source of debt benefits is direct. It is ambiguous if the source is the tax shield. Though negative in a wide parameter region, it can be positive, too.

Table 3: Model Implication and Features Comparison

	Comparative Statics of Industry Indebtedness Measure			Model Features	
	$\frac{\partial \text{Leverage } D/V}{\partial \text{Debt Benefits}}$	$\frac{\partial \text{Level } D}{\partial \text{Debt Benefits}}$	$\frac{\partial \text{Indebtedness}}{\partial \text{Redeployability}}$	Endogenous Asset Price	Hetero-geneity
Williamson 1988	D/V not derived	Positive [a]	Positive [b]	No	No
Harris-Raviv 1990	D/V derived, but benefits unexplored [c]	Benefits unexplored	Positive [d]	No	No
Shleifer-Vishny 1992	D/V not derived	Negative within parameter region. Positive across. [e]	Positive [f]	Mostly [g]	Exogenous [h]
Acharya-Vish-wanathan 2011	D/V not derived	Negative for existing firms. Positive for new firms. [i]	Redeployability online only. No comparative statics.	Yes	Exogenous
Our Model	Positive when debt benefits τ are small. Negative when large. [j]	Deemphasized due to empirical near-un-identifiability. Ambiguous.	Negative when acquisition channel dominates. Positive when liquidation channel dominates.	Yes	Endogenous when assets are indivisible

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(a) When debt is simpler to implement, more firms choose debt over equity (cf. pg. 579-581).

(b) Equity complexity is necessary for specific hard-to-transfer assets. Firms prefer debt when assets are easy to liquidate (cf. pg. 579-581).

(c) The benefits of debt are that payment/non-payment and audits in default provide signals of asset quality (cf. pg. 329).

(d) Only one firm. Redeployability is a discount in liquidation relative to fundamental value (cf. pg. 340).

(e) Debt helps avoid negative NPV investment and is firm specific. (Own) debt decreases within and increases between equilibrium regions (cf. pg. 1354).

(f) Discussed only informally relative to Williamson 1988 (cf. pg. 1359).

(g) Endogenous decision to sell to insider/outsider. Only when sold to the outsider is the price dependent on debt levels. Otherwise exogenous. (maybe cf. pg. 1353).

(h) The model cannot accommodate identical firms.

(i) They investigate responses to increases in the good asset's expected quality (not relative to bad investment) and show that it decreases debt levels on the intensive margin but increases on the extensive margin.(cf. pg. 120, para. 4).

(j) Measured as net (parameter τ). The value continues to increase even though firms begin to max out leverage.

Figure 1: Game Tree for the Acquisition Model

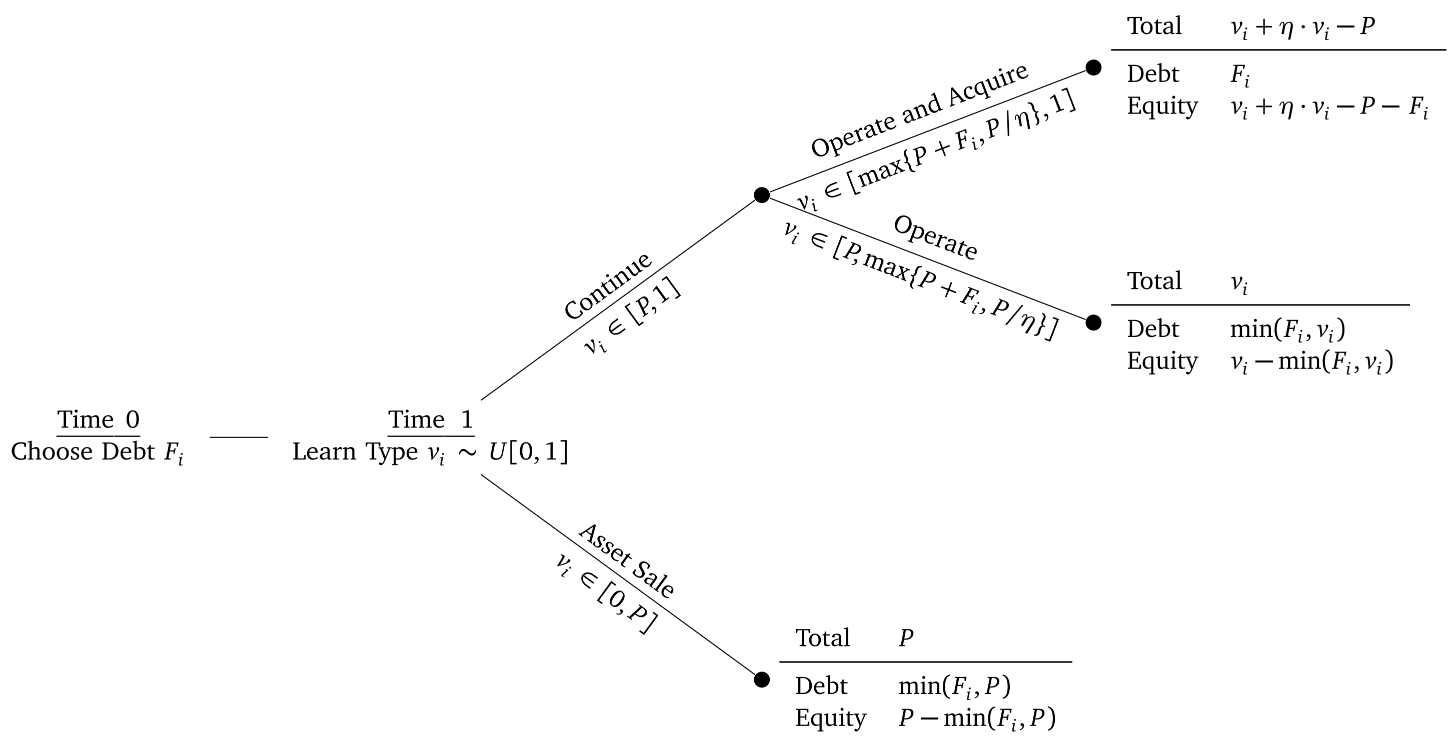
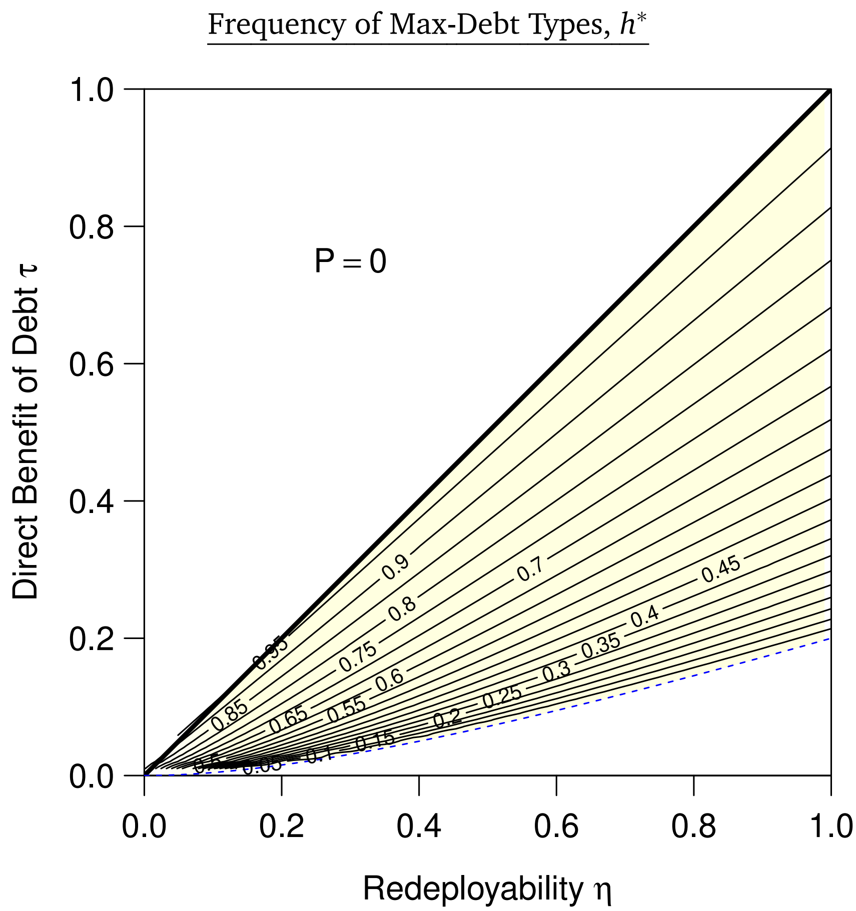


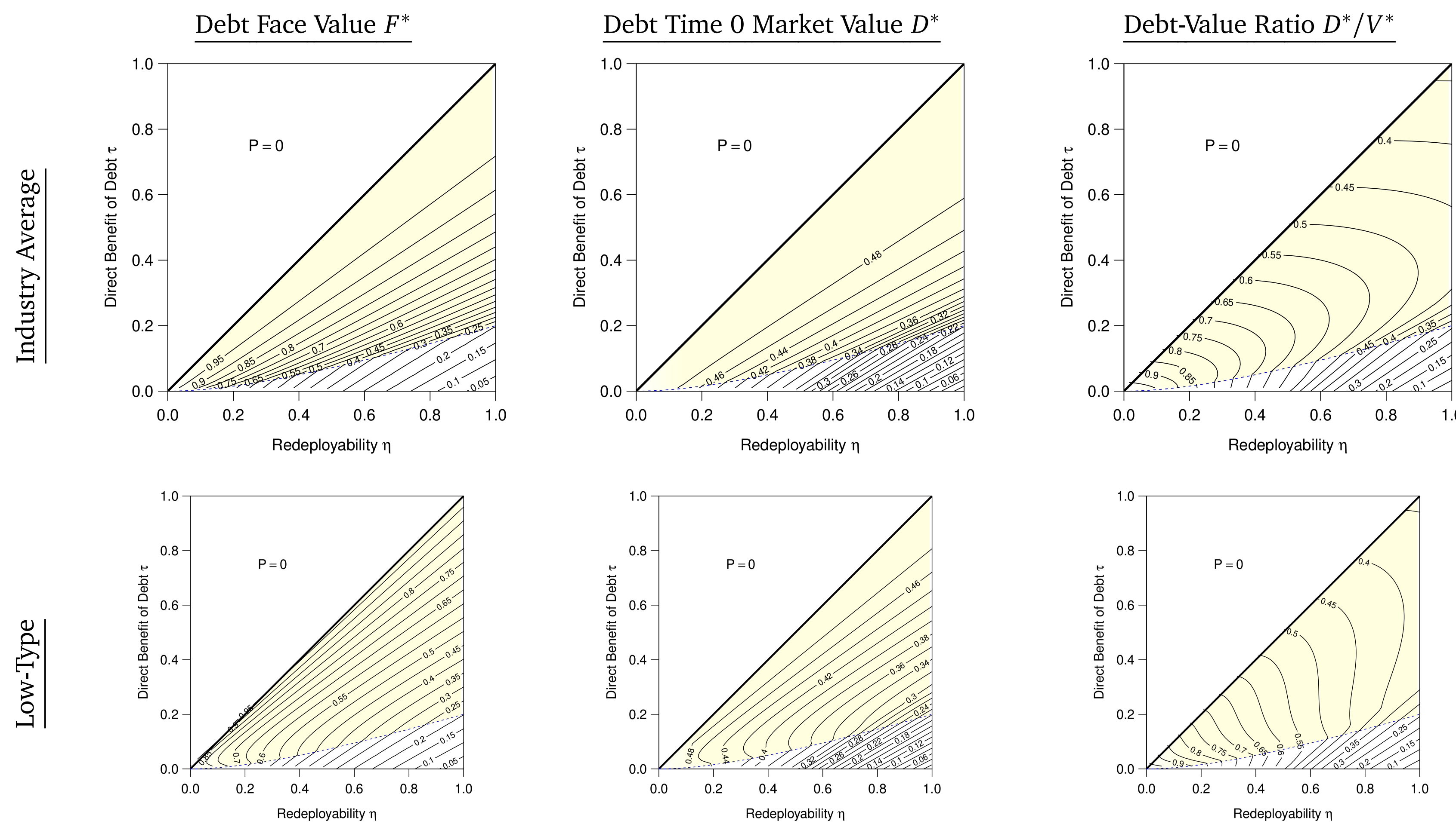
Figure 2: Comparative Statics for Heterogeneity in the Acquisition-Only Model ($\phi = 0$)



Explanation: This is a contourplot with the fraction of heterogeneous firms as the dependent variable. The yellow area contains the two-type equilibria (as defined in Theorem 1 on Page 15). The area above the diagonal is uninteresting, as all firms choose $F_i = 1$ and the price is 0. The area on the bottom right has all firms act alike.

Interpretation: Heterogeneity arises unless redeployability is high and debt benefits are low. It is common for intermediate values of redeployabilities and direct debt benefits.

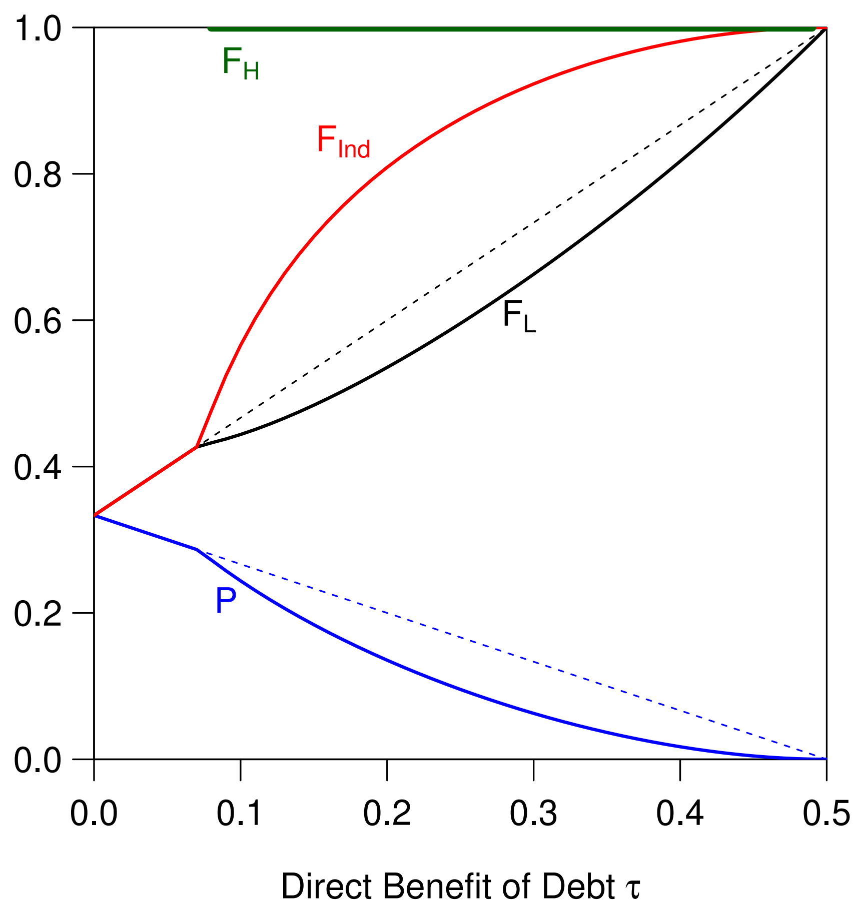
Figure 3: Comparative Statics for the Acquisition-Only Model Leverage (ϕ=0)



Explanation: These are contourplots. The yellow area contains the two-type equilibria (as defined in Theorem 1 on Page 15). Patterns that have "∩" or "∪" shapes indicate ambiguous comparative statics in redeployability. "⊂" or "⊃" shapes indicate ambiguous comparative statics in direct debt benefits.

Interpretation: Left and Middle: The face value F^* and current market value of debt (D^*) increase [everywhere for the industry, almost everywhere for the low-debt firm] monotonically in debt benefits τ and decrease monotonically in redeployability η. Right: The debt-to-value ratio is ambiguous in debt benefits τ, and decreasing monotonically in redeployability η.

Figure 4: Peer Effects on Debt Choice



Explanation: In this figure, η=1/2.

Interpretation: For high τ, some firms choose a high-debt strategy (F_H=1). Therefore, industry debt (F_{Ind}) is higher and the equilibrium price (P) is lower than what would have occurred if all firms had chosen a low-debt strategy (represented by the dashed lines). Other firms recognize that more valuable buying opportunities will become available and have an incentive to choose debt (F_L) below what is optimal if industry debt was lower.

Figure 5: Game Tree for the Full Model ($F_i > P$)

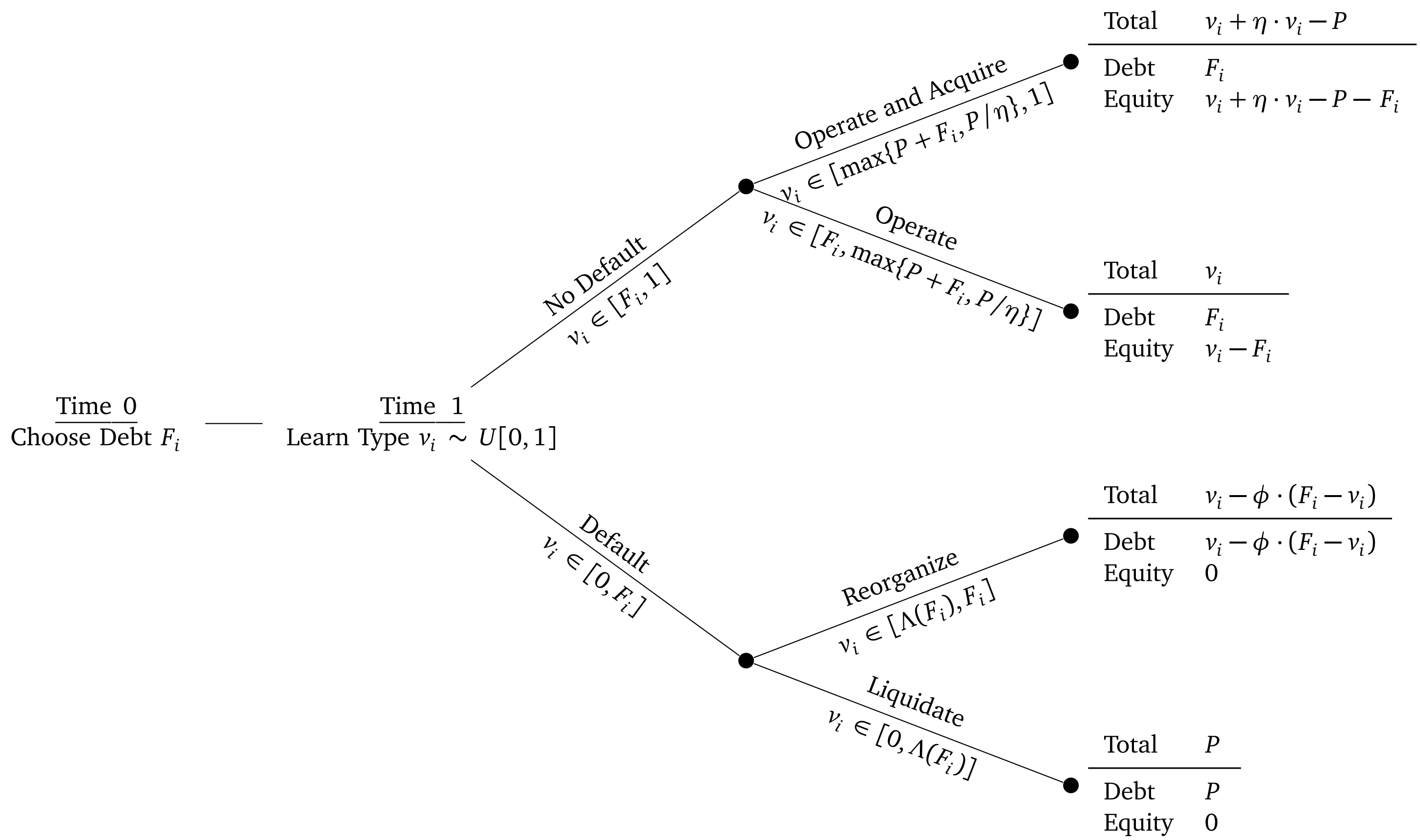
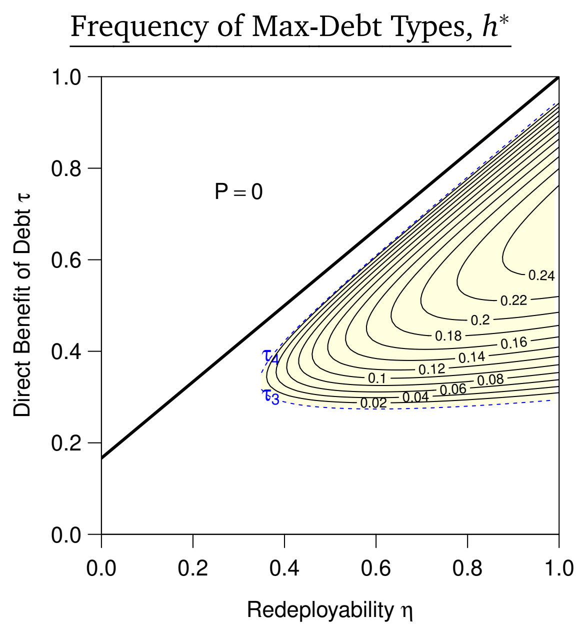


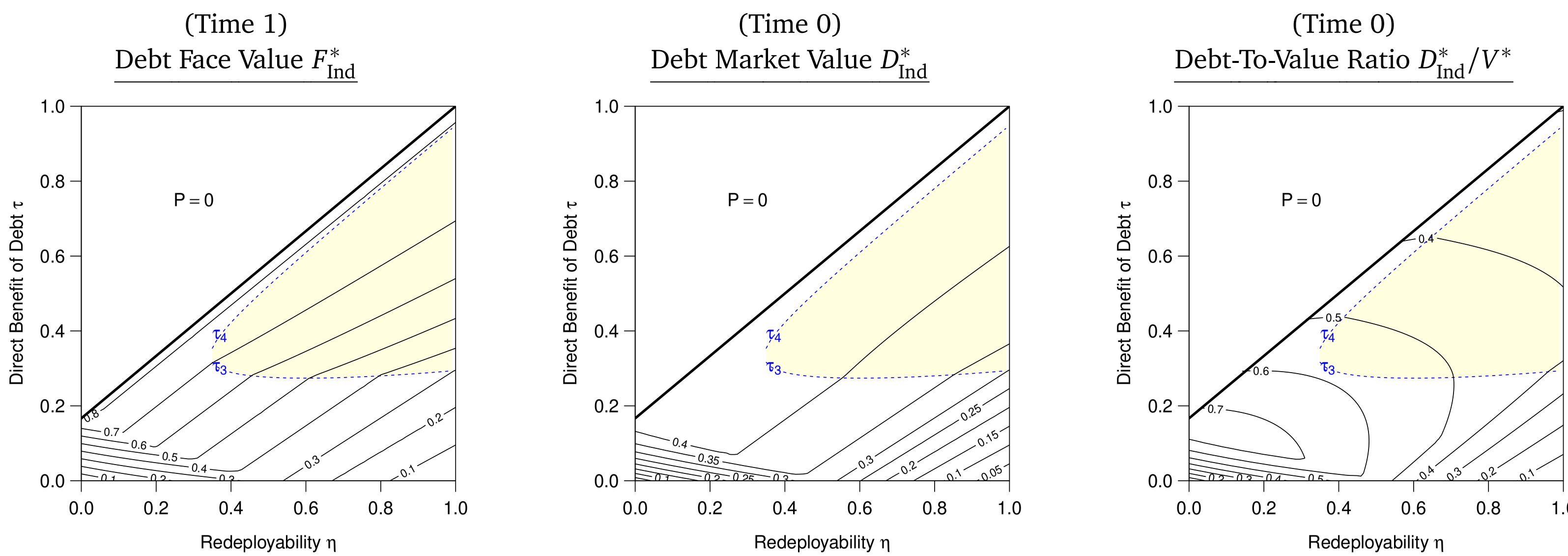
Figure 6: Comparative Statics for Heterogeneity when ϕ=0.25



Explanation: This is a contourplot. The yellow area contains the two-type equilibria (as defined in Appendix Section B on Page 73). Patterns that have "∩" or "∪" shapes indicate ambiguous comparative statics in redeployability. "⊂" or "⊃" shapes indicate ambiguous comparative statics in direct debt benefits.

Interpretation: Heterogeneity still arises for intermediate values of debt benefits and large values of redeployability. However, the heterogeneity region is now smaller than it was when ϕ=0 in Figure 2.

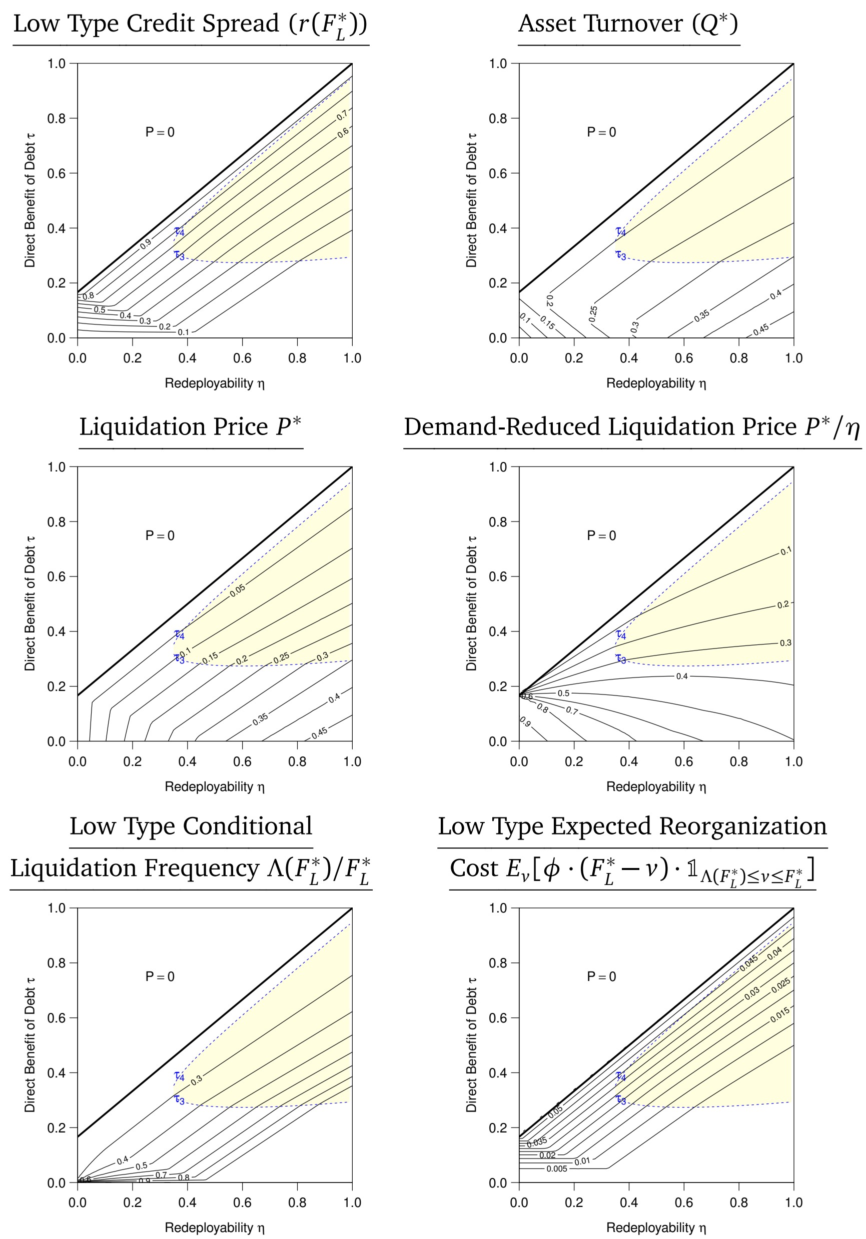
Figure 7: Comparative Statics for Industry Leverage when ϕ=0.25



Explanation: These are contourplots. The yellow area contains the two-type equilibria (as defined in Appendix Section B on Page 73). Patterns that have "∩" or "∪" shapes indicate ambiguous comparative statics in redeployability. "⊂" or "⊃" shapes indicate ambiguous comparative statics in direct debt benefits.

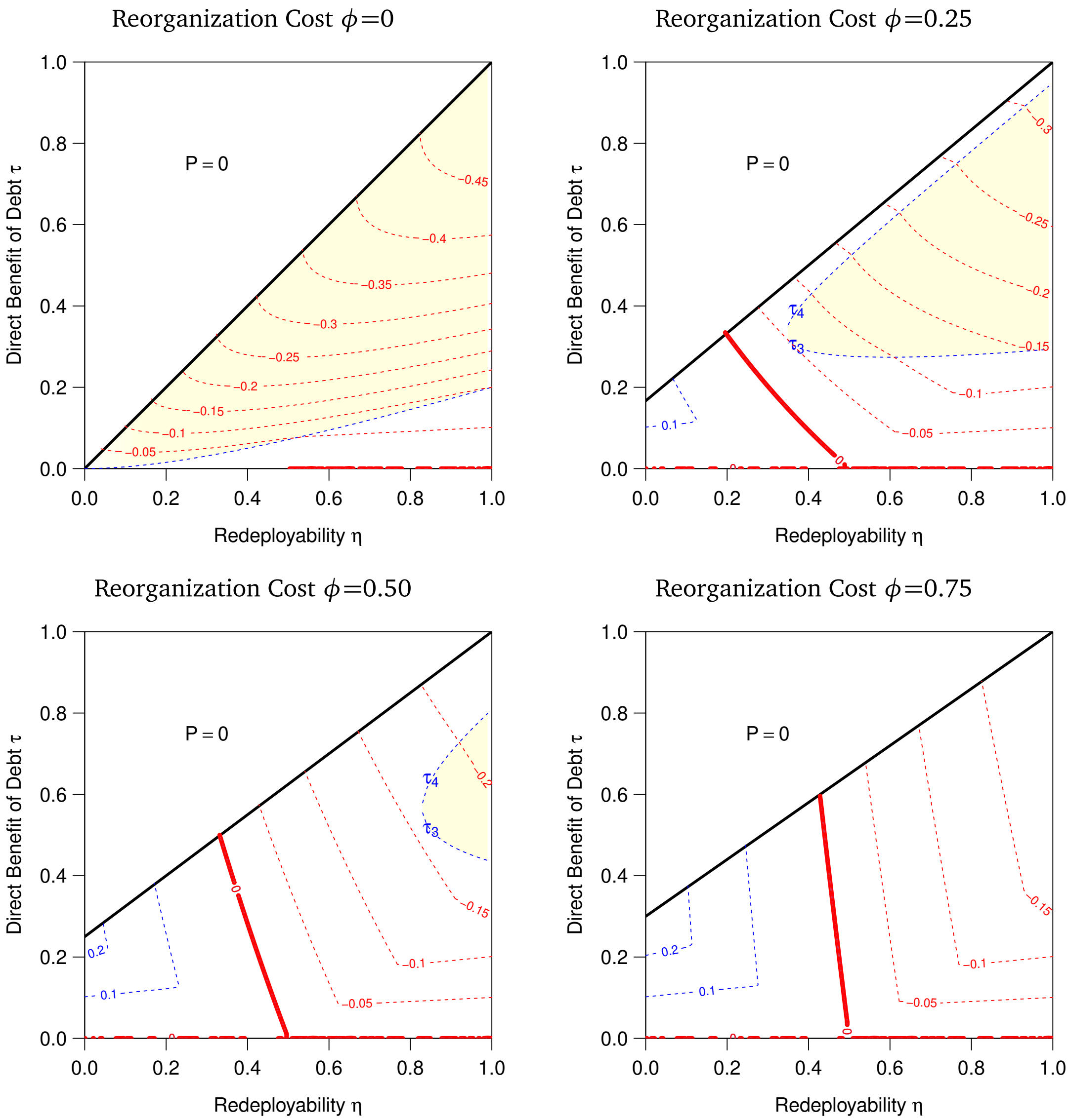
Interpretation: Note somewhere $F^*_{\text{Ind}} = h^* \cdot 1 + (1 - h^*) \cdot F^*_L$. —— Industry debt increases monotonically with direct debt benefits τ. However, it can first increases and then decrease in redeployability η. Not shown, the low-debt firm (rather than industry debt) shows a very similar pattern, with only small changes in the yellow region.

Figure 8: Ancillary Comparative Statics for ϕ=0.25



Explanation: These are contourplots. The yellow area contains the two-type equilibria (as defined in Appendix Section B on Page 73). Patterns that have "∩" or "∪" shapes indicate ambiguous comparative statics in redeployability. "⊂" or "⊃" shapes indicate ambiguous comparative statics in direct debt benefits.

Figure 9: Allocational Efficiency



Explanation: These are contourplots. The yellow area contains the two-type equilibria, delineated by τ_3 and τ_4 (as in the appendix). The fat line shows the parameters for τ and η where equilibrium results in first-best redeployment.

Interpretation: The area to the left of the fat line has too much transfer activity (Q^*). The area to the right of the fat line has too little transfer activity. If there are no reorganization costs (ϕ=0), there is always too little redeployment.

A Proof of Theorem 1

Each firm is competitive and takes the price, P, as given. The marginal benefit of debt is τ for all values of debt, F. The marginal cost of debt falls into three regions:

1. For $F \in [0, P(1/\eta - 1)]$ the marginal cost is zero

2. For $F \in [P(1/\eta - 1), 1 - P]$ the marginal cost is $\eta(F + P) - P$

3. For $F \in [1 - P, 1]$ the marginal cost is zero

In Region 1, increasing debt is not costly to the firm because the marginal project is negative NPV. In Region 2, increasing debt is costly because the firm must forego positive NPV projects. In Region 3, the debt level is so high that the firm cannot finance the acquisition of the asset even if it is the highest productivity type, $V_i = 1$. Therefore, increasing debt further results in no additional costs to the firm.

Since the marginal benefit of debt is positive (equal to τ) it follows that it is never optimal for the firm to choose a debt level in Region 1, i.e., $F \leq P/\eta - P$. Furthermore, since the marginal cost of debt jumps down to zero at $F = 1 - P$ there may be a mixed equilibrium in which some firms choose debt in Region 2 while others choose debt in Region 3. Clearly, in such an equilibrium, firms in Region 3 will choose $F_H = 1$ since the marginal benefit of debt exceeds the marginal cost for all debt choices in Region 3.

The first-order condition for an optimal (interior) debt choice is:

$$F_L(P) = P/\eta - P + \tau/\eta.$$

The second-order condition is clearly satisfied.

Pure-strategy interior equilibrium

Without reorganization costs the firm liquidates at time 1 if and only if $V_i \leq P$, therefore, the supply of the asset is P. In a pure-strategy equilibrium, the demand is $1 - P - F_L(P)$, therefore, we have the unique market clearing price

$$P^* = \frac{\eta - \tau}{1 + \eta}$$

and the unique interior debt choice

$$F_L^* = \frac{2\tau + 1 - \eta}{1 + \eta}.$$

Mixed-strategy equilibrium

We now consider the possibility of a mixed equilibrium. The value of a firm choosing $F_L(P)$ is

$$\begin{aligned} V_L(P) &= 0.5\cdot(1+P^2)+\int_{P+F_L}^{1}(\eta V-P)dV+\tau F_L \\ &= 0.5\cdot(1+P^2)+\frac{(\eta^2+P^2-2P\eta-\tau^2)}{2\eta}+\tau F_L, \end{aligned}$$

and the value of a firm choosing $F_H = 1$ is

$$V_1(P) = 0.5\cdot(1+P^2)+\tau.$$

Setting $V_L(P) = V_1(P)$ and solving for P yields the unique market clearing price in a mixed-strategy equilibrium:

$$P = \eta-\tau+\eta\tau-\sqrt{\eta^2\tau^2+2\eta\tau(\eta-\tau)}.$$

There is another candidate P, but it is greater than the pure-strategy equilibrium price. We know that this price could never be supported in equilibrium, as introducing high-debt firms both reduces the demand and increases the supply of the liquidated asset.

We now find the boundaries of the mixed-strategy equilibrium. Since prices are continuous we know $P^*(\tau^c) = P(\tau^c)$ (i.e., prices in the pure-strategy and mixed-strategy equilibria are equal at the boundaries). Solving for τ^c gives:

$$\tau^c = \frac{2\eta^2+\eta\pm\eta(1+\eta)}{3\eta+2} \Rightarrow \{\tau_{c_1},\tau_{c_2}\} = \left\{\frac{\eta^2}{3\eta+2},\eta\right\}.$$

Therefore, for $\tau \in \left(\frac{\eta^2}{3\eta+2},\eta\right]$ there is a mixed-strategy equilibrium with proportion h^* of firms choosing $F_H = 1$, proportion $1-h^*$ of firms choosing $F_L(P^*) = P^*/\eta-P^*+\tau/\eta$, and the price

$$P^* = \eta-\tau+\eta\tau-\sqrt{\eta^2\tau^2+2\eta\tau(\eta-\tau)}.$$

The supply of the asset is P^* and in a mixed-strategy equilibrium the demand is $(1-h^*)\cdot(1-P^*-F_L)$, therefore, we can solve for the unique proportion:

$$h^* = \frac{1-2P^*-F_L(P^*)}{1-P*-F_L(P^*)}.$$

Pure-strategy extreme equilibrium

For $\tau > \eta$ there is a pure-strategy equilibrium in which all firms choose $F^* = 1$ and the equilibrium price is $P^* = 0$. In this region, the marginal benefit always exceed the marginal cost. In this region, the demand for the asset is zero, as the financing constraint always binds. The supply is also zero, since the manager always prefers to keep the asset worth $0 \leq V_i$ instead of selling it for nothing.

Uniqueness

All together, we can characterize which type of symmetric equilibrium will obtain by looking at the exogenous parameters. For $\tau \in \left[0, \frac{\eta^2}{3\eta+2}\right]$, we have a unique, pure-strategy interior equilibrium. For $\tau \in \left(\frac{\eta^2}{3\eta+2}, \eta\right]$, we have a unique, mixed-strategy equilibrium. For $\tau \in (\eta, 1]$, we have a unique, pure-strategy extreme equilibrium. It cannot be that there is a mixed-strategy equilibrium in either of the pure-strategy regions, as it would require $h^* < 0$ or $h^* > 1$ to support the equilibrium price, which is not possible. Also, there cannot be either of the pure-strategy equilibria in the mixed-strategy region. Here, the fraction h^* is chosen to make firms indifferent between high and low debt. If all the firms chose high debt, it would cause prices to fall and make F_L more attractive. Conversely, if all of the agents selected low debt, prices would rise and F_H would be preferable.

We also established that the equilibrium debt levels and prices are unique functions of the exogenous parameters. Therefore, for any given set of exogenous parameters, we can identify the unique symmetric equilibrium.

B Extension of Theorem 2 to the Full Parameter Space

A Statement

In the text, we covered a limited parameter space for illustration. This appendix states the theorem and proof for the model's complete parameter space.

$$
\begin{aligned}
\text{Let } \tau_0 &= \frac{\phi(1-2\eta)}{(1+\eta)(1+\phi)+\phi(1-2\eta)}, \\
\tau_1 &= \frac{2\eta^2(\phi+1)-\phi+\eta(\phi+1)-\sqrt{(\eta+1)^2(\phi+1)(\eta^2\phi+\eta^2-2\phi-\eta\phi)}}{3\eta(\phi+1)+3\phi+2}, \\
\tau_2 &= \frac{(2\eta-1)(2\eta+\phi+2\eta\phi)}{2+3\phi} \\
&- \frac{\sqrt{(2\eta-1)^2(1+\phi)(4\eta^2(1+\phi)+\eta\phi-2(\eta+\phi))}}{2+3\phi}, \\
\tau_3 &= \frac{2\eta^2(1-\phi^2)+\eta(1+9\phi+\phi^2-5\phi^3)+2\phi+12\phi^2+7\phi^3}{2+14\phi+20\phi^2+9\phi^3+3\eta(1-\phi)(1+\phi)^2} \\
&- \frac{\sqrt{(1+\phi)(1+\eta+5\phi-\eta\phi)^2(\eta^2(1+\phi)(1+3\phi^2)-3\eta\phi^2(2+\phi)-2\phi^3)}}{2+14\phi+20\phi^2+9\phi^3+3\eta(1-\phi)(1+\phi)^2}, \\
\tau_4 &= \frac{2\eta^2(1-\phi^2)+\eta(1+9\phi+\phi^2-5\phi^3)+2\phi+12\phi^2+7\phi^3}{2+14\phi+20\phi^2+9\phi^3+3\eta(1-\phi)(1+\phi)^2} \\
&+ \frac{\sqrt{(1+\phi)(1+\eta+5\phi-\eta\phi)^2(\eta^2(1+\phi)(1+3\phi^2)-3\eta\phi^2(2+\phi)-2\phi^3)}}{2+14\phi+20\phi^2+9\phi^3+3\eta(1-\phi)(1+\phi)^2}.
\end{aligned}
$$

Region 1: $\eta \geq 1/2$ **and** $\phi < \frac{3\eta-2}{6-3\eta}$

- ▹ If $0 \leq \tau \leq \tau_1$ there exists a unique, pure-strategy equilibrium with price $P^* = \frac{\eta-\tau}{1+\eta}$ in which all firms choose $F_L^* = \frac{1-\eta+2\tau}{1+\eta}$.
- ▹ If $\tau_1 < \tau \leq \tau_2$ there exists a unique, mixed-strategy equilibrium with price

$$
\begin{aligned}
P^* &= \frac{\phi\eta-(1+\phi)[\tau-\eta(1+\tau)]}{1+\phi(1+\eta)} \\
&- \frac{\sqrt{\eta(\phi+1)(2\tau(\eta\phi+(\eta-\tau)(1+\phi))+\eta\tau^2(\phi+1)-\phi(1+\tau-\eta)^2)}}{1+\phi(1+\eta)}.
\end{aligned}
$$

in which the proportion $1-h^*$ of firms choose $F_L^* = \frac{\tau}{\eta} + \frac{(1-\eta)}{\eta} P^*$ and the proportion h^* of firms choose $F_H^* = 1$, where

$$h^* = \frac{(1+\phi)\cdot(\eta-\tau-(1+\eta)\cdot P)}{\eta\phi+(1+\phi)(\eta-\tau)-(1+\phi(1+\eta))\cdot P}.$$

- If $\tau_2 < \tau \leq \tau_4$ there exists a unique, mixed-strategy equilibrium with price

$$P^* = \frac{\phi(1+2\phi(1-\tau)-3\tau)+\eta(1+\phi)(1+\tau+\phi(2+\tau))-\tau}{1+(6-3\eta)\phi(1+\phi)}$$
$$- \frac{\sqrt{(1+\phi)(\eta+\phi+\eta\phi)\begin{pmatrix} 3\eta^2\phi(1+\phi)-2(\phi(\tau-1)+\tau)^2 \\ +\eta(\phi(\tau-1)+\tau)(2+\phi(\tau-1)+\tau) \end{pmatrix}}}{1+(6-3\eta)\phi(1+\phi)}. \tag{9}$$

in which the proportion $1-h^*$ of firms choose $F_L^* = \frac{\tau(1+\phi)}{(\eta+\phi+\eta\phi)} + \frac{(1-\eta)(1+\phi)+\phi}{(\eta+\phi+\eta\phi)} P^*$ and the proportion h^* of firms choose $F_H^* = 1$, where

$$h^* = \frac{(1+\phi)\cdot(\eta+\phi+\eta\phi-\tau(1+2\phi)-P\cdot(1+\eta+5\phi-\eta\phi))}{(1+2\phi)\cdot(\eta+\phi+\eta\phi-\tau(1+\phi))-P\cdot(1+5\phi-\eta\phi+5\phi^2-\eta\phi^2)}. \tag{10}$$

- If $\tau_4 < \tau \leq \frac{\eta+\phi+\eta\phi}{1+2\phi}$ there exists a unique, pure-strategy equilibrium with price $P^* = \frac{\eta+\phi+\eta\phi-\tau(1+2\phi)}{1+\eta+5\phi-\eta\phi}$ in which all firms choose $F_L^* = \frac{1+2\phi+\tau+(\tau-\eta)(1+\phi)}{1+\eta+5\phi-\eta\phi}$.

Region 2: $\eta \geq 1/2$, $\phi \geq \frac{3\eta-2}{6-3\eta}$, **and** $\eta^2(1+\phi)(1+3\phi^2)-3\eta\phi^2(2+\phi)-2\phi^3 \geq 0$

- If $0 \leq \tau \leq \frac{2\eta-1}{3}$ there exists a unique, pure-strategy equilibrium with price $P^* = \frac{\eta-\tau}{1+\eta}$ in which all firms choose $F_L^* = \frac{1-\eta+2\tau}{1+\eta}$.
- If $\frac{2\eta-1}{3} < \tau \leq \tau_3$ or if $\tau_4 < \tau \leq \frac{\eta+\phi+\eta\phi}{1+2\phi}$ there exists a unique, pure-strategy equilibrium with price $P^* = \frac{\eta+\phi+\eta\phi-\tau(1+2\phi)}{1+\eta+5\phi-\eta\phi}$ in which all firms choose $F_L^* = \frac{1+2\phi+\tau+(\tau-\eta)(1+\phi)}{1+\eta+5\phi-\eta\phi}$.
- If $\tau_3 < \tau \leq \tau_4$ there exists a unique, mixed-strategy equilibrium with price (9) in which the proportion $1-h^*$ of firms choose $F_L^* = \frac{\tau(1+\phi)}{(\eta+\phi+\eta\phi)} + \frac{(1-\eta)(1+\phi)+\phi}{(\eta+\phi+\eta\phi)} P^*$ and the proportion h^* of firms choose $F_H^* = 1$, where h^* is (10).

Region 3: $\eta \geq 1/2$, $\phi \geq \frac{3\eta-2}{6-3\eta}$ **and** $\eta^2(1+\phi)(1+3\phi^2)-3\eta\phi^2(2+\phi)-2\phi^3 < 0$

- If $0 \leq \tau \leq \frac{2\eta-1}{3}$ there exists a unique, pure-strategy equilibrium with price $P^* = \frac{\eta-\tau}{1+\eta}$ in which all firms choose $F_L^* = \frac{1-\eta+2\tau}{1+\eta}$.
- If $\frac{2\eta-1}{3} < \tau \leq \frac{\eta+\phi+\eta\phi}{1+2\phi}$ there exists a unique, pure-strategy equilibrium with price $P^* = \frac{\eta+\phi+\eta\phi-\tau(1+2\phi)}{1+\eta+5\phi-\eta\phi}$ in which all firms choose $F_L^* = \frac{1+2\phi+\tau+(\tau-\eta)(1+\phi)}{1+\eta+5\phi-\eta\phi}$.

Region 4: $\eta < 1/2$ **and** $\eta^2(1+\phi)(1+3\phi^2) - 3\eta\phi^2(2+\phi) - 2\phi^3 \geq 0$

- ▷ If $0 < \tau \leq \tau_0$ there exists a unique, pure-strategy equilibrium with price $P^* = \frac{\eta(1-\tau)}{1+\eta}$ in which all firms choose $F_L^* = \frac{\tau(1+\eta+\phi)+\phi\eta}{\phi(1+\eta)}$.
- ▷ If $\tau_0 < \tau \leq \tau_3$ or if $\tau_4 \leq \tau \leq \frac{\phi+\eta(1+\phi)}{1+2\phi}$ there exists a unique, pure-strategy equilibrium with price $P^* = \frac{\eta+\phi+\eta\phi-\tau(1+2\phi)}{1+\eta+5\phi-\eta\phi}$ in which all firms choose $F_L^* = \frac{1+2\phi+\tau+(\tau-\eta)(1+\phi)}{1+\eta+5\phi-\eta\phi}$.
- ▷ If $\tau_3 < \tau \leq \tau_4$ there exists a unique, mixed-strategy equilibrium with price (9) in which the proportion $1-h^*$ of firms choose $F_L^* = \frac{\tau(1+\phi)}{(\eta+\phi+\eta\phi)} + \frac{(1-\eta)(1+\phi)+\phi}{(\eta+\phi+\eta\phi)} P^*$ and the proportion h^* of firms choose $F_H^* = 1$, where h^* is (10).

Region 5: $\eta < 1/2$ **and** $\eta^2(1+\phi)(1+3\phi^2) - 3\eta\phi^2(2+\phi) - 2\phi^3 < 0$

- ▷ If $0 < \tau \leq \tau_0$ there exists a unique, pure-strategy equilibrium with price $P^* = \frac{\eta(1-\tau)}{1+\eta}$ in which all firms choose $F_L^* = \frac{\tau(1+\eta+\phi)+\phi\eta}{\phi(1+\eta)}$.
- ▷ If $\tau_0 < \tau \leq \frac{\eta+\phi+\eta\phi}{1+2\phi}$ there exists a unique, pure-strategy equilibrium with price $P^* = \frac{\eta+\phi+\eta\phi-\tau(1+2\phi)}{1+\eta+5\phi-\eta\phi}$ in which all firms choose $F_L^* = \frac{1+2\phi+\tau+(\tau-\eta)(1+\phi)}{1+\eta+5\phi-\eta\phi}$.

Region 6:

- ▷ If $\frac{\eta+\phi+\eta\phi}{1+2\phi} < \tau \leq 1$ there exists a unique, pure-strategy equilibrium with price $P^* = 0$ in which all firms choose $F_L^* = \min\left\{1, \frac{\tau(1+\phi)}{\eta(1+\phi)+\phi}\right\}$.

B Proof

Proof when $\eta \geq 1/2$

We first consider the case $\eta \geq 1/2$. Each firm is competitive and takes the price, P, as given. For now, we assume $P > 0$. We will consider the possibility that $P = 0$ later in the proof. The marginal benefit of debt is τ for all values of debt, F. If $P > 0$ the marginal cost of debt falls into four regions:

1. For $F \in [0, P \cdot (1/\eta - 1)]$ the marginal cost of debt is 0
2. For $F \in (P \cdot (1/\eta - 1), P]$ the marginal cost of debt is $\eta F - P \cdot (1-\eta)$
3. For $F \in (P, 1-P)$ the marginal cost of debt is $\eta F - P \cdot (1-\eta) + (F-P) \cdot \phi/(1+\phi)$

4. For $F \in [1-P, 1]$ the marginal cost of debt is $(F-P) \cdot \phi/(1+\phi)$

Importantly, the marginal cost of debt is weakly increasing and continuous in F over the first three regions but then jumps down at $F = 1-P$ (since the financing constraint is no longer binding) after which it increases again. Therefore, for any given marginal benefit τ, there are at most two possible optimal debt choices, one where $F < 1-P$, and one where $1-P < F \leq 1$. Consequently, there is the possibility of both pure-strategy equilibria and mixed-strategy equilibria in which some firms choose low debt and others choose high debt. We must consider three cases: (i) $0 \leq \tau < (2\eta-1) \cdot P$, (ii) $(2\eta-1) \cdot P \leq \tau \leq \eta - P + (1-2P)\phi/(1+\phi)$, and (iii) $\eta - P + (1-2P)\phi/(1+\phi) < \tau \leq 1$.

Case 1: $0 \leq \tau < (2\eta-1) \cdot P$

If $\tau < (2\eta-1) \cdot P$ then firms choose either $F_L \in [P \cdot (1/\eta - 1), P)$ or $F_H \in [1-P, 1]$ where $F_L = \tau/\eta + P \cdot (1/\eta - 1)$ and $F_H = \min\{1, P + \tau \cdot (1+\phi)/\phi\}$.

Pure-strategy equilibria

There cannot exist a symmetric equilibrium with $P > 0$ in which all firms choose F_H because the aggregate demand for the risky asset would be zero but the supply is positive $[(P+\phi F)/(1+\phi)]$. Therefore, if there exists a symmetric equilibrium in the case $\tau < (2\eta-1) \cdot P$, then all firms choose $F_L = \tau/\eta + P \cdot (1/\eta - 1)$. The demand for the liquidated asset is then $1 - P - F_L$ and the supply of the liquidated asset is P since for $F_L < P$ which implies $\tau < (2\eta-1) \cdot P$ it is optimal to liquidate the asset for all $V \leq P$. Equating supply and demand gives the unique market clearing price $P^* = (\eta - \tau)/(1+\eta)$.

Importantly, note that if $P^* = (\eta-\tau)/(1+\eta)$ then we must have $\tau < (2\eta-1)/3$ to be in a symmetric equilibrium in the case $\tau < (2\eta-1) \cdot P$.

Mixed-strategy equilibria

There is also the possibility of a mixed-strategy equilibrium (a fraction of firms choosing F_L and the remaining fraction of firms choosing F_H). Firms choosing F_L have ex ante value

$$V_L = \int_0^P P dV + \int_P^1 V dV + \int_{P+F_L}^1 (\eta V - P) dV + \tau \cdot F_L$$

and by substituting $F_L = \tau/\eta + P\cdot(1/\eta - 1)$ yields

$$V_L = 0.5(\eta + (P-1)^2 + (P+\tau)^2/\eta - 2P\tau).$$

Firms choosing F_H have ex ante value

$$V_H = \int_0^{\Lambda} P dV + \int_{\Lambda}^{F_H} [V - \phi\cdot(F_H - V)]dV + \int_{F_H}^{1} V dV + \tau\cdot F_H.$$

where $\Lambda = (P + \phi F)/(1+\phi)$.

If $F_H = P + \tau\cdot(1+\phi)/\phi$ then ex ante value is

$$V_H = 0.5\cdot(P+\tau)^2 + 0.5\cdot\tau^2/\phi + 0.5,$$

but if $F_H = 1$ then ex ante value is

$$V_1 = 0.5\cdot(P+\phi)^2/(1+\phi) + 0.5\cdot(1-\phi) + \tau.$$

The following result shows that in a mixed-strategy equilibrium the high-type always chooses $F_H = 1$.

Lemma 1: If $\tau < (2\eta - 1)/3$ then in a mixed-strategy equilibrium the high type chooses $F_H = 1$.

Proof: Proof by contradiction. Suppose $F_H = P + \tau\cdot(1+\phi)/\phi < 1$. Then we have:

$$G(P) \equiv V_L(P) - V_H(P) = 0.5\cdot(\eta + (P-1)^2 + (P+\tau)^2\cdot(1-\eta)/\eta - 2P\tau - \tau^2/\phi - 1).$$

Note, $G'(P) = P/\eta - 1 + \tau\cdot(1/\eta - 2) \leq P/\eta - 1 \leq 0$ where the first inequality follows from our assumption that $1/\eta - 1 \leq 1$ and the second from the fact that $P \leq \eta$ as the price for the asset will never exceed its maximum value. Therefore, G(P) is decreasing in P. Furthermore, in a mixed-strategy equilibrium the price is bounded above by the pure-strategy equilibrium price (i.e.

$P \leq (\eta - \tau)/(1+\eta))$ because the introduction of some high-debt firms both reduces the demand and increases the supply of the liquidated asset. Therefore,

$$\begin{aligned}
G(P) &\geq G((\eta-\tau)/(1+\eta)) \\
&= 0.5 \cdot \left[\left(\frac{1+\tau}{1+\eta} \right)^2 (1+\eta-\eta^2) + \eta - 2\tau \left(\frac{\eta-\tau}{\eta+1} \right) - \frac{\tau^2}{\phi} - 1 \right] \\
&> 0.5 \cdot \left[\left(\frac{1+\tau}{1+\eta} \right)^2 (1+\eta-\eta^2) + \eta - 2\tau \left(\frac{\eta-\tau}{\eta+1} \right) - \tau(1-\tau) - 1 \right] \\
&\geq 0 \quad \forall \tau
\end{aligned}$$

where the second inequality follows from the fact that if $F_H^* < 1$ and $P > 0$ then $\tau < \phi/(1+\phi)$ which implies $\phi > \tau/(1-\tau)$; and the third inequality is easily verified numerically. But this contradicts the optimality of F_H^*.

By Lemma 1, we must only compare V_L to V_1 to find a mixed-strategy equilibrium.

Conjecture the existence of a pure-strategy equilibrium in which $F_L = \tau/\eta + P \cdot (1/\eta - 1)$ and $P^* = (\eta - \tau)/(1+\eta)$. Substituting P^* into our expressions for V_L and V_1 implies:

$$H(\tau) \equiv V_L(\tau) - V_1(\tau) = \frac{\eta(\eta-\tau)^2(\phi+1) + \phi(\tau+1)^2 - 2(\eta+1)(\phi+1)\tau(\eta-\tau)}{2(\eta+1)^2(\phi+1)}$$

Therefore,

$$H'(\tau) = \frac{\phi - \eta(1+\phi) - 2\eta^2(1+\phi) + (2+3\phi+3\eta(1+\phi))\tau}{(1+\eta)^2(1+\phi)}$$

and

$$H''(\tau) = \frac{2+3\phi+3\eta(1+\phi)}{(1+\eta)^2(1+\phi)}$$

Note that $H'\left(\frac{2\eta-1}{3}\right) = -\frac{2}{3(1+\eta)(1+\phi)} < 0$ and $H''(\tau) > 0$ for all τ. Therefore, $H'(\tau) < 0$ for all $\tau \leq \frac{2\eta-1}{3}$.

Also, $H\left(\frac{2\eta-1}{3}\right) = \frac{2-3\eta+\phi\cdot(6-3\eta)}{18(1+\phi)} \geq 0$ if and only if $\phi \geq \frac{3\eta-2}{6-3\eta}$. Note that if $\eta < \frac{2}{3}$, then $\frac{3\eta-2}{6-3\eta} < 0$ and $H\left(\frac{2\eta-1}{3}\right) \geq 0$ for any ϕ.

Therefore, if $\phi \geq \frac{3\eta-2}{6-3\eta}$ and $\tau \leq \frac{2\eta-1}{3}$ then $H(\tau) \geq 0$ and all firms optimally choose $F_L = \tau/\eta + P \cdot (1/\eta - 1)$ and the conjectured equilibrium price of $P = (\eta - \tau)/(1+\eta)$ is confirmed by the firms' debt decisions.

However, if $\phi < \frac{3\eta-2}{6-3\eta}$, then the conjectured pure-strategy equilibrium is confirmed only for $\tau \leq \tau_1$ where $H(\tau_1) \equiv 0$. For $\tau > \tau_1$ there is a mixed-strategy equilibrium in which some firms choose $F_L = \tau/\eta + P \cdot (1/\eta - 1)$ and others choose $F_H = 1$ (by Lemma 1). Solving $F(\tau_1) = 0$ yields

$$\tau_1 = \frac{2\eta^2(\phi+1) - \phi + \eta(\phi+1) - \sqrt{(\eta+1)^2(\phi+1)(\eta^2\phi + \eta^2 - 2\phi - \eta\phi)}}{3\eta(\phi+1) + 3\phi + 2}$$

(Note: There is another solution to $H(\tau_1) = 0$ where $H(\tau)$ again becomes positive beyond that point. However, $H'(\tau) < 0$ for all $\tau \leq \frac{2\eta-1}{3}$ so we know $H(\tau) < 0$ for all $\tau_1 < \tau < \frac{2\eta-1}{3}$.)

For $\tau > \tau_1$ there is a unique, mixed-strategy equilibrium which is constructed by finding P that equates $V_L = V_1$, which is quadratic in P. There are two solutions, but only one where P is less than the pure-strategy price (which must be true in equilibrium as argued above) and it is

$$\begin{aligned} P^* = & \frac{\phi\eta - (1+\phi)[\tau - \eta(1+\tau)]}{1+\phi(1+\eta)} \\ & - \frac{\sqrt{\eta(\phi+1)(2\tau(\eta\phi + (\eta-\tau)(1+\phi)) + \eta\tau^2(\phi+1) - \phi(1+\tau-\eta)^2)}}{1+\phi(1+\eta)}, \end{aligned}$$

Let h be the fraction of firms choosing $F = 1$. The demand for the risky asset is then $(1-h)\cdot(1-P-F_L)$ and the supply of the risky asset is $(1-h)\cdot P + h\cdot(P+\phi\cdot 1)/(1+\phi)$, therefore, market clearing requires a unique proportion of high debt firms:

$$h^* = \frac{(1+\phi)\cdot(\eta - \tau - (1+\eta)\cdot P)}{\eta\phi + (1+\phi)(\eta-\tau) - (1+\phi(1+\eta))\cdot P},$$

Finally, if there is a mixed-strategy equilibrium at the upper boundary we know that $P < (\eta-\tau)/(1+\eta)$ and therefore $\tau < (2\eta-1)/3$ at the boundary. Equating $\tau_2 = (2\eta-1)\cdot P^*(\tau_2)$ yields the upper boundary in this case:

$$
\begin{aligned}
\tau_2 &= \frac{(2\eta-1)(2\eta+\phi+2\eta\phi)}{2+3\phi} \\
&- \frac{\sqrt{(2\eta-1)^2(1+\phi)(4\eta^2(1+\phi)+\eta\phi-2(\eta+\phi))}}{2+3\phi}
\end{aligned}
$$

(Note: There is another root but it is greater than $(2\eta-1)/3$ when $\eta \geq 1/2$ so we can ignore it.)

Case 2: $(2\eta-1)\cdot P \leq \tau \leq \eta - P + (1-2P)\phi/(1+\phi)$

If $(2\eta-1)\cdot P \leq \tau \leq \eta - P + (1-2P)\phi/(1+\phi)$ then firms choose either $F_L \in [P, 1-P)$ or $F_H \in [1-P, 1]$ where $F_L = \frac{\tau(1+\phi)}{\eta+\phi+\eta\phi} + \frac{(1-\eta)(1+\phi)+\phi}{(\eta+\phi+\eta\phi)}P$ and $F_H = \min\{1, P+\tau\cdot(1+\phi)/\phi\}$.

Pure-strategy equilibria

Again, there cannot exist a pure-strategy equilibrium with $P > 0$ in which all firms choose F_H because the aggregate demand for the risky asset would be zero but the supply is positive $[(P+\phi F)/(1+\phi)]$. Therefore, in a pure-strategy equilibrium firms choose

$$
F = \frac{\tau(1+\phi)}{\eta+\phi+\eta\phi} + \frac{(1-\eta)(1+\phi)+\phi}{(\eta+\phi+\eta\phi)}P.
$$

The demand for the liquidated asset is $1-P-F$ and the supply of the liquidated asset is

$$
\Lambda = \frac{P+\phi F}{1+\phi} = \frac{P\cdot((1-\phi)\eta+2\phi)+\phi\tau}{\eta+\phi+\eta\phi}.
$$

Equating supply and demand gives the unique market clearing price

$$
P^* = \frac{\eta+\phi+\eta\phi-\tau(1+2\phi)}{1+\eta+5\phi-\eta\phi}.
$$

Substituting into the expression for F yields

$$
F^* = \frac{1-\eta-\eta\phi+2\phi+\tau(2+\phi)}{1+\eta+5\phi-\eta\phi}.
$$

Mixed-strategy equilibria

As in Case 1, there is the possibility of a mixed-strategy equilibrium. Firms choosing F_L have ex ante value

$$V_L = \int_0^\Lambda P dV + \int_\Lambda^{F_L} [V - \phi(F_L - V)] dV + \int_{F_L}^1 V dV + \int_{P+F_L}^1 (\eta V - P) dV + \tau \cdot F_L$$

and by substituting $F_L = \frac{\tau(1+\phi)}{\eta+\phi+\eta\phi} + \frac{(1-\eta)(1+\phi)+\phi}{(\eta+\phi+\eta\phi)} P$ yields

$$\begin{aligned} V_L &= \left(\frac{1+\eta+6\phi-3\eta\phi}{2(\eta+\phi+\eta\phi)}\right) P^2 - \left(1 - \frac{\tau(1-\eta+2\phi-\eta\phi)}{\eta+\phi+\eta\phi}\right) P \\ &+ \left(1 + \frac{\eta(\eta-1)+\phi(\eta^2-1)+\tau^2(1+\phi)}{2(\eta+\phi+\eta\phi)}\right). \end{aligned} \tag{11}$$

Firms choosing F_H have ex ante value

$$V_H = \int_0^\Lambda P dV + \int_\Lambda^{F_H} [V - \phi \cdot (F_H - V)] dV + \int_{F_H}^1 V dV + \tau \cdot F_H.$$

If $F_H = P + \tau \cdot (1+\phi)/\phi$ then ex ante value is

$$V_H = 0.5 \cdot (P+\tau)^2 + 0.5 \cdot \tau^2/\phi + 0.5 \tag{12}$$

but if $F_H = 1$ then ex ante value is

$$V_1 = 0.5 \cdot (P+\phi)^2/(1+\phi) + 0.5 \cdot (1-\phi) + \tau. \tag{13}$$

The next result shows that in a mixed-strategy equilibrium the high type always chooses $F_H = 1$.

Lemma 2: In a mixed-strategy equilibrium the high type chooses $F_H = 1$.

Proof: Proof by contradiction. Suppose $F_H^* = P + \tau \cdot (1+\phi)/\phi < 1$. Since we assume $P > 0$, this implies $\tau < \phi/(1+\phi)$. We have:

$$\begin{aligned} G(P) &\equiv V_L(P) - V_H(P) \\ &= \left(\frac{1+(5-4\eta)\phi}{2(\eta+\phi+\eta\phi)}\right) P^2 - \left(1 - \frac{\tau(1+\phi)(1-2\eta)}{\eta+\phi+\eta\phi}\right) P + \left(\frac{\eta}{2} - \frac{\eta\tau^2(1+\phi)^2}{2\phi(\eta+\phi+\eta\phi)}\right) \end{aligned}$$

Therefore,

$$\begin{aligned} G'(P) &= \left(\frac{1+(5-4\eta)\phi}{\eta+\phi+\eta\phi}\right)P-\left(1-\frac{\tau(1+\phi)(1-2\eta)}{\eta+\phi+\eta\phi}\right) \\ &\leq \left(\frac{1+(5-4\eta)\phi}{\eta+\phi+\eta\phi}\right)\left(\frac{\eta(1+\phi)+\phi-\tau(1+2\phi)}{(5-\eta)\phi+1+\eta}\right)-\left(1-\frac{\tau(1+\phi)(1-2\eta)}{\eta+\phi+\eta\phi}\right) \\ &\leq 0 \end{aligned}$$

where the first inequality follows from the fact that in a mixed-strategy equilibrium the equilibrium price is less than the pure-strategy equilibrium price (i.e., $P \leq \frac{\eta+\phi+\eta\phi-\tau(1+2\phi)}{1+\eta+5\phi-\eta\phi}$) and the second inequality holds for all $\eta \geq 1/2$. Therefore,

$$\begin{aligned} G(P) &\geq G\left(\frac{\eta+\phi+\eta\phi-\tau(1+2\phi)}{1+\eta+5\phi-\eta\phi}\right) \\ &> 0 \quad \forall \tau < \phi/(1+\phi). \end{aligned}$$

where the last inequality is easily verified numerically. But this contradicts the optimality of F_H^*.

By Lemma 2, we must only compare V_L to V_1 to find a mixed-strategy equilibrium. We have

$$\begin{aligned} V_L - V_1 &= \frac{\left(\phi(\phi+1)(2-3\eta)+(2\phi+1)^2\right)}{2(\phi+1)(\eta\phi+\eta+\phi)}P^2 \\ &- \frac{\left((2\phi+1)(\eta+\phi+\eta\phi-\tau(1+\phi))+\eta(\phi+1)^2\tau\right)}{(\phi+1)(\eta\phi+\eta+\phi)}P \\ &+ \frac{((\eta-\tau)(1+\phi)+\phi)^2}{2(\phi+1)(\eta\phi+\eta+\phi)} \end{aligned} \tag{14}$$

Conjecture the existence of a pure-strategy equilibrium in which case $P^* = \frac{\eta+\phi+\eta\phi-\tau(1+2\phi)}{1+\eta+5\phi-\eta\phi}$. Substituting P^* into our expressions for V_L and V_1 implies:

$$\begin{aligned} H(\tau) = &\frac{2+14\phi+20\phi^2+9\phi^3+3\eta(1-\phi)(1+\phi)^2}{2(1+\phi)(1+\eta+5\phi-\eta\phi)^2}\tau^2 \\ &-\frac{2\eta^2(1-\phi^2)+\phi(2+12\phi+7\phi^2)+\eta(1+9\phi+\phi^2-5\phi^3)}{(1+\phi)(1+\eta+5\phi-\eta\phi)^2}\tau \\ &+\frac{\eta^3(\phi-1)^2(1+\phi)+\phi^2(2+11\phi)+2\eta\phi(1+8\phi+\phi^2)-4\eta^2\phi(-1+2\phi+2\phi^2)}{2(1+\phi)(1+\eta+5\phi-\eta\phi)^2} \end{aligned} \tag{15}$$

We have two possibilities to consider. First, suppose there is a pure-strategy equilibrium at the upper boundary of Case 1, $\tau = (2\eta - 1) \cdot P$. In this case, we know that $P = (\eta - \tau)/(\eta + 1)$ which implies $\tau = (2\eta - 1)/3$. We also know that $\phi \geq \frac{3\eta-2}{6-3\eta}$. Therefore, at the transition to this case we have $H\left(\frac{2\eta-1}{3}\right) = \frac{2-3\eta+\phi\cdot(6-3\eta)}{18(1+\phi)} \geq 0$ for $\phi \geq \frac{3\eta-2}{6-3\eta}$. Furthermore,

$$H'\left(\frac{2\eta-1}{3}\right) = -\frac{2+(10-6\eta)\phi + 6(1-\eta)\phi^2}{3(1+\eta(1-\phi)+5\phi)(1+\phi)} < 0,$$

and

$$H''(\tau) = \frac{2+14\phi+20\phi^2+9\phi^3+3\eta(1-\phi)(1+\phi)^2}{(1+\phi)(1+\eta+5\phi-\eta\phi)^2} > 0.$$

We see then that $H(\tau)$ is an upward facing parabola in τ. At the lower boundary of Case 2, where $\tau = \frac{2\eta-1}{3}$, $H(\tau)$ is positive but decreasing.

Solving $H(\tau) = 0$ yields two solutions:

$$\begin{aligned} \tau_3, \tau_4 &= \frac{2\eta^2(1-\phi^2)+\eta(1+9\phi+\phi^2-5\phi^3)+2\phi+12\phi^2+7\phi^3}{2+14\phi+20\phi^2+9\phi^3+3\eta(1-\phi)(1+\phi)^2} \\ &\pm \frac{\sqrt{(1+\phi)(1+\eta+5\phi-\eta\phi)^2(\eta^2(1+\phi)(1+3\phi^2)-3\eta\phi^2(2+\phi)-2\phi^3)}}{2+14\phi+20\phi^2+9\phi^3+3\eta(1-\phi)(1+\phi)^2} \end{aligned}$$

If $\eta^2(1+\phi)(1+3\phi^2) - 3\eta\phi^2(2+\phi) - 2\phi^3 < 0$, the roots of the solution of $H(\tau) = 0$ are complex so $H(\tau) > 0$ for all τ. Therefore, for $\frac{2\eta-1}{3} < \tau \leq 1$, there is a pure-strategy equilibrium where all firms choose $F^* = \frac{1+2\phi+\tau+(\tau-\eta)(1+\phi)}{1+\eta+5\phi-\eta\phi}$ and $P^* = \frac{\eta+\phi+\eta\phi-\tau(1+2\phi)}{1+\eta+5\phi-\eta\phi}$. However, our free disposal assumption implies $P^* \geq 0$ which requires $\tau \leq \frac{\eta+\phi+\eta\phi}{1+2\phi}$. We thus consider pure-strategy equilibrium in the range $\frac{2\eta-1}{3} < \tau \leq \frac{\eta+\phi+\eta\phi}{1+2\phi}$.

If $\eta^2(1+\phi)(1+3\phi^2) - 3\eta\phi^2(2+\phi) - 2\phi^3 \geq 0$, then τ_3 and τ_4 are real. Therefore, for $\tau_3 < \tau \leq \tau_4$, we have $H(\tau) < 0$ and a mixed-strategy equilibrium where P^* equates $V_L = V_1$. There are two solutions, but only one where P is less than the pure-strategy price (which must be true in equilibrium as argued above) and it is

$$\begin{aligned} P^* &= \frac{\phi(1+2\phi(1-\tau)-3\tau)+\eta(1+\phi)(1+\tau+\phi(2+\tau))-\tau}{1+\phi(6-3\eta)(1+\phi)} \\ &- \frac{\sqrt{(1+\phi)(\eta+\phi+\eta\phi)\left(\begin{array}{c} 3\eta^2\phi(1+\phi)-2(\phi(\tau-1)+\tau)^2 \\ +\eta(\phi(\tau-1)+\tau)(2+\phi(\tau-1)+\tau) \end{array}\right)}}{1+\phi(6-3\eta)(1+\phi)}. \end{aligned}$$

The fraction h of firms choosing $F = 1$ supporting the price P is found by equating demand for the risky asset $(1-h)\cdot(1-P-F_L)$ with supply $(1-h)\cdot(P+\phi F_L)/(1+\phi)+h\cdot(P+\phi\cdot 1)/(1+\phi)$ where $F_L = \frac{\tau(1+\phi)}{\eta+\phi+\eta\phi} + \frac{(1-\eta)(1+\phi)+\phi}{(\eta+\phi+\eta\phi)}P$. Therefore, we have the unique proportion to clear the market:

$$h^* = \frac{(1+\phi)\cdot(\eta+\phi+\eta\phi-\tau(1+2\phi)-P(1+5\phi+\eta-\eta\phi))}{(1+2\phi)(\eta+\phi+\eta\phi-\tau-\phi\tau)-P(1+5\phi(1+\phi)-\eta\phi(1+\phi))}.$$

It has been verified numerically that $\tau_4 \leq 1$ and $P^*(\tau_4) > 0$. This means that for $\tau > \tau_4$, there will be a range of pure-strategy equilibria with positive prices. Therefore, for $\frac{2\eta-1}{3} < \tau \leq \tau_3$ and $\tau_4 < \tau \leq \frac{\eta+\phi+\eta\phi}{1+2\phi}$ we have a pure-strategy equilibrium where $F^* = \frac{1+2\phi+\tau+(\tau-\eta)(1+\phi)}{1+\eta+5\phi-\eta\phi}$ and $P^* = \frac{\eta+\phi+\eta\phi-\tau(1+2\phi)}{1+\eta+5\phi-\eta\phi}$

Second, suppose there is a mixed-strategy equilibrium at the upper boundary of Case 1, i.e. $\tau_2 = (2\eta-1)\cdot P^*(\tau_2)$. Then we know that $\phi < \frac{3\eta-2}{6-3\eta}$. It can be verified that $H(\tau_2) < 0$, and, following the arguments above, $\{\tau_3, \tau_4\}$ are the same as described above. However, it can be verified that $\tau_3 < \tau_2$ if $\phi < \frac{3\eta-2}{6-3\eta}$, thus there is a mixed-strategy equilibrium for all $\tau_2 < \tau \leq \tau_4$. In the mixed-strategy equilibrium, $F_L = \frac{\tau(1+\phi)}{\eta+\phi+\eta\phi} + \frac{(1-\eta)(1+\phi)+\phi}{(\eta+\phi+\eta\phi)}P^*$ and $\{P^*, h^*\}$ are characterized above. It is still the case that $\tau_4 \leq 1$ and $P^*(\tau_4) > 0$. Therefore, for $\tau_4 < \tau \leq \frac{\eta+\phi+\eta\phi}{1+2\phi}$ we have a pure-strategy equilibrium where $F^* = \frac{1+2\phi+\tau+(\tau-\eta)(1+\phi)}{1+\eta+5\phi-\eta\phi}$ and $P^* = \frac{\eta+\phi+\eta\phi-\tau(1+2\phi)}{1+\eta+5\phi-\eta\phi}$

We must also check that all these equilibria fall under Case 2, which requires $\tau \leq \eta - P + (1-2P)\phi/(1+\phi)$. We know that $P^* \leq \frac{\eta+\phi+\eta\phi-\tau(1+2\phi)}{1+\eta+5\phi-\eta\phi}$, the pure-strategy equilibrium price, for all equilibria. Therefore, it is sufficient to show that:

$$\tau \leq \eta - \frac{\eta+\phi+\eta\phi-\tau(1+2\phi)}{1+\eta+5\phi-\eta\phi} + \left(1 - \frac{2(\eta+\phi+\eta\phi-\tau(1+2\phi))}{1+\eta+5\phi-\eta\phi}\right)\frac{\phi}{1+\phi}$$

which is true if and only if

$$\tau \leq \frac{\eta+2\phi-\eta\phi}{1-\phi}.$$

However, this is satisfied for all $\tau \leq \frac{\eta+\phi+\eta\phi}{1+2\phi}$. Therefore, the equilibria described above all fall within Case 2.

Case 3: $\eta - P + (1-2P)\phi/(1+\phi) < \tau \leq 1$

If $\eta - P + (1-2P)\phi/(1+\phi) < \tau \leq 1$ then all firms choose $F > 1-P$ which implies that the aggregate demand for the liquidated asset is zero. But, since supply is positive when the price is positive, this case is incompatible with an equilibrium $P^* > 0$.

Equilibrium with $P^* = 0$

We assume free disposal therefore we know $P^* \geq 0$. We now consider the possibility that $P^* = 0$ which can occur in our model because we assume limited liability (i.e., the continuation value of a firm in distress is bounded below by zero). If $P = 0$ the four regions for the marginal cost of debt collapse into one:

- For $F \in [0,1]$ the marginal cost of debt is $\eta \cdot F + F \cdot \phi/(1+\phi)$

Consequently, if $P = 0$ there can only exist a pure-strategy equilibrium in which all firms choose $F^* = \min\left\{1, \frac{\tau(1+\phi)}{\eta(1+\phi)+\phi}\right\}$. Therefore, the aggregate demand for the asset is $1-P^*-F^* = 1-F^* \geq 0$. The aggregate supply of the asset, however, is indeterminate. In particular, because of limited liability the firm will be indifferent between liquidation at $P^* = 0$ and continuation for all $V_i \in \left[0, \frac{\phi F^*}{1+\phi}\right]$. Therefore, the price $P^* = 0$ can be supported in equilibrium if $1-F^* \leq \frac{\phi F^*}{1+\phi}$ or $\tau \geq \frac{\eta(1+\phi)+\phi}{1+2\phi}$.

Proof when $\eta < 1/2$

We now consider the case $\eta < 1/2$. For now, assume $P > 0$. We will consider the possibility that $P = 0$ at the end of the proof. If $P > 0$ the marginal cost of debt now falls into four regions:

1. For $F \in [0,P]$ the marginal cost of debt is 0
2. For $F \in [P, P \cdot (1/\eta - 1)]$ the marginal cost of debt is $(F-P)\phi/(1+\phi)$
3. For $F \in [P \cdot (1/\eta - 1), 1-P]$ the marginal cost of debt is $\eta F - P(1-\eta) + (F-P)\phi/(1+\phi)$
4. For $F \in [1-P, 1]$ the marginal cost of debt is $(F-P)\phi/(1+\phi)$

We must consider three cases: (i) $0 \leq \tau < (P/\eta - 2P)\phi/(1+\phi)$, (ii) $(P/\eta-2P)\phi/(1+\phi) \leq \tau \leq \eta - P + (1-2P)\phi/(1+\phi)$, and (iii) $\eta - P + (1-2P)\phi/(1+\phi < \tau \leq 1$.

Case 1: $0 \leq \tau < (P/\eta - 2P)\phi/(1+\phi)$

If $0 \leq \tau < (P/\eta - 2P)\phi/(1+\phi)$ then $F \in [P, P(1/\eta - 1)]$ and all firms equate the marginal cost of debt in this region to the marginal benefit which implies

$$F(P) = P + \frac{\tau(1+\phi)}{\phi}.$$

In this region, all firms for whom the asset is positive NPV ($\eta V \geq P$) will be able to obtain financing to purchase the asset. The demand for the liquidated asset is then $1 - P/\eta$ and the supply of the liquidated asset is $(P + \phi F)/(1+\phi) = P + \tau$. Equating supply and demand gives the equilibrium price

$$P^* = \frac{\eta(1-\tau)}{1+\eta}$$

and, therefore,

$$F^* = \frac{\tau(1+\eta+\phi)+\phi\eta}{\phi(1+\eta)}.$$

To determine the values of τ included in this case, substitute P^* into the expression

$$\tau < (P/\eta - 2P)\phi/(1+\phi) \quad \Rightarrow \quad \tau \leq \frac{\phi(1-2\eta)}{(1+\eta)(1+\phi)+\phi(1-2\eta)} \equiv \tau_0.$$

Case 2: $(P/\eta - 2P)\phi/(1+\phi) \leq \tau \leq \eta - P + (1-2P)\phi/(1+\phi)$

If $(P/\eta - 2P)\phi/(1+\phi) \leq \tau \leq \eta - P + (1-2P)\phi/(1+\phi)$ then firms choose either $F_L \in [P(1/\eta - 1), 1-P)$ or $F_H \in [1-P, 1]$ where $F_L = \frac{\tau(1+\phi)}{\eta+\phi+\eta\phi} + \frac{(1-\eta)(1+\phi)+\phi}{(\eta+\phi+\eta\phi)}P$ and $F_H = \min\{1, P + \tau \cdot (1+\phi)/\phi\}$.

Pure-strategy equilibria

There cannot exist a pure-strategy equilibrium with $P > 0$ in which all firms choose F_H because the aggregate demand for the risky asset would be zero but the supply is positive $[(P + \phi F)/(1+\phi)]$. Therefore, in a pure-strategy equilibrium, firms choose

$$F = \frac{\tau(1+\phi)}{\eta(1+\phi)+\phi} + \frac{(1-\eta)(1+\phi)+\phi}{\eta(1+\phi)+\phi}P.$$

The demand for the liquidated asset is $1 - P - F$ and the supply of the liquidated asset is

$$\Lambda = \frac{P+\phi F}{1+\phi} = \frac{P \cdot [(1-\phi)\eta + 2\phi] + \phi\tau}{\eta(1+\phi)+\phi}.$$

Equating supply and demand gives the equilibrium price

$$P^* = \frac{\eta + \phi + \eta\phi - \tau(1+2\phi)}{1+\eta+5\phi-\eta\phi}.$$

Substituting into the expression for F yields

$$F^* = \frac{1+2\phi+\tau+(\tau-\eta)(1+\phi)}{1+\eta+5\phi-\eta\phi}.$$

Mixed-strategy equilibria

Again, there is the possibility of a mixed-strategy equilibrium. The proof here follows closely the proof in Case 2 when $\eta \geq 1/2$. Firms choosing $F_L = \frac{\tau(1+\phi)}{\eta+\phi+\eta\phi} + \frac{(1-\eta)(1+\phi)+\phi}{(\eta+\phi+\eta\phi)}P$ have ex ante value V_L as described in equation (11), firms choosing $F_H = P + \tau \cdot (1+\phi)/\phi$ have ex ante value V_H as described in equation (12), and firms choosing $F_H = 1$ have ex ante value V_1 as described in equation (13).

It is straightforward to show that Lemma 2 applies in the case $\eta < 1/2$ when $\tau_0 \leq \tau < \phi/(1+\phi)$. Therefore, in a mixed-strategy equilibrium the high type always chooses $F_H = 1$. Therefore, we must only compare V_L to V_1 to find a mixed-strategy equilibrium. We also have $V_L - V_1$ as described in equation (14) and $H(\tau)$ as described in equation (15).

We know there is a pure-strategy equilibrium at the upper boundary of case 1, $\tau = \tau_0$. Therefore, at the transition to this region we have $H(\tau_0) \geq 0$. Furthermore, it can be shown numerically that for all $\phi \in [0,1]$ and all $\eta \in [0,1/2]$ that

$$H'(\tau_0) < 0,$$

and

$$H''(\tau) = \frac{2+14\phi+20\phi^2+9\phi^3+3\eta(1-\phi)(1+\phi)^2}{(1+\phi)(1+\eta+5\phi-\eta\phi)^2} > 0.$$

We see then that $H(\tau)$ is an upward facing parabola in τ. At the lower boundary of Case 2, where $\tau = \tau_0$, $H(\tau_0)$ is positive but decreasing.

Solving $H(\tau) = 0$ yields τ_3, τ_4 as before and the remainder of the proof is identical to Case 2 when $\eta \geq 1/2$.

Case 3: $\eta - P + (1-2P)\phi/(1+\phi) < \tau \leq 1$

If $\eta - P + (1-2P)\phi/(1+\phi) < \tau \leq 1$ then all firms choose $F > 1-P$ which implies that the aggregate demand for the liquidated asset is zero. But, since supply is positive when the price is positive, this case is incompatible with an equilibrium $P^* > 0$.

Equilibrium with $P^* = 0$

Finally, as before, if $P = 0$ the four regions for the marginal cost of debt collapse into one:

▹ For $F \in [0,1]$ the marginal cost of debt is $\eta \cdot F + F \cdot \phi/(1+\phi)$

Consequently, if $P = 0$ there can only exist a pure-strategy equilibrium in which all firms choose $F^* = \min\left\{1, \frac{\tau(1+\phi)}{\eta(1+\phi)+\phi}\right\}$. Following the argument in the proof when $\eta \geq 1/2$, the price $P^* = 0$ can be supported in equilibrium if $\tau \geq \frac{\eta(1+\phi)+\phi}{1+2\phi}$.

Uniqueness

We've established above that the equilibrium debt choices, prices, and high-type proportion are unique functions of the exogenous parameters in each of the equilibrium types (pure strategy interior/extreme and mixed-strategy). Also, the necessary conditions for each of the equlibria form non-overlapping regions. It cannot be that a given exogenous parameter value supports multiple types of symmetric equilibria. Therefore, the equilibium is unique for any given parameter values.

C Extension: Immediate Use of Debt Benefits

We now show that the solution to our model is isomorphic to one in which the firm can use the immediate benefit τF to pay off debt and purchase liquidated assets.

If $F(1-\tau) < P$ the value of the firm is now

$$\int_0^P P dV + \int_P^1 V dV + \int_{P+F(1-\tau)}^1 \max\{0, \eta V - P\} dV + \tau F.$$

The supply of the risky asset is P and the demand is now $1 - P - (1-\tau)F$.

Therefore, if we let $F' = (1-\tau)F$ the value of the firm is

$$\int_0^P P dV + \int_P^1 V dV + \int_{P+F'}^1 \max\{0, \eta V - P\} dV + \frac{\tau}{1-\tau} \cdot F'.$$

The supply of the risky asset is P and the demand is now $1 - P - F'$.

If $F(1-\tau) \geq P$ the value of the firm is

$$\int_0^{\Lambda'} P dV + \int_{\Lambda'}^{F(1-\tau)} [V - \phi(F - (V + \tau F)] dV + \int_{(1-\tau)F}^1 V dV + \int_{P+F(1-\tau)}^1 \max\{0, \eta V - P\} dV + \tau F.$$

The supply of the risky asset is $\Lambda' = \frac{P+\phi(1-\tau)F}{1+\phi}$ and the demand is now $1 - P - (1-\tau)F$.

Again, if we let $F' = (1-\tau)F$ the value of the firm is

$$\int_0^{\Lambda'} P dV + \int_{\Lambda'}^{F'} [V - \phi(F' - V)] dV + \int_{F'}^1 V dV + \int_{P+F'}^1 \max\{0, \eta V - P\} dV + \frac{\tau}{1-\tau} \cdot F'.$$

The supply of the risky asset is $\Lambda' = \frac{P+\phi F'}{1+\phi}$ and the demand is now $1 - P - F'$.

In sum, the solution to our original model yields $\{P(\tau'), F'(\tau')\}$ where $\tau' = \tau/(1-\tau)$. To convert to the equilibrium $\{P(\tau), F(\tau)\}$ note that $\tau = \tau'/(1+\tau')$ and $F = F'/(1-\tau)$. The latter expression for the face value of debt implies that the comparative statics for debt choices with respect to model parameters are not necessarily the same as in the base model. Although we don't report the results here, our main qualitative results continue to hold in this extension; in particular, debt levels and ratios may increase or decrease in η and the comparative statics for debt levels can be different than for debt-to-value ratios.

D Extension: Industry Booms and Busts

In this section we extend our model to consider uncertainty in the economy. For certain parameter regions of this model, it is possible that we find either pro-cyclical or counter-cyclical reallocations of assets.

Suppose we extend the model to include uncertainty about the distribution of asset qualities, $v_i \in [0, \gamma]$. With probability a, $\gamma = 1 - \Delta$ and with probability $1 - a$, $\gamma = 1 + \Delta$. In this model, the firms will choose their debt level F and mixture probability h before realizing the state of the economy, but $P(\gamma)$ will be determined after the state of the economy has been realized.

Reallocation

We know that the amount of reallocation will equal the mass of firms who are acquiring assets. This is equal to:

$$Q(\gamma) = (1 - h) \cdot \min\left\{ \frac{\gamma - P(\gamma) - F}{\gamma}, \frac{\gamma - P(\gamma)/\eta}{\gamma} \right\}$$

In our base model, the marginal cost of debt may fall in several regions depending on the debt level. However, the boundaries of these regions depend on the realization of γ. Thus, optimal debt levels will be determined by equating the marginal benefit of debt, τ, with the expected marginal cost. We will assume for simplicity that Δ is sufficiently small such that the optimal debt level falls in the same marginal cost region in either case (i.e. there are not distress costs in the bust or the boom.)

Pro-Cyclical

Paralleling Case 1 where $\eta \geq 1/2$ of Appendix B, in this region we have no distress costs, and our only concern is foregone acquisition costs. Conditional on γ, we know that the supply is $\frac{P(\gamma)}{\gamma}$ and the demand is $\frac{\gamma - P(\gamma) - F}{\gamma}$. Therefore, $P(\gamma) = \frac{\gamma - F}{2}$. Plugging this back in to our supply we find that reallocation here is $Q(\gamma) = \frac{1}{2} - \frac{F}{2\gamma}$ and it is the case that $Q(1 + \Delta) > Q(1 - \Delta)$. We have in this region, the case where booms lead to more reallocation.

For this equilibrium to exist, there must exist parameters such that $F < P(\gamma)$ for both values of γ, which has been verified to exist numerically.

Counter-Cyclical

Paralleling Case 1 where $\eta < 1/2$ of Appendix B, in this region we have only distress costs and no foregone acquisition costs (still acquisitions, just NPV constraint binds). Conditional on γ, we know that the supply is $\frac{P(\gamma)+\phi F}{\gamma(1+\phi)}$ and the demand is $\frac{\gamma-P(\gamma)/\eta}{\gamma}$. Therefore, $P(\gamma) = \frac{\eta(\gamma(1+\phi)-\phi F)}{1+\eta+\phi}$. Plugging this back in to our demand we find that reallocation here is $Q(\gamma) = 1 - \frac{1+\phi}{1+\eta+\phi} + \frac{\phi F}{\gamma(1+\eta+\phi)}$ and it is the case that $Q(1+\Delta) < Q(1-\Delta)$. We have in this region the case where booms lead to less reallocation.

Again, for this equilibrium to exist, there must exist parameters such that $F < P(\gamma)(1/\eta - 1)$ for both values of γ, which has been verified to exist numerically.

E Tax Shields as Sources of Debt Benefits

In this section, we allow the debt benefits to be partly or fully tax-related. t is now the total benefit of debt, inclusive of the direct benefits (e.g. signaling or agency related) and the tax shield benefits. $g \in [0,1]$ is the share of the debt benefits that are from tax shields. The value of the firm for a given level of debt is

(16)

$$V(P, F_i) = t \cdot F_i + (1 - g \cdot t) \begin{cases} \int_0^P P\, dv + \int_P^1 v\, dv + \int_{\min\{P+F_i,1\}}^1 \max\{0, \eta \cdot v - P\}\, dv & \text{if } F_i < P \\ \int_0^{\Lambda(F_i)} P\, dv + \int_{\Lambda(F_i)}^{F_i} [v - \phi \cdot (F_i - v)]\, dv + \int_{F_i}^1 v\, dv + \int_{\min\{P+F_i,1\}}^1 \max\{0, \eta \cdot v - P\}\, dv & \text{if } F_i \geq P \end{cases}$$

The tax rate on the total earnings of the firm is $g \cdot t$, and the total benefits of debt are $t \cdot F_i$. This means that the tax shield is $g \cdot t \cdot F_i$, and other direct debt benefits are $(1-g) \cdot t \cdot F_i$. If $g = 0$, it is the model in the main text. If $g = 1$, all of the benefits of debt come from the tax shield. Our specification allows full deductibility of the debt expense, in all states–possibly by selling the tax-loss credits. It also assumes that the proceeds in liquidation, P, are fully taxable. This is equivalent to the asset being fully depreciated with a tax basis of zero.

Dividing equation 16 by $(1 - g \cdot t)$ returns to our original model with $\tau = t/(1 - g \cdot t)$. However, the “firm value” in our original model is now $\tilde{V} = V/(1 - g \cdot t)$. The optimal F_i^* is the same for either problem (with the remapped τ), because dividing by a constant does not change the optimum. Also, the rescaling of firm value does not affect any other equilibrium quantities such as P^* or h^*. However, we only solved the original model for $\tau \leq 1$. To preserve this, we also require that $t/(1 - g \cdot t) \leq 1$ or $t \leq 1/(1+g)$.

The sign of most of our comparative statics are preserved, but the quantitative plots (numbers) have to be remapped. For comparative statics with respect to η and ϕ, the comparative statics are the same as the old model with the remapping $\tau = t/(1-g\cdot t)$. The comparative statics with respect to the new argument t can be found with the chain rule where $\tau(t) = t/(1-g\cdot t)$:

$$\frac{\partial F(\tau(t))}{\partial t} = \frac{\partial F(\tau(t))}{\partial \tau}\cdot\frac{\partial \tau(t)}{\partial t} = \frac{\partial F(\tau(t))}{\partial \tau}\cdot\frac{1}{(1-g\cdot t)^2}$$

We have $\frac{\partial F(\tau(t))}{\partial \tau}$ from our original model, so the sign is preserved. It is still the case that the debt face value is increasing in t. The same chain rule argument works for our other equilibrium quantities, *except* where we use V^* explicitly. This is V^* and D^*/V^*.

The $\partial V^*/\partial t$, where $V^* \equiv \tilde{V}^*\cdot(1-g\cdot t)$, is

$$\begin{aligned}\frac{\partial V^*}{\partial t} &= (1-g\cdot t)\cdot\frac{\partial \tau(t))}{\partial t}\cdot\frac{\partial \tilde{V}^*(\tau(t))}{\partial \tau} - g\cdot\tilde{V}^*(\tau(t))\\ &= \frac{1}{1-g\cdot t}\cdot\frac{\partial \tilde{V}^*(\tau(t))}{\partial \tau} - g\cdot\tilde{V}^*(\tau(t))\end{aligned}$$

Although $\partial\tilde{V}^*/\partial\tau$ in our original model is always positive, $\partial V^*/\partial t$ is *not* generally positive, especially when $g \approx 1$. Instead, and somewhat surprisingly, $\partial V^*/\partial t$ becomes ambiguous: It is now negative for low tax rates, but $\partial V^*/\partial t$ is still positive for high tax rates in equilibrium. The top left of Figure 10 shows that this is not an obscure region, but a widespread phenomenon for high tax rates. This is mostly due to the fact that, in this region, the face value of debt exceeds the expected EBIT of the firm. The firm expects to receive more in tax-loss credits than it expects to have to pay out in taxes.

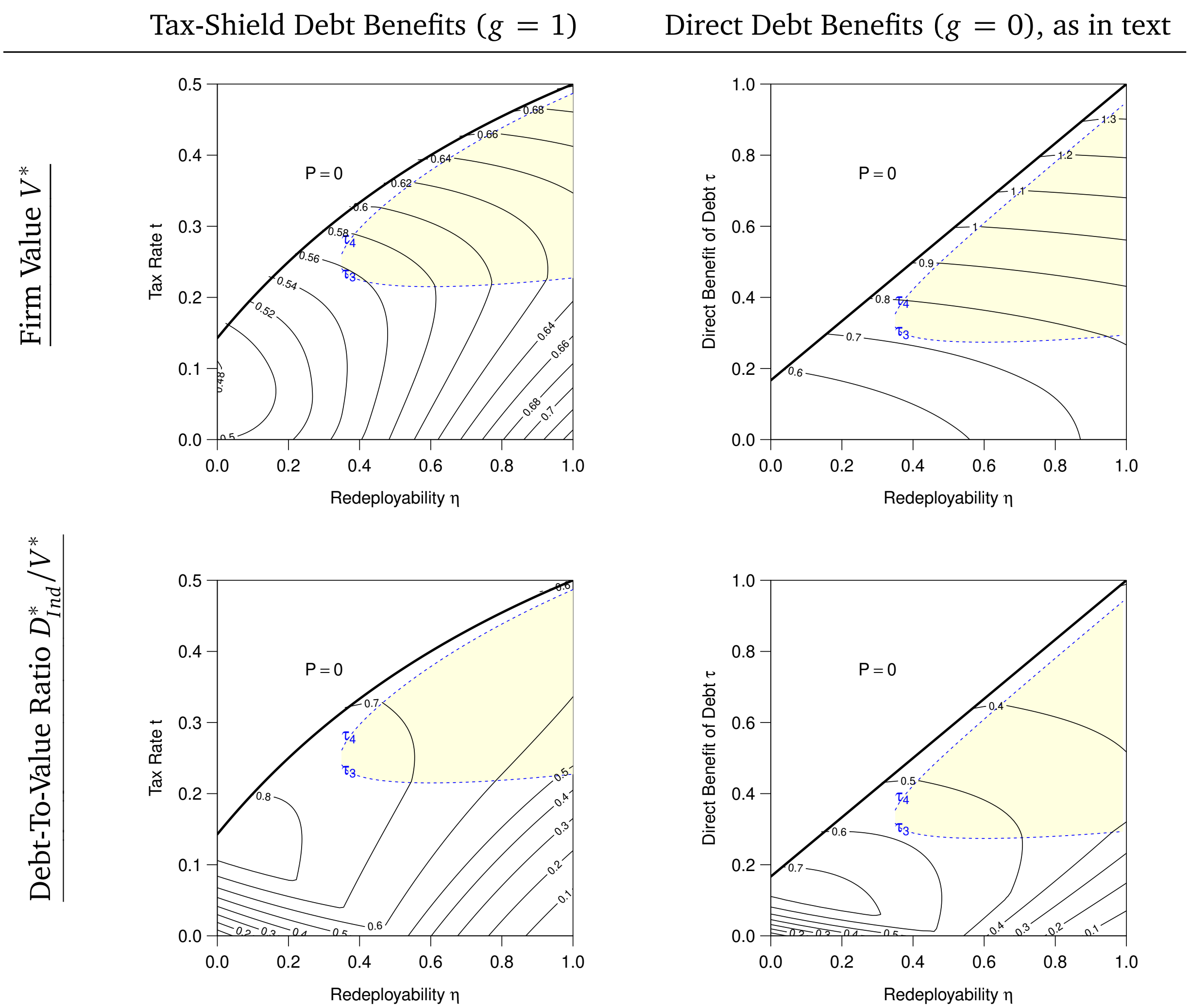
With even V^* being ambiguous in t, it is a lesser surprise that D^*/V^* also retains the ambiguity in both t and τ. This can be seen as follows: Let what we have be $a(\tau) = D^*(\tau)/\tilde{V}^*(\tau)$ and what we want be $b(t) = D^*(\tau(t))/V^*(t) = D^*(\tau(t))/(\tilde{V}^*(\tau(t))(1-g\cdot t)) = a(\tau(t))\cdot 1/(1-g\cdot t)$. Then

$$\begin{aligned}\frac{\partial D^*(\tau(t))/V^*(\tau(t))}{\partial t} = \frac{\partial b}{\partial t} &= a'(\tau(t))\cdot\frac{\partial \tau(t)}{\partial t}\cdot\frac{1}{1-g\cdot t} + a(\tau(t))\cdot\frac{g}{(1-g\cdot t)^2}\\ &= a'(\tau(t))\cdot\frac{1}{(1-g\cdot t)^3} + a(\tau(t))\cdot\frac{g}{(1-g\cdot t)^2}\end{aligned}$$

Thus, D^*/V^* is not necessarily decreasing in actual tax rates, either.

In sum, our model retains all the same comparative statics, regardless of the source of the debt benefits, except where we discuss it in our paper—specifically, in the aforementioned $\partial V^*/\partial \tau$ case. (Although the quantitative regions can also change, none change so dramatic as to undo or now deserve a “rare” designation.)

Figure 10: Value and Debt-Ratio When Benefits are Tax Shields



Explanation: These are contourplots, as in the main text.

Interpretation: When there are taxes and debt benefits derive from the tax shield, value becomes ambiguous in the tax rate.