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Authors: Xiaoshuai Fan, Ying-Ju Chen, Christopher S. Tang

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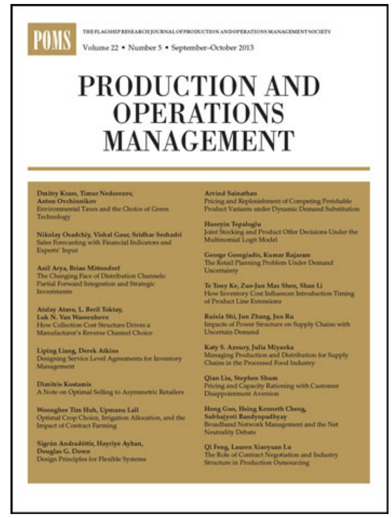

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To Bribe or Not in a Procurement Auction under Disparate Corruption Pressure*

Xiaoshuai Fan (Corresponding author)† Ying-Ju Chen‡ Christopher S. Tang§

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Abstract

We examine a (large) manufacturer's bribery decision (to bribe or not to bribe) arising from a procurement auction under "disparate corruption pressure" when another (small) manufacturer is known to offer the auctioneer (i.e., the intermediary) a bribe in exchange for the "right of first refusal". We discover that the large manufacturer should refuse to pay bribes at all times in order to prevent from leaking its private cost information to the small manufacturer and prevent from intensifying the competition. However, even when the large manufacturer is disadvantaged for refusing to bribe, we show that it can benefit from this corrupted auction when the difference in production efficiency or the bribe is high so that the "positive force" (i.e., cost advantage) derived from the right of first refusal dominates the information disadvantage. Hence, under a specific condition, the large manufacturer has no incentive to expose the collusion between the intermediary and the corrupt manufacturer. Such a "silence tactic" provides a plausible explanation for the prevalence of corrupt auctions in practice.

Keywords: Procurement auction, Right of first refusal, Disparate corruption pressure, Collusion.

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†School of Business, Southern University of Science and Technology, Shenzhen 518055, Guangdong, China; e-mail: fanxs@sustech.edu.cn.

‡School of Business and Management & School of Engineering, The Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong; tel: +852-2358-7758; fax: +852-2358-2421; email: imchen@ust.hk.

§UCLA Anderson School, 110 Westwood Plaza, Los Angeles, CA 90095, USA; e-mail: chris.tang@anderson.ucla.edu.

1 Introduction

Corruption and bribery arising from procurements are rampant in developing countries. The temptation to pay and receive bribes and the difficulties of identifying corrupted procurement processes are two major causes for corruption and bribery. According to the World Bank, bribery is involved in roughly $1.5 trillion government procurement contracts, and the corrupted process in (public/private) procurement has received attention from governments and non-profit organizations. For example, OECD Anti-Bribery Convention,[1] CoE (Council of Europe) Convention on Corruption,[2] and UNCAC (United Nations Convention against Corruption),[3] are committed to combating corruption in procurement. Also, non-government organizations, such as GIACC (Global Infrastructure Anti-Corruption Center),[4] are making substantial efforts in providing valuable resources to help private sectors and government to understand, identify, and prevent corruption. According to CMDA/GRS Governance and Corruption Study published by the World Bank,[5] enterprises often face "disparate corruption pressures". Figure 1 displays the illicit requirement of payment when companies use government services. This study reports the experiences of 600 managers and 590 public officials in Sierra Leone published by World Bank, which reveals that small enterprises are more frequent and more likely to "offer" a bribe.[6]

Observe from Figure 1 that small scale companies are under stronger pressure to pay bribes than large companies. Reasons for this phenomenon are diverse. For example, the small manufacturers are eager to win the contracts to survive in the market, or they have lower bargain power relative to the intermediary, and hence they have to pay bribes to participate in the procurement auction. The issue of "disparate corruption pressure" is common in procurement auctions, but it has not been examined in the research literature. This observation motivates us to make an initial attempt to examine its impact on the bribery decision (to bribe or not to bribe) and bidding strategies (how much to bid).

[1]OECD Anti-Bribery Convention was adopted in 1999. It communalizes bribery of foreign officials and prohibits accounting manipulation. More details are available at `https://www.oecd.org/daf/anti-bribery/ConvCombatBribery_ENG.pdf`.

[2]It entered into force in 2002. More details are available at `http://www.coe.int/en/web/conventions/full-list/-/conventions/treaty/173`.

[3]UNCAC was adopted in 2003 by more than 160 state signatories. It calls for to adopt preventive and punitive measures against corruption. More details are available at `https://www.unodc.org/unodc/en/treaties/CAC/`.

[4]See `http://www.giaccentre.org/`.

[5]Readers are referred to `http://siteresources.worldbank.org/INTANTICORRUPTION/Resources/383901-1317672198081/sl_gacreport.pdf`.

[6]Since the corrupt auction involves covert operations and no participants (bidders) will frankly admit that they have been involved in the corrupt auction and paid bribes to the intermediary and won contracts that they do not deserve. Hence, it is difficult to collect data regarding the bribery behavior of bidders with different scales. However, data related to government corruption is public, and companies are willing to report the fact that they need to pay bribes in order to get some services. Hence, we use the government data to reveal the phenomenon that the bribe pressure is diverse among different scales of companies.

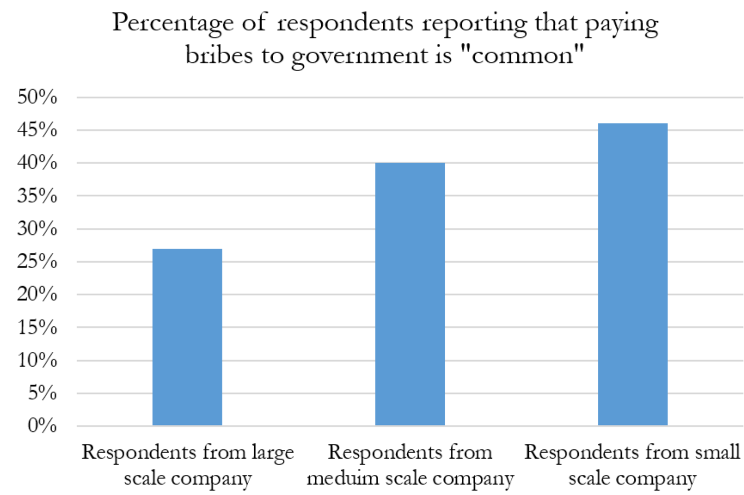

Figure 1: Enterprise's experience in Sierra Leone (referring to CMDA/GRS Governance and Corruption Study page 77 published by World Bank)

In many public/private procurement auctions, the common practice is using the sealed-bid first-price auction. In spite of the advantages (e.g., inducing truth-telling equilibrium, and generating allocation efficiency) of the sealed second-price auction, such procedures are extremely rare in practice (see Rothkopf et al. (1990), Ausubel et al. (2006)) for several reasons.[7] Therefore, to study the bribery decision (to bribe or not to bribe) and bidding strategies in the corrupt auction, we examine first-price auction with asymmetric bidders. However, based on the auction literature, it is well-known that the optimal bidding strategy of the first-price asymmetric auction (with multiple bidders and general cost distributions) is analytically unsolvable, even though it can be characterized by a system of ODEs (ordinary differential equations) (see Marshall et al. 1994, Maskin and Riley 2000, Lee 2008, etc). The analytical solution is tractable only for two-bidder case with uniform distributions (see Lee 2008 and Kaplan and Zamir 2012).

The above observations motivate us to focus on an auction that involves two asymmetric bidders, each of whom incurs a cost that is drawn from a uniform distribution. Also, there is a corrupt intermediary (i.e., auctioneer) who offers the "*right of first refusal*" in exchange for bribes (i.e., kickbacks). Specifically, when a bidder offers bribes for the right of first refusal, the intermediary will reveal other submitted bids to the briber (i.e., the "corrupt" manufacturer who bribes the intermediary) so that the briber can decide whether to win the contract by submitting a winning bid that is slightly below the current lowest bid. Corruption could take different forms. The reason why we focus on the "right of first refusal" is that it is commonly used in the practice that the intermediary

[7]For example, the second-price auction may induce collusion among bidders; and bidders may be reluctant to reveal their true values, especially when that information may adversely affect the subsequent negotiations or transactions.

offers "the right of first refusal" in return for a payment from a briber.[8] The widely usage of the "right of first refusal" as a form of favoritism is because it is easy to implement. Auctioneers only need to decide on which bidder to favor. At the same time, bidders can easily take the "right of first refusal" into consideration when developing their bidding strategies.

Based on the phenomenon of "disparate corruption pressure", in the base model, we assume that a small manufacturer is a committed briber and a large manufacturer is a non-committed briber. (In Section 7, we shall examine the robustness of our results with respect to the assumption that small manufacturer is a committed briber.) Due to the scale efficiency, the small manufacturer commonly has an *ex ante* higher production cost relative to the large manufacturer. Hence, we consider the small manufacturer as an inefficient one ("weak" manufacturer) and the large manufacturer as an efficient one ("strong" manufacturer).[9] Hereafter we use W (abbreviated from "weak") to denote the inefficient small manufacturer and use S (abbreviated from "strong") to represent the efficient large manufacturer.

Admittedly, the two-bidder, uniform distribution setup is a simplification of the practical situation (with multiple bidders and general cost distributions) and constitutes an obvious limitation of our model. Nevertheless, this particular modeling framework is well-known to be the only tractable setting in the economics literature for two asymmetric bidders and is well-known to be a good starting point for understanding the nature of bribery in the procurement context (see Lee 2008 and Kaplan and Zamir 2012). Furthermore, our work represents the first attempt to analyze an endogenous bribery decision which creates a signaling problem in the procurement auction. Hence, in addition to addressing the research questions in the bribery procurement context, our work makes a theoretical contribution to the auction literature.

By examining the large manufacturer's bribery decision (i.e., to bribe or not to bribe) and the bidding strategies of both manufacturers, we attempt to answer the following questions:

1. What is the large manufacturer's bribery decision (i.e., to bribe or not to bribe) and the bidding decisions of both manufacturers?
2. How would different forms (i.e., proportional and fixed) of bribe affect the optimal bribery decision of the large manufacturer?
3. In the event when the large manufacturer decided not to bribe, should he expose this corrupted auction process?

[8] Readers can refer to Noonan (1987) for a history of bribery and Rose-Ackerman (1975) for a general survey of corruption. Also, readers can refer to Lee (2008) and Burguet et al. (2007) which record several practical instances for the implementation of the "right of first refusal" in corrupt auctions.

[9] Note that these two manufacturers have the same production efficiency is just a special case of our work, which is analyzed in Section 6.1.

Three key findings can be summarized as follows. First, we find that it is optimal for the manufacturer S to refuse bribery at all times in order to prevent leaking its cost information to the manufacturer W. The underlying reason is that the bribery decision is a precise signal of S's production cost. When S agrees to offer a bribe, W can precisely infer S's equilibrium bribery decision and hence update W's belief on S's production cost. The *ex ante* weaker in the auction where S's cost distribution has been updated will bid more aggressively, which drives S's profit down. Hence, to avoid leaking cost information and intensifying the competition, the best strategy for S is to refuse bribery at all times, irrespective of the realization the production cost. Second, under S's optimal bribery decision, we observe that "the right of first refusal" has two separate effects on competition: a) it tilts the competition in favor of W through revealing S's bid to W, and b) it strengthens S's cost advantage due to the fact that W has to pay bribes when W wins the contract. The former induces a "negative force" (information disadvantage) on S which deteriorates S's competitive power against W through leaking S's bid; but the latter provides a "positive force" (cost advantage) through raising W's effective cost to enhance S's profit. Our analysis reveals that, when the difference in the production efficiency or the bribe is high, the "positive force" derived from the right of first refusal dominates its "negative force". Hence, manufacturer S benefits from the corruption between W and the intermediary. This result directly leads to our third point: even though the large manufacturer had not been involved into the corruption and was disadvantaged by the intermediary, S may benefit from such a corrupted auction. Hence, there is no incentive for S to expose such illegal practice, which will encourage the popularity of corruption in auctions. This result can be viewed as a plausible explanation for the ubiquitous existence and concealment of corrupt procurement auctions.

The structure of this paper is as follows. We begin with a review of the literature in Section 2. In Section 3, we introduce the model settings and the benchmark case of an auction without bribery. The optimal bidding strategy and bribery decision are determined in Sections 4 and 5, respectively. Subsequently, to investigate the impact of corruption, we compare S's expected payoffs associated with the corrupt auction and the non-corrupt auction in Section 6. Section 7 examines the robustness of our results with respect to the assumption that the small manufacturer is a committed briber. Section 8 extends our model by considering a fixed bribe case to study the impact of different forms of bribe on the optimal bidding and bribery decisions. Also, in Section 9, we investigate the case where the intermediary offers the right of first refusal back and forth between bidders when both bidders bribe. Finally, conclusions are shown in Section 10.

2 Literature Review

This paper is related to the operations management literature in auction and procurement contract (e.g., Balseiro and Gur (2019), Candogan et al. (2015), Duenyas et al. (2013), Chen (2007), Wan and Beil (2014), Wan and Beil (2009)). Duenyas et al. (2013) showed that a simple modified version of the standard open-descending auction is optimal to obtain supplies. Wan and Beil (2009) combined with a request-for-quotes (RFQ) reverse auction and supplier qualification screening to study the optimal sourcing contract. Our work aims to combine the corrupt agents (both the suppliers and the intermediaries) with a first-price asymmetric procurement auction to investigate supplier's economic incentives in bribery decision. Singh (2017) is a closely related paper which analyzed the corruption in a competitive market. However, our work distinguishes it from focusing on the auction mechanism, which is commonly used in the procurement context.

Furthermore, our study is related to the economics literature in corrupt procurement auctions. This research stream analyzes the bidding behavior and buyer's/seller's expected profit in the presence of corruption, and investigates how to improve the allocation distortion and buyer's payoff by designing selection rules and/or selecting certain reserve prices (Laffont and Tirole 1991, Burguet and Che 2004, Lengwiler and Wolfstetter 2000). The first sub-stream examines the manipulation of quality measurements.[10] Laffont and Tirole (1991) assume that the auctioneer has the "manipulation power" to distort assessment standards to favor a specific bidder. Based on this structure, Burguet and Che (2004) design an optimal scoring rule for the project owner to limit the manipulation power of an auctioneer. The second sub-stream is "bid rigging" (Arozamena and Weinschelbaum 2009, Burguet et al. 2007, Ingraham 2005, Bikhchandani et al. 2005, Choi 2009) where auctioneer reveals other bidders' quotations in order to help the pre-determined (due to bribery or favoritism) "favored" bidder to win the contract by submitting an advantageous bid. The third sub-stream arises from a setting in which the favored bidder is not pre-determined. After receiving all bids, the auctioneer will approach to the winning bidder by allowing him to "adjust" his bid so that he can win at a more favorable term. In exchange for this favoritism, the bidder has to offer a bribe (c.f, Menezes and Monteiro 2006, Lengwiler and Wolfstetter 2010). Another way to ascertain the favored bidder is through "bribe competition" (c.f. Compte et al. (2005), Burguet and Che (2004)): the auctioneer will favor the bidder who offers the highest bribe. In Compte et al. (2005), they prove that under the unbounded bribe competition, the results regarding bidders' expected payoffs and allocation outcome will be the same as the first-price auction without bribery.

The above research focuses on analyzing the interaction between the optimal bidding strategy and the corruption manner (bid rigging or manipulating quality assessment) *and* the influence

[10] Here we only focus on the literature in which auctioneers are involved in corruption. In fact, there exists corruption caused by bidders' collusion through price fixing (Porter and Zona 1993) or internal bribe (Eső and Schummer 2004).

of corruption on the contract allocation. Also, in this stream of corrupt auction literature, it is commonly assumed that *all bidders* are unethical as committed bribers so that the bribery decision (to bribe or not to bribe) is moot. The main difference between our work and the way corruption is modeled in the above literature is that we do not assume that bidders are identical in terms of the bribery decision and allow the heterogeneity in bidders' bribery decisions due to the "disparate corruption pressure". To capture such a phenomenon, we construct an asymmetric setting under which the small manufacturer is a committed briber but not the large one. We intend to examine the bribery decision and the bidding strategy of manufacturers arising from procurement auction under "disparate corruption pressure".

Our study involves the analysis of an asymmetric auction arising from not only the *ex ante* asymmetry in production efficiency and but also the fact that the small manufacturer is a committed briber but not the large manufacturer. While the large manufacturer observes no information about the small manufacturer's decision, the small manufacturer (i.e., the committed briber) can first observe the large manufacturer's bribery decision (and its bid if it refuses to bribe), and use this information to update its prior belief about the large manufacturer's cost. This asymmetric information creates the backdrop for an asymmetric auction. Lee (2008) and Burguet et al. (2007) studied *ex ante* asymmetric bidders along with "pre-determined" favored bidder. Unlike Lee's model, there is no favored bidder if the large manufacturer also agrees to bribe. This also gives rise to *ex post* asymmetry where the small manufacturer updates it belief through interpreting the large manufacturer's bribery decision, and the analysis is more complex due to the dynamic interaction between the bribery decision and the bidding strategy. Overall, we find that the large manufacturer should refuse bribery so that it can safeguard its cost information.

3 Model Preliminaries

We consider a procurement supply chain comprising a procurer, an intermediary (i.e., a procurement service provider) who acts as an auctioneer and two bidders S and W. Manufacturer S is a strong manufacturer and has a large market share in the supply market. In contrast, W is a weak manufacturer and has a small market share. Incorporating the phenomenon of "disparate corruption pressure" in the practice, we assume that W is a committed briber and S is a non-committed briber. That is to say, W is committed to offering bribes to the intermediary in order to exchange for "the right of first refusal", but S could strategically determine its bribery decision. (In Section 7, we shall check the rationality of this set up by endogenizing the small manufacturer's bribery decision.) The intermediary conducts a sealed-bid first-price auction on behalf of the procurer, and the lowest bidder wins the procurement contract from the procurer. We do not consider the second-price auction since it is rare in practice. Furthermore, in our context with the right of first

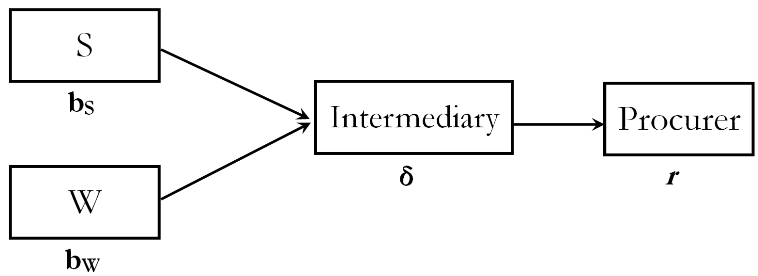

Figure 2: Structure of the sealed-bid first-price auction

refusal, the second-price auction does not admit bribery. Hence, the bribery decision becomes trivial. Therefore, in the sequel, we focus on the first-price auction.

The auction structure is exhibited in Figure 2, where b_S, b_W are the bids submitted by manufacturers S and W, respectively. The reserve price r is set by the procurer and the procurer will sign a contract with the winning bidder only when the winning bid $\min\{b_S, b_W\} \leq r$. We use δ to denote the "bribe share" so that the bribe to be paid to the intermediary is equal to the bribe share δ times the winning bid. The bribe share δ is determined by an "open secret" in the market. Hence, bribers know how much they should pay to the intermediary, and intermediary knows how much they could receive from the briber. (The reader is referred to Lai and Tang (2014) which states that the bribe share is around 3% $\sim$ 5% of the bid.) Anyone who breaks the "secret rule" will be ostracized or retaliated by losing "reputation" in this corrupted market. To check the robustness of results, in Section 8, we consider the case where the bribe is a fixed amount instead of a proportion of the winning bid; and in Appendix A, we analyze the case where S and W pay different bribery fees.

Intermediary. The intermediary charges a bribe which is proportional to the contract price from manufacturers and the portion (i.e., the bribe share δ) is exogenously given. In return for the bribe, the intermediary will offer the briber the *right of first refusal*: the briber can receive its opponent's bid before submitting its own bid (or alternatively, the briber can resubmit its bid after observing its opponent's bid).

In our model, if S declines to bribe, then the intermediary will offer the right of first refusal to W. However, if S agrees to bribe, in our base model, we focus on the case where the intermediary has no incentive to reveal S's (or W's) bid to W (or S) because such revelation will intensify competition, which will lower the amount of the winning bid and further the corresponding bribe. Therefore, once S agrees to bribe, in our base model W and S compete fairly in the procurement auction but the winner has to pay a bribe to the intermediary as promised. Here, "fair" refers to that when S agrees to offer a bribe, no one will be awarded the right of first refusal but it does not mean that it is a fully fair competition. This is because when S agrees to offer a bribe, W will not receive any bid information about S from the intermediary; hence, W will know that S has offered a bribe to the intermediary as well. This bribery decision of S is a precise signal of S private cost and therefore, W can update its belief regarding S's cost distribution, which grants information advantage to the manufacturer W. However, in Section 9, we investigate the case where the intermediary offers the right of first refusal back and forth between bidders when both bidders bribe.[11]

Note that to avoid the situation that the briber reneges on the promise of paying bribes after winning the contract, the intermediary accepts the briber's bid (and then submit it to the procurer) upon the requirement of receiving the promised bribe in advance. That is to say, knowing the fact that the briber will win the contract, the intermediary will extract the bribe before going through with the auction procedure.

Manufacturers. There are two bidders (S and W) participating in the procurement auction, where W is known to be a committed briber (who will bribe the intermediary), and S is known to be a non-committed briber (who may or may not bribe the intermediary). This common knowledge implies that S knows W will get the right of first refusal from the intermediary unless S is willing to bribe the intermediary. At the same time, W knows that: (a) if S refuses to bribe, then W will get the right of first refusal; and (b) if S agrees to bribe, then W has to compete fairly with S. [12]

We assume that the production cost of the strong manufacturer S follows a uniform distribution over $[0, 1]$, i.e., $c_S \sim U[0, 1]$; and the production cost of the weak manufacturer W follows a uniform distribution over $[a, 1 + a]$, i.e., $c_W \sim U[a, 1 + a]$, where $a \in (0, 1)$ captures the production inefficiency of W relative to S. We assume that these two distributions are common knowledge. We use $F_i(\cdot)$ and $f_i(\cdot)$, $i \in [S, W]$, to denote the CDF and PDF associated with the cost distributions of S and W. The actual production costs c_S and c_W are private information.

Procurer. Corruption in auction should never be an issue without intermediation. However, intermediary is indispensable since procurers often have little knowledge about different manufacturers located around the world. As such, even though it is easy to conduct an auction to collect bids from manufacturers, it is inefficient and ineffective for procurers to identify and certify different contract manufacturers in terms of their production capabilities (in terms of quantity and quality), on time delivery, reliability, safety, etc. For these reasons, procurers have to rely

[11] In Appendix B, we consider a "coalition" case where both manufacturers offer a bribe and collude with the intermediary to push up the contract price to the reserve price r in order to extract the highest price from the procurer.

[12] Because we assume away the bribery competition between S and W, when both S and W offer bribes, the intermediary will not tilt the competition to benefit either manufacturer; otherwise, it will drive bids down and hence decrease the bribe the intermediary can get. Even if we incorporate the bribery competition, our main results will continue to hold.

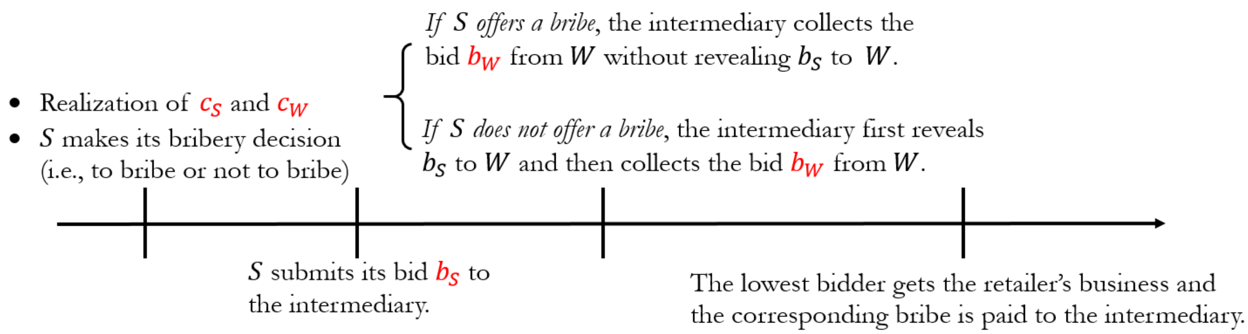

Figure 3: Timeline of the sequential auction

on the intermediary (i.e., procurement service providers) to provide end-to-end solutions. The procurer can set a reserve price r that is known by the intermediary and manufacturers and will reject the contract once the bid is higher than the reserve price. To be effective, the reserve price should satisfy $a < r < 1 + a$, where the parameter a will be elaborated when we introduce the manufacturers' cost distribution.

Timeline. Next, we summarize the timeline of the events. Due to the presence of disparate corruption pressure as presented in Section 1, we consider the case when W is a committed briber. Because W is a committed briber, the resulting bribery game (i.e., to bribe or not to bribe) has to be sequential and S has to decide to bribe or not to bribe. In Section 10, we discuss the optimal bribery strategy when the bribery game is simultaneous when disparate corruption pressure is absent.

However, the auction game (i.e., the bidding decisions of both bidders) can be sequential or simultaneous. To begin, let us first consider the case when S and W submit their bids sequentially and the sequence of the events is depicted in Figure 3:

1. S and W observe their own private production costs c_S and c_W, respectively; S makes its bribery decision (to bribe or not to bribe) and submits a bid b_S to the intermediary.

2. (a) If S agrees to bribe, then W submits its bid b_W (without observing b_S). (b) If S refuses to bribe, then W submits its bid b_W (after observing b_S) so that W receives the right of first refusal.

3. The intermediary presents the submitted bids (b_S, b_W) to the procurer, and the contract is offered to the lowest bidder.

Next, let us consider the case when S and W submit their bids simultaneously and the event of sequence associated with this simultaneous auction is described in Figure 4:

1. S and W observe their realized production costs c_S and c_W, respectively; S makes its bribery decision (to bribe or not to bribe).
2. S and W submit their bids to the intermediary simultaneously.
3. (a) If S offers a bribe, then the procurement contract is awarded to the lowest bidder;
   (b) If S refuses to bribe, then the intermediary has incentive to reveal S's bid to W and allows W to "revise" its bid if so desired. Finally, the lowest bidder wins.

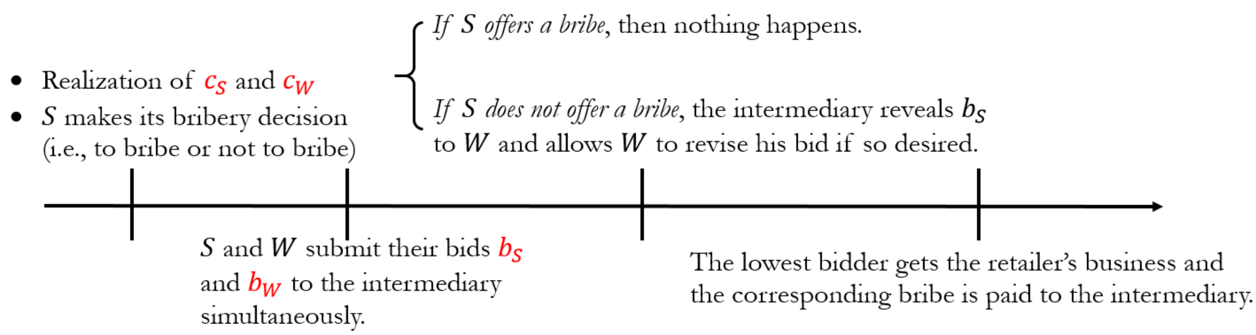

Figure 4: Timeline of the simultaneous auction

In the simultaneous auction as shown in Figure 4, W has to submit an "initial bid" first without observing S's bribery decision. However, in the sequential auction as depicted in Figure 3, W has already acquired the information of S's bribery decision before submitting its bid b_W. On the surface, it seems that these two sequences generate different information for W and may result in different outcomes. However, upon closer examination, both simultaneous and sequential auctions will lead to the same outcome (in terms of the winning bid and winning bidder). This is because in the simultaneous auction, W can "revise and resubmit" its bid if S refuses to bribe (i.e., W is awarded the right of first refusal by the intermediary because W is the only briber in this case.) Hence, when S refuses to bribe, W will receive the same information in the simultaneous auction (Figure 3) and the sequential auction (Figure 4). Next, if S agrees to bribe, W will receive the same information (i.e., no new information about S) and W will form the same belief on S's cost distribution in the simultaneous auction (Figure 3) and the sequential auction (Figure 4). By noting that the simultaneous auction will yield the same result as in the sequential auction, it suffices for us to focus on the sequential auction as depicted in Figure 3.

3.1 Benchmark case: optimal bidding strategy without bribery

We first examine a benchmark case that involves "no bribery" so that manufacturers participate in an asymmetric auction. We use $b_i(c_i), i \in [S, W]$, to denote the bidding strategies of the asymmetric auction where $c_S \sim U[0,1]$ and $c_W \sim U[a, 1+a]$. Also, we use $\gamma_S(c_S)$ ($\gamma_W(c_W)$) to represent the bidding strategies of the symmetric auction with two S (W). In this section, we will show some properties of the asymmetric auction and provide some results which will be used later.

Without corruption, the strong manufacturer S's problem is as follows:

$$\begin{aligned} \max_b \quad & (1 - F_W(b_W^{-1}(b)))(b - c_S) \\ s.t. \quad & b \le r; \end{aligned}$$

and the weak manufacturer W's problem is as follows:

$$\begin{aligned} \max_b \quad & (1 - F_S(b_S^{-1}(b)))(b - c_W) \\ s.t. \quad & b \le r. \end{aligned}$$

In the literature, Lebrun (1996) and Kaplan and Zamir (2012) have proved the following lemma.

Lemma 1. *The closure of the set of equilibrium bids is $[\underline{b}, \overline{b}]$ wherein both bidders have positive probability of winning.*

With the minimum bid $\underline{b}$ and the maximum bid $\overline{b}$, the optimal bidding strategy is characterized by the following pair of differential equations:

$$\begin{cases} \frac{f_W(b_W^{-1}(b))}{1-F_W(b_W^{-1}(b))}(b_W^{-1})'(b) = \frac{1}{b-b_S^{-1}(b)} \\ \frac{f_S(b_S^{-1}(b))}{1-F_S(b_S^{-1}(b))}(b_S^{-1})'(b) = \frac{1}{b-b_W^{-1}(b)} \end{cases} \tag{1}$$

with boundary conditions of:

$$b_W(a) = b_S(0) = \underline{b}; \text{ and } b_W(\overline{b}) = b_S(\min\{1, r\}) = \overline{b}. \tag{2}$$

Next, we prove some lemmas which demonstrate the properties of the asymmetric auction and also they will be used when we analyze the corrupt auction. Similar results have been illustrated in Maskin and Riley (2000). However, the differences between our work and Maskin and Riley (2000) are that we involve a reserve price in the auction and that we consider a procurement auction (i.e., "reverse auction") rather than an auction for selling products.

Lemma 2. *Considering the case where $c_S \sim U[0,1]$ and $c_W \sim U[a, 1+a]$, the equilibrium results have the following properties:*

1. *$F_S(b_S^{-1}(b)) > F_W(b_W^{-1}(b))$, for $b \in (\underline{b}, \bar{b})$;*
2. *$b_W^{-1}(b) > b_S^{-1}(b)$, for $b \in (\underline{b}, \bar{b})$;*
3. *$b_W^{-1}(b) > \gamma_W^{-1}(b)$, for $b \in (\underline{b}, \bar{b})$;*

4. $\gamma_S^{-1}(b) > b_S^{-1}(b)$, *for* $b \in (\underline{b}, \bar{b})$,

where $b_S^{-1}(b)$ *and* $b_W^{-1}(b)$ *are the optimal inverse bidding function of S and W in the asymmetric auction, and* $\gamma_W^{-1}(b)$ *(*$\gamma_S^{-1}(b)$*) is the optimal inverse bidding function in the symmetric auction with two W (S).*

From Lemma 2, we observe several results regarding the equilibrium bidding strategy of the asymmetric auction. The first statement of Lemma 2 indicates that the strong manufacturer S's bid distribution is first-order stochastically dominated by the weak manufacturer W. That is to say, W's bid is stochastically larger than S's bid. However, from the second statement, we know that at the same cost level, W will bid more aggressively (i.e., offer a lower bid) in order to compete with S. Hence, knowing that S is *ex ante* stronger, the manufacturer W, under the pressure of losing the contract with a high probability, will initiatively lower its marginal profit by offering a lower bid. Furthermore, the last two statements convey that the strong manufacturer S (the weak manufacturer W) bids less (more) aggressively when competing with a weak W (a strong S) than competing with another strong manufacturer S (weak manufacturer W).

4 Optimal Bidding Strategy with Bribery

We now model a corrupt procurement auction in which W is a committed briber, but S will decide strategically whether to bribe or not after observing its own cost c_S as depicted in Figure 3. Because S's bribery decision is "endogenously" dependent on its realized cost c_S, S's bribery decision contains a "signal" about S's production cost. Therefore, if S refuses to bribe, then W can observes S's bid b_S directly. However, if S agrees to bribe, W cannot observe b_S, but could use S's bribery decision to infer a more precise cost distribution regarding S. Specifically, when S agrees to offer a bribe, S submits its bid based on its prior belief on W's cost, i.e., $c_W \sim U[a, 1+a]$. However, while W can only observe that S has agreed to bribe, it can use this information to update his belief about S's cost. Hence, W can submit its bid based on its posterior belief about S's cost (which is denoted by a probability density function $h(c_S)$ to be defined in Section 4.2). Therefore, when S agrees to bribe, W and S engage in an "asymmetric" first-price sealed bid auction in which S believes that W's cost distribution is $U[a, 1+a]$, and W has an updated belief about S's cost, which follows a probability density function $h(c_S)$ that is different from that of $U[0, 1]$. By considering the fact that W observes S's bid if S refuses to bribe and believes S's cost distribution is $h(c_S)$ when S agrees to bribe, we can use backward induction to derive the optimal bribery decision for S as follows:

1. In Section 4.1, we study the optimal bidding strategies for each manufacturer when S refuses to bribe so that W could observe S's bid directly.

2. In Section 4.2, we focus on the equilibrium of S's bribery decision with the "threshold structure". Then, we investigate the optimal bidding strategies for each manufacturer for the case when S agrees to bribe. When S offers a bribe, an asymmetric auction occurs because of the informational advantage of W over S (because W knows S's cost c_S follows $h(c_S)$). Based on the threshold structure of S's bribery decision, we can derive W's posterior belief on S's production cost (i.e., $h(c_S)$), which is essential for determining the optimal bidding strategies for both manufacturers and S's expected revenue under such a bribery decision.

3. By comparing the expected revenues associated with the cases when S agrees and refuses to bribe, we determine S's optimal bribery decision in Section 5.

4.1 When S refuses to bribe

If the manufacturer S refuses to bribe in a corrupt auction, then the intermediary will favor W by disclosing S's bid b_S to W so that W can decide whether to outbid S and win the contract. First, let us consider the case when S is an unqualified bidder (i.e., $c_S > r$). In this case, depending on the value of c_W and accounting for the bribe δb_W to be paid to the intermediary, it is easy to check that W's optimal bid is:[13]

$$b_W^N = \begin{cases} r & \text{if } r > \frac{c_W}{1-\delta} \\ +\infty \text{ (i.e., do not bid)} & \text{if } r \le \frac{c_W}{1-\delta}. \end{cases} \tag{3}$$

Next, when S submits a bid $b_S \le r$, W determines its bid by solving the following optimization problem:

$$\max_{b_W}((1-\delta)b_W - c_W) \cdot \mathbb{1}_{b_W \le b_S}. \tag{4}$$

Accordingly, W's optimal bid b_W is:

$$b_W^N = \begin{cases} b_S - \epsilon & \text{if } b_S > \frac{c_W}{1-\delta} \\ +\infty \text{ (i.e., do not bid)} & \text{if } b_S \le \frac{c_W}{1-\delta}. \end{cases} \tag{5}$$

As stated in (3) and (5), we can conclude that: (a) if $\min\{b_S, r\} > \frac{c_W}{1-\delta}$, W is the winner and W's winning contract is established at the price of $\min\{b_S, r\}$; (b) if $b_S \le \min\{\frac{c_W}{1-\delta}, r\}$, S is the winner and the final contract price is b_S; and (c) if $r \le \min\{\frac{c_W}{1-\delta}, b_S\}$, then these two manufacturers are disqualified.

Anticipating W's best responses as stated in (3) and (5), S has to select his bid carefully. First, if S is unqualified with $c_S > r$, then S will not submit any bid and the issue is moot. However, if S is a qualified bidder with $c_S \le r$ and bids b_S that has $b_S \le \frac{c_W}{1-\delta}$, then S can infer from (5) that

[13] The superscript N indicates the case where the strong manufacturer S declines offering a bribe.

he will win the contract at price b_S. (Otherwise, S's bid $b_S > \frac{c_W}{1-\delta}$ and W will win the contract.) Therefore, S's optimal bidding decision is the solution to the following maximization problem:

$$\max_{b_S} \quad \mathbb{E}(\pi_S(b_S)) = (b_S - c_S)\Pr(b_S \le \min\{\frac{c_W}{1-\delta}, r\}). \tag{6}$$

By noting that the problem is only relevant when S is a qualified bidder with $c_S \le r$ (so that $b_S \le r$), the problem shown in (6) can be simplified as:

$$\begin{aligned} \max_{b_S} \quad & \mathbb{E}(\pi_S(b_S)) = (b_S - c_S)\Pr(b_S \le \frac{c_W}{1-\delta}) = (b_S - c_S)(1 - F_W((1-\delta)b_S)) \\ s.t. \quad & b_S \le r. \end{aligned} \tag{7}$$

Let $J(x) = x - \frac{1-F_W(x)}{f_W(x)} = 2x - a - 1$ where $x \sim U[a, 1+a]$. The function $J(\cdot)$ is known as "virtual valuation" in the auction literature. In this case, the first-order condition of (7) satisfies:

$$\frac{d\pi_S(b_S)}{db_S} = [1 - F_W((1-\delta)b_S)] - (1-\delta)(b_S - c_S)f_W((1-\delta)b_S) = f_W((1-\delta)b_S)[(1-\delta)c_S - J((1-\delta)b_S)] \tag{8}$$

and the SOC (second-order condition) of (7) satisfies:

$$\frac{d^2\pi_S(b_S)}{db_S^2} = -(1-\delta)f_W((1-\delta)b_S)J^{'}((1-\delta)b_S) < 0,$$

where the inequality is due to the fact that $J^{'}(\cdot) > 0$. Hence, the SOC confirms that $\pi_S(b_S)$ is concave so that we can obtain the optimal b_S^* through FOC in (8), getting

$$b_S^* = \frac{J^{-1}((1-\delta)c_S)}{1-\delta}.$$

By using the fact that $J(x) = 2x - a - 1$, we get:

Proposition 1. *When S refuses to bribe, its optimal bidding strategy should satisfy:*

$$b_S^N(c_S) = \begin{cases} \frac{c_S}{2} + \frac{1+a}{2(1-\delta)} & \text{if } 0 < c_S \le (2r - \frac{1+a}{1-\delta})^+ \\ r & \text{if } (2r - \frac{1+a}{1-\delta})^+ < c_S \le r \\ \infty (\text{i.e., do not bid}) & \text{if } c_S > r. \end{cases} \tag{9}$$

Also, manufacturer S's bidding strategy $b_S^N(c_S)$ increases in the reserve price r and in the bribe share δ.

Consider the case without the reserve price (corresponding to the case when $r = \infty$), S's bid $b_S^N(c_S)$ is increasing in its cost c_S and also in the bribe share δ. This is because W's competitive power is weakened due to the bribe paid to the intermediary. Hence, S can afford to submit a higher bid without markedly lowering its winning probability as the bribe share δ increases. Proposition 1 also shows that the reserve price r will exert pressure on S to submit a lower bid.

4.2 When S agrees to bribe

As explained in Section 3, when S agrees to bribe, W and S will compete fairly: the lowest bidder wins and pays a bribe to the intermediary as promised. Also, because S observes nothing, S's belief on W's cost remains as $U[a, 1+a]$. However, as depicted in Figure 3, W does observe S's bribery decision (i.e., S agrees to bribe) but not S's bid. Because this information contains a signal about S's cost structure, W can use this signal to update its belief on S's cost from $U[0,1]$ to $h(c_S)$ and use this updated belief to determine its bid. We focus on the equilibrium of S's bribery decision with the "threshold structure", i.e., S agrees to offer a bribe when $c_S \in [L, P]$ where $0 \leq L < P \leq (1-\delta)r$.[14] Hence, the updated distribution of S's production cost when S agrees to offer a bribe is characterized by $c_S \sim U[L, P]$ and

$$h(c_S) = \frac{1}{P-L}, \; H(c_S) = \frac{c_S - L}{P-L}, \text{for } c_S \in [L, P].$$

Therefore, if W does not receive S's bid from the intermediary, then W will know that S has already agreed to bribe and that he has to compete with S "fairly". Also, W can infer that $c_S \in [L, P]$, and can derive a posterior belief on the S's cost distribution (i.e, the PDF and CDF of c_S is $h(c_S)$ and $H(c_S)$).

To examine the property of the optimal bidding strategy for both manufacturers who engage in the asymmetric auction arising from a situation when S agrees to bribe, we shall apply the result from lemma 1 and use the interval $[\underline{b}^Y, \overline{b}^Y]$, where $a \leq \underline{b}^Y \leq \overline{b}^Y \leq r$, to represent the set of equilibrium bids for both manufacturers.[15] Manufacturer S and W's equilibrium bidding strategies in the case where S agrees to offer a bribe are represented by $b_S^Y(c_S)$ and $b_W^Y(c_W)$, respectively.

From the equilibrium of the bribery decision defined by $[L, P]$ and the equilibrium bidding strategy bounded by the interval $[\underline{b}^Y, \overline{b}^Y]$ when S agrees to offer a bribe, P is the upper limit of c_S at which S agrees to bribe and the bid $\overline{b}^Y$ satisfies $b_S^Y(c_S = P) = \overline{b}^Y$. Also, by definition of P that is the upper limit of c_S at which S agrees to bribe, S should be indifferent between offering a bribe and not offering a bribe when $c_S = P$. Furthermore, by definition of $\overline{b}^Y$ that is the highest bid, W should be indifferent between bidding $\overline{b}^Y$ and not to bid. By noting that W's payoff is $(1-\delta)\overline{b}^Y - c_W$ when he bids $\overline{b}^Y$and wins and W's payoff is 0 when he decides not to bid, we can conclude that $b_W^Y(c_W = (1-\delta)\overline{b}^Y) = \overline{b}^Y$. In summary, the highest bid $\overline{b}^Y$ is directly related to the

[14]The condition $P \leq (1-\delta)r$ ensures that the manufacturer S who agrees to bribe is a qualified bidder. In other words, his total cost including the production cost and the amount of bribe does not exceed the reserve price r in the asymmetric auction. This equilibrium structure (i.e., $[L, P]$) will be analytically derived in Section 5.

[15]The superscript Y indicates the case where the strong manufacturer S agrees to offer a bribe.

cost c_S and c_W as follows (i.e., boundary conditions of the equilibrium bidding strategy):

$$b_S^Y(c_S = P) = \bar{b}^Y, \text{ and } b_W^Y(c_W = (1-\delta)\bar{b}^Y) = \bar{b}^Y. \tag{10}$$

Applying the same method developed in Kaplan and Zamir (2012), we can derive the exact value of the upper bound $\bar{b}^Y$ which is stated in Lemma 3.

Lemma 3. *The exact value of the upper bound $\bar{b}^Y$ can be described as follows:*

1. *If $r > \frac{1+a+P}{2(1-\delta)}$, then $\bar{b}^Y = \frac{1+a+P}{2(1-\delta)}$;*
2. *If $r \le \frac{1+a+P}{2(1-\delta)}$, then $\bar{b}^Y = r$,*

where P is the highest cost for c_S such that S agrees to bribe, and $\bar{b}^Y$ is the highest bid associated with the case when $c_S = P$.

Consider the asymmetric auction where $c_S \sim U[L, P]$ and $c_W \sim U[a, 1+a]$. Before deriving the equilibrium structure of $[L, P]$, we cannot check whether the cost distribution of manufacturer W stochastically dominates that of S or not. Hence, it is even impossible to analyze the properties of the equilibrium strategy in this case. Next, we will elaborate the optimal briery decision for S, i.e., deriving the cost stricture $[L, P]$ in which S agrees to bribe.

5 Optimal Bribery Decision

In this section, we aim to determine S's optimal bribery decision: to bribe or not to bribe. To do so, we need to first determine S's conditional expected payoff associated wih the cost realization c_S for the case when he refuses to bribe and for the case when he agrees to bribe. Then we can determine S's optimal bribery decision by direct comparison of these conditional expected payoffs.

Let us first determine S's expected profit for the case when S refuses to bribe. By using the optimal bidding strategy as stated in (9), we can express S's optimal expected profit $\mathbb{E}(\pi_S^N(c_S))$ shown in (7) for any cost given value of c_S as:

$$\begin{aligned}\mathbb{E}(\pi_S^N(c_S)) &= (b_S^N(c_S) - c_S)\,\Pr\Big(\frac{c_W}{1-\delta} \ge b_S^N(c_S)\Big) \\ &= \begin{cases} \frac{(1+a-(1-\delta)c_S)^2}{4(1-\delta)} & \text{if } 0 \le c_S \le (2r - \frac{1+a}{1-\delta})^+ \\ (r - c_S)(1 + a - (1-\delta)r) & \text{if } (2r - \frac{1+a}{1-\delta})^+ < c_S \le r \\ 0 & \text{if } c_S > r. \end{cases}\end{aligned} \tag{11}$$

Next, we determine S's conditional expected profit at the cost realization c_S for the case when S agrees to bribe indirectly due to the underlying complexity associated with the corresponding asymmetric auction. To begin, let us recall from (10) that: (a) $\bar{b}^Y$ is the bid that S will submit when its cost $c_S = P$; i.e., $b_S^Y(c_S = P) = \bar{b}^Y$; and (b) $\bar{b}^Y$ is the bid that W is indifferent between bidding or not when $c_W = (1-\delta)\bar{b}^Y$. Observation (b) implies that W will not submit its bid when $c_W > (1-\delta)\bar{b}^Y$ so that S will win the contract with an admissible bid. Combining these observations with the explicit form of $\bar{b}^Y$ as stated in Lemma 3, we can compute manufacturer S's expected profit $\mathbb{E}(\pi_S^Y(c_S = P))$ when S's bid $b_S = \bar{b}^Y$, where

$$\begin{aligned}\mathbb{E}(\pi_S^Y(c_S = P)) &= ((1-\delta)\bar{b}^Y - P)\Pr(c_W > (1-\delta)\bar{b}^Y\}) \\ &= \begin{cases} \frac{(1+a-P)^2}{4} & \text{if } r > \frac{1+a+P}{2(1-\delta)} \\ (r(1-\delta)-P)(1+a-(1-\delta)r) & \text{if } r \leq \frac{1+a+P}{2(1-\delta)}. \end{cases}\end{aligned} \tag{12}$$

Also, by recalling that S is indifferent between bribing and no bribing when $c_S = P$, we get:

$$\mathbb{E}(\pi_S^Y(c_S = P)) = \mathbb{E}(\pi_S^N(c_S = P)). \tag{13}$$

Observe from (11) and (12) that $\mathbb{E}(\pi_S^N(P)) > \mathbb{E}(\pi_S^Y(P))$ for any value of $P \in [0, (1-\delta)r]$ so that S should refuse to bribe when $c_S \in [0, (1-\delta)r]$. It remains to consider the case when $c_S \in ((1-\delta)r, r]$. Suppose S agrees to bribery. Then, for any "admissible" bid $b \in [0, r]$, S's payoff for agreeing to pay a bribe is equal to $b - \delta b - c_S \leq r - \delta r - c_S < 0$. However, if S refuses to bribe, S can earn a non-negative payoff as presented in (11). Therefore, S should also refuse to bribe when $(1-\delta)r < c_S \leq r$. Combining this observation with the result that S should refuse to bribery when $c_S \leq (1-\delta)r$, we get:

Theorem 1. *Under the proportional bribe case (i.e., the bribe is proportional to the value of the winning bid), it is optimal for the manufacturer S to refuse bribery at all times.*

Theorem 1 asserts that the optimal bribery decision for S is to refuse bribery at all times. The underlying reasons are as follows. If S agrees to bribe, W can observe this bribery decision and use this signal to develop a more accurate estimate about c_S so that W can gain from this information advantage by making a better bidding decision in the resulting asymmetric auction. Furthermore, if S agrees to bribe, the manufacturer who perceives that he is *ex ante* less efficient will bid more aggressively as stated in Lemma 2 which intensifies the competition in the asymmetric auction and lowers each manufacturer's expected payoff. To avoid this intensified competition in the asymmetric auction and the information disadvantage for S, it is optimal for S to refuse to bribe at all times.

To illustrate the above analysis, Figure 5 explicitly shows the evolution of the bribery decision

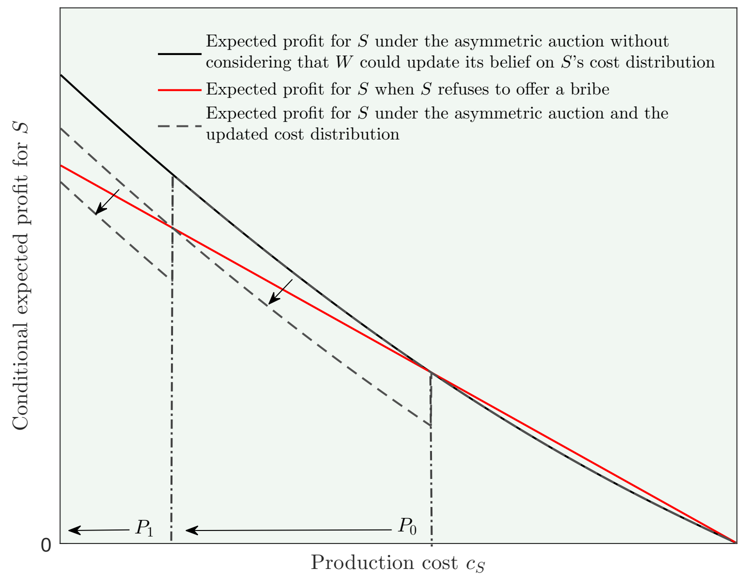

Figure 5: The evolution of the bribery decision

for S and explicates why refusing bribery is the optimal strategy. Without considering that W could update its belief on S's production cost using S's bribery decision, when S agrees to offer a bribe, S and W participate into an asymmetric auction where $c_S \sim U[0,1]$ and $c_W \sim U[a, 1+a]$. (The winner in the asymmetric auction needs to pay a bribe equal to δ times the winning bid to the intermediary.) Comparing it with the case when S refuses bribery, we find that there is a threshold P_0 such that it is beneficial for S to offer a bribe when $c_S < P_0$ (supposing that information updating is infeasible). However, in effect, W is able to precisely infer S's equilibrium decision that only when $c_S < P_0$, S agrees to offer a bribe. Accordingly, W could update its belief on S's cost distribution from $U[0,1]$ to $U[0, P_0]$. Based on Lemma 2, we know that the *ex ante* weak manufacturer will bid more aggressively in order to lower its losing probability. Hence, after updating S's cost distribution, W will offer a lower bid than the case when his belief on S's cost distribution is $U[0,1]$. Such a bid undercutting will increase W's winning probability but will lower S's expected profit when S agrees to bribe. Hence it pushes the threshold down to P_1. Similarly, when the threshold is equal to P_1, W could update its belief again and then adjusts its bidding strategy accordingly which further cuts S's expected profit down. Finally, refusing bribery fully dominates the strategy of offering a bribe and it is optimal for S is to refuse bribery at all times.

In summary, even though being disadvantaged by the intermediary, S chooses to refuse bribery at all times in order to prevent from leaking its cost information to W and to lessen the tension of competition. In the next section, we will investigate the influence of corruption on S's *ex ante* expected profit. Will S's profit decrease in the presence of the right of first refusal awarded to W? Will S disclose the corruption between W and the intermediary to defend its interest? We will explore these issues next.

6 Profit Analysis: the Incentive of Corruption Disclosure

After deriving S's optimal bribery decision, we can compare the *ax ante* expected profit for S associated with the asymmetric auction without corruption and associated with the corrupt auction (under which the optimal bribery decision for S is to refuse bribery at all times). It helps to demonstrate the impact of awarding the right of first refusal to W on S's profit and helps to explore whether S will disclose the corruption between W and the intermediary or not. If S's profit under the corrupt case is dominated by the case without corruption, then S has strong incentive to accuse W and the intermediary of colluding together to grab illicit profits and to rely on the legal force to eliminate the corruption in the auction. In contrast, if S benefits from the corruption even though being disadvantaged by the intermediary through revealing its bid to W, S may choose to keep silence about the corruption behavior. Eventually, it will encourage corruption in the auction.

Due to the complexity of the first-price asymmetric auction, it is hardly possible to get S's *ex ante* expected profit under the auction without corruption. Hence, we could not directly compare the profits under the corruption case and the non-corruption case. To theoretically derive the comparison results, we first construct an upper bound of S's *ex ante* expected profit under the first-price asymmetric auction (i.e., non-corrupt auction).[16]

Lemma 4. *In the asymmetric auction without corruption where $c_S \sim U[0,1]$ and $c_W \sim U[a, 1+a]$, $0 < a < 1$, and when the reserve price is r, the strong manufacturer S's profit is higher under the second-price auction than that under the first-price auction; i.e., $E_{c_S}\left[\mathbb{E}(\pi_S^{SPA}(c_S))\right] > E_{c_S}\left[\mathbb{E}(\pi_S^{FPA}(c_S))\right]$.*

Comparing the *ex ante* expected profits of S associated with the second-price auction and associated with the corrupt auction (under which S's optimal bribery decision is to refuse bribery), we can derive the sufficient condition of that S's *ex ante* expected profit under the corrupt auction is higher than that under the non-corrupt auction.[17]

Theorem 2. *When $\delta > \frac{1}{3} - \frac{a^3}{3r^3}$, the* ex ante *expected profit of S associated with the corrupt auction is*

[16] A similar result has been proved by Maskin and Riley (2000) without involving a reserve price.
[17] The superscript *SPA* and *FPA* denote the Second-Price Auction and the First-Price Auction without corruption.

higher than that associated with the non-corrupt auction. More formally, we have that if $\delta > \frac{1}{3} - \frac{a^3}{3r^3}$, then

$$E_{c_S}\left[\mathbb{E}(\pi_S^N(c_S))\right] > E_{c_S}\left[\mathbb{E}(\pi_S^{SPA}(c_S))\right] > E_{c_S}\left[\mathbb{E}(\pi_S^{FPA}(c_S))\right].$$

Under the corrupt auction, S's optimal bribery decision is to refuse bribery at all times. The case of S refusing bribery has been analyzed in Section 4.1 and the conditional expected profit for S at the cost realization c_S is presented in (11). Hence, the *ex ante* expected profit under the corrupt auction is captured by the term $E_{c_S}\left[\mathbb{E}(\pi_S^N(c_S))\right]$. On the other hand, when there is no corruption in the procurement auction, S and W participate in the asymmetric first-price auction which has been explored in the benchmark case presented in Section 3.1 and the *ex ante* expected profit is characterized by the term $E_{c_S}\left[\mathbb{E}(\pi_S^{FPA}(c_S))\right]$. From Theorem 2, we can directly derive Corollary 1 that under a certain condition (i.e., $\delta > \frac{1}{3} - \frac{a^3}{3r^3}$), S has no economic incentive to reveal the corruption between W and the intermediary because even though being disadvantaged by the intermediary due to refusing bribery, S benefits from the corruption between W and the intermediary.

Corollary 1. *When $\delta > \frac{1}{3} - \frac{a^3}{3r^3}$, S has no intention to reveal the corruption between W and the intermediary. Furthermore, such an intention is enhanced as the increase of the bribe share δ and the difference in the production efficiency (i.e., a).*

The intuition for Corollary 1 is as follows. Without considering the right of first refusal and in the presence of the difference in the production efficiency $a, 0 < a < 1$ (which captures the asymmetry between S and W), S takes a cost advantage over W. As a result, S will bid less aggressively than the case of competing with another strong S (based on the fourth statement of Lemma 2). In other words, when a is high, the competition with W is lessened from the perspective of the strong S and S could enjoy a higher *ex ante* expected profit. Hence, without the right of first refusal, the competition is tilted in favor of S due to its cost advantage. Next, let us focus on the right of first refusal. Based on our analysis in Section 5, it has been clearly state that refusing bribery is the optimal strategy for S, and under this optimal bribery decision, W will be awarded the right of first refusal. We find that this right has two separate forces on the competition between S and W. Firstly, that the intermediary favors W through revealing S's bid to W will enhance W's competitive power in the auction which neutralizes S's cost advantage and reduce (or may inverse when a and δ are small) the extent of the asymmetry between S and W. We call it "negative force" on S's cost advantage. Secondly, to get the right of first refusal, W needs to commit paying bribes upon winning the procurer's contract. Hence, it strengthens S's cost advantage over W and we call it "positive force" on S's cost advantage. When the extent of the asymmetry is high (i.e., a is high) or when the bribe share δ is high, the negative force is fully overcome by the positive force generated from the right of first refusal. Accordingly, even though being disadvantaged by the intermediary, S still benefits from the right of first refusal and will keep silence about the corruption in the auction (even though without being involved into the corruption). Such a "silence tactic" will encourage the corruption in the market eventually. Next, to explicitly present the above analysis, we investigate a special case where $a = 0$. In such a case, we take away the *ex ante* asymmetry between S and W and it will help us to focus on analyzing the impact of the right of first refusal.

6.1 A special case: $a = 0$

Considering the case where $a = 0$ so that both S and W has the same cost structure; i.e., $c_S \sim U[0, 1]$ and $c_W \sim U[0, 1]$. We can easily derive the optimal bidding strategy in the first-price auction without corruption (denoted by $b_i^0(c)$):

$$b_i^0(c_i) = \begin{cases} \frac{2r - r^2 - c_i^2}{2(1-c_i)} & \text{if } c_i \leq r \\ \infty \text{ (i.e., do not bid)} & \text{if } c_i > r. \end{cases} \tag{14}$$

Also, we can applying the optimal bidding strategy presented in (9) to derive the bidding function in the corrupt auction when $a = 0$, that is:

$$b_S^N(c_S) = \begin{cases} \frac{c_S}{2} + \frac{1}{2(1-\delta)} & \text{if } 0 < c_S \leq (2r - \frac{1}{1-\delta})^+ \\ r & \text{if } (2r - \frac{1}{1-\delta})^+ < c_S \leq r \\ \infty \text{(i.e., do not bid)} & \text{if } c_S > r. \end{cases} \tag{15}$$

With the optimal bid $b_S^0(c_S)$ given in (14) when bribery does not exist and the optimal bid $b_S^N(c_S)$ stated in (15) when bribery is allowed but S refuses (which is the optimal bribery strategy), we now can directly examine and compare S's profits under the corrupt and non-corrupt auctions. In preparation, let us compare these two optimal bids:

Lemma 5. *Consider a special case where $a = 0$. Relative to the benchmark case of no bribery, S will always submit a higher bid in the corrupt auction, i.e., $b_S^N(c_S) > b_S^0(c_S)$.*

Observe from Lemma 5 that, if S wins the contract, then S will obtain a higher *ex post* profit in the corrupt auction than the auction without bribery, i.e., $b_S^N(c_S) > b_S^0(c_S)$. Next, we investigate S's *ex ante* expected profit. From the perspective of S's expected payoff, by using our results as stated in (14) and (15) along with the optimal bid b_W as stated in (3) and (5), we get:

$$E_{c_S}\left[\mathbb{E}(\pi_S^N(c_S))\right] = \begin{cases} \frac{r^2(1-(1-\delta)r)}{2} & \text{if } r \leq \frac{1}{2(1-\delta)} \\ \frac{-1+6(1-\delta)r-6(1-\delta)^2r^2+2(1-\delta)^3r^3}{12(1-\delta)^2} & \text{if } r > \frac{1}{2(1-\delta)}. \end{cases} \tag{16}$$

Comparing with the first-price auction without corruption where $E_{c_S}\left[\mathbb{E}(\pi_S^0(c_S))\right] = \frac{r^2}{2} - \frac{r^3}{3}$, we get:

Corollary 2. *Consider a special case where $a = 0$. It is sufficient that when the bribe share $\delta > \frac{1}{3}$, the large manufacturer S will obtain a higher* ex ante *expected profit in a corrupt auction than in an auction without corruption.*

Corollary 2 specifies a condition under which the strong manufacturer S can benefit from the corrupt auction. To gain some insights, let us look at an extreme case first where $\delta = 0$, in which the manufacturer W can enjoy the right of first refusal but does not need to pay bribes (i.e., $\delta = 0$) to the intermediary. In other words, under the case of $\delta = 0$, the right of first refusal only produces a "negative force" on S through tilting the competition in favor of W. Intuitively, we could obtain that in such an extreme case, S's *ex ante* expected payoff in the corrupt auction is strictly lower than that under the auction without corruption due to an unfair treatment from the intermediary. More formally, we could get that

$$E_{c_S}\left[\mathbb{E}(\pi_S^N(c_S))\right] = \begin{cases} \frac{r^2(1-r)}{2} & \text{if } r \leq \frac{1}{2} \\ \frac{-1+6r-6r^2+2r^3}{12} & \text{if } r > \frac{1}{2} \end{cases} < E_{c_S}\left[\mathbb{E}(\pi_S^0(c_S))\right].$$

However, when the intermediary charges a positive bribe share (i.e., $\delta > 0$), the right of first refusal generates both "negative force" and "positive force" for S. We observe that S's *ex ante* expected profit is increasing in δ, i.e., $\frac{E_{c_S}\left[\mathbb{E}(\pi_S^N(c_S))\right]}{\partial\delta} > 0$. When the bribe share $\delta > \frac{1}{3}$, the" positive force" derived from the right of first refusal which strengthens S's cost advantage dominates the "negative force" which tilts the competition in favor of W. Accordingly, S will obtain a higher profit in the presence of the corruption than that without corruption when the bribe share is large. For this reason, when the bribe share δ is large, there is no economic incentive for the strong manufacturer S to expose the illicit transaction between the weak manufacturer and the intermediary. This result provides a plausible explanation for the concealment of corrupt auctions.

7 Robustness Check for the Assumption that W is a Committed Briber

In the base model, we have analyzed manufacturer S's optimal bribery decision by assuming that W is a committed briber. Letting W be the committed briber is for capturing the phenomenon of the small manufacturers offering bribes more frequently than the large manufacturers in reality, as shown in Figure 1 published by World Bank. But the question of why the small manufacturers are willing to be a committed briber is not addressed till now. Hence, in this section, we intend to see if there are some economic incentives for small manufacturers to be committed bribers. There may be some practical reasons for small manufacturers offering bribes more often. For example, small manufacturers have lower bargain power relative to the intermediary, and hence they have to pay bribes in order to be allowed to participate in the procurement auction; or small manufacturers are eager to expand their business and hence they are more willing to offer bribes. If we can show that the small manufacturer has strong economic incentives for W to be a committed briber (without considering these practical driving forces), then our results will be significantly more convincing.

We consider a sequential bribery game under which the small manufacturer W moves first to make its bribery choice (i.e., W's bribery decision is "endogenously" determined), and then the large manufacturer S decides whether to bribe or not after observing the small one's decision. Note that the large manufacturer, due to the consideration of reputation and the negative effect of potential disclosure of such bribery by the small manufacturer, will not bribe when the small one keeps itself away from bribery. But when the small manufacturer bribes, the large manufacturer strategically determines whether to bribe or not. Based on the analysis in Section 5, we already know that the best strategy for S is refusing to offer bribes at all times. Next, we calculate W's ex-ante expected payoff under two subgames: 1) one is when the small manufacturer W offers a bribe and 2) the other is when the small manufacturer W does not offer a bribe. Through comparing W's payoff under the above two subgames, we can derive W's optimal bribery decision (i.e., to bribe to not to bribe).

Firstly, let us consider the subgame in which W bribes the intermediary. In this case, W's ex-ante expected payoff is

$$\mathbb{E}[\Pi_W(c_W)] = E_{c_W}[(1-\delta)(b_W - c_W)\cdot \mathbb{1}_{b_W<b_S}].$$

Furthermore, knowing S's optimal bidding strategy $b_S(c_S)$ in (9) and W's best bidding function $b_W(c_W)$ in(5), we get W's ex-ante expected payoff as follows:

$$\mathbb{E}[\Pi_W(c_W)] = \begin{cases} \frac{1}{2}((1-\delta)r-a)^2 & \text{if } 2r - \frac{1+a}{1-\delta} \le 0 \\ \frac{1}{24(1-\delta)}[(2r - \frac{1+a}{1-\delta} + 1 - a)^3 - (1-a)^3] + \frac{1}{2}((1-\delta)r-a)^2 & \text{if } 2r - \frac{1+a}{1-\delta} > 0. \end{cases} \tag{17}$$

Next, let us consider the subgame in which W refuses to bribe. In this case, we can apply the optimal bidding strategy for asymmetric auctions derived in Kaplan and Zamir (2012) to numerically calculate W's ex-ante expected profit. We consider a special case of $r = 0.9, \delta = 0.1$. Note that in practice, the bribery is around $3\% \sim 5\%$ industry by industry. That is to say, letting $\delta = 10\%$ already captures the extreme case (or the worst case in practice) where it has high probability that W is reluctant to be a committed briber since W needs to pay a high portion of its revenue to the intermediary. Focusing on the case where $r = 0.9, \delta = 0.1$, we numerically show the comparison results regarding W's ex-ante expected profits associated with the cases of W refusing and agreeing to bribe in Figure 6. We observe that when the production inefficiency a is not too high, manufacturer W can get a higher ex-ante expected profit when committing to be a briber than joining a fair asymmetric auction with S. However, when a is extremely high (e.g., $a > r(1-\delta)$), committing to be a briber means that W certainly loses the contract and hence the best strategy for W is refusing to bribe.

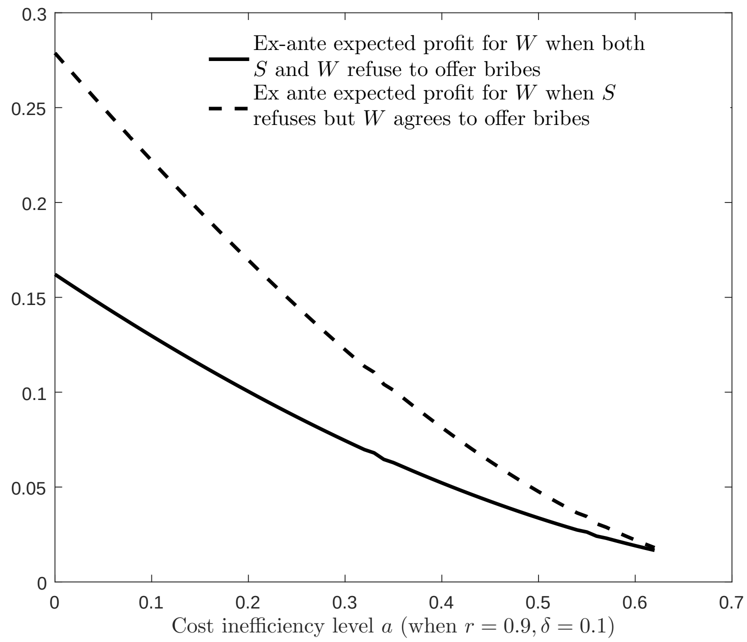

Figure 6: Comparison results on W's ex-ante expected profits associated the cases of refusing and agreeing to bribe

The intuition is as follows. When W refuses to offer a bribe, due to ex-ante production inefficiency, manufacturer W has a cost disadvantage when competing with S. So, in order to increase the winning probability, W will bid aggressively and hence, even though W wins, the profit margin is quite low. On the other hand, when W offers a bribe, we know that W's effective cost $\frac{c_W}{1-\delta}$ is increased due to the existence of bribery fee. Since only when the effective cost is lower than the reserve price, i.e., $\frac{c_W}{1-\delta} < r$, W is a qualified bidder to compete with S. Therefore, when W bribes, the probability of W to be a qualified bidder is lower. However, W gets the information advantage from the right of first refusal. As a result, once W wins, W gets a high profit margin, which could not only compensate the bribe paid to the intermediary but also increase the ex-ante expected profit owned by W. In a word, when a is smaller, the probability for W to be an unqualified bidder is not high. Therefore, the profit margin gained from the right of first refusal dominates the negative effect of increasing the possibility of early quit from the competition. In other words, W will benefit from committing to being a briber as long as the production inefficiency a is not very high. For the case when a is not very high, we now know it is optimal for W to be a committed briber. Consequently, it is optimal for S to refuse bribery at all times and our key result continues to hold.

8 Extension 1: Fixed Bribe Case

In the base model as depicted in Figure 3, we assume that the bribe is proportional to the winning bid. To examine the robustness of our results, we now examine the case when the bribe is based on a fixed amount κ.

8.1 When S refuses to bribe

First, when S refuses to bribe, we can use the same approach as presented in Section 4.1 to show that the optimization problem faced by S is,

$$\max_{b_S} \mathbb{E}(\pi(b_S)) = (b_S - c_S)\Pr(b_S \leq \min\{c_W + \kappa, r\}).$$

Also, using the same approach, the optimal bidding strategy for S is:

$$b_S^N(c_S) = \begin{cases} \frac{1+a+\kappa+c_S}{2} & \text{if } 0 < c_S \leq (2r-1-a-\kappa)^+ \\ r & \text{if } (2r-1-a-\kappa)^+ < c_S \leq r \\ \infty \text{ (i.e., do not bid)} & \text{if } c_S > r. \end{cases}$$

Additionally, the expected profit under the fixed bribe case when S refuses to bribe is:

$$\mathbb{E}(\pi_S^N(c_S)) = \begin{cases} \frac{(1+a+\kappa-c_S)^2}{4} & \text{if } 0 < c_S \leq (2r-1-a-\kappa)^+ \\ (r-c_S)(1+a+\kappa-r) & \text{if } (2r-1-a-\kappa)^+ < c_S \leq r \\ 0 & \text{if } c_S > r. \end{cases} \tag{18}$$

8.2 When S agrees to bribe

When S agrees to bribe, we can use the same approach as presented in Section 4.2 to deal with the case where the ”effective” production costs are $c_i + \kappa$, $i \in \{S, W\}$. We use κ to denote the exact amount of the bribe paid to the intermediary. Suppose the threshold structure of the bribery decision under which S accepts offering a bribe is $c_S \in [L, P]$; and the set of equilibrium bids is $[\underline{b}^\kappa, \overline{b}^\kappa]$. By using a similar analysis as presented in Section 4.2, we get:

Lemma 6. *The exact value of the upper bound $\overline{b}^{\kappa}$ is:*

1. *If $r > \frac{1+a+P+2\kappa}{2}$, then $\overline{b}^{\kappa} = \frac{1+a+P+2\kappa}{2}$;*
2. *If $r \leq \frac{1+a+P+2\kappa}{2}$, then $\overline{b}^{\kappa} = r$,*

where P is the highest cost of c_S such that S agrees to bribe, and $\overline{b}^{\kappa}$ is the highest bid associated with the case when $c_S = P$.

Accordingly, manufacturer S's expected profit when its cost reaches the upper bound of the set in which S chooses offering a bribe (i.e., $c_S = P$) is:

$$\begin{aligned}\mathbb{E}(\pi_S^Y(P)) &= (\overline{b}^{\kappa} - P - \kappa)\,\Pr(c_W > \overline{b}^{\kappa} - \kappa\}) \\ &= \begin{cases} \frac{(1+a-P)^2}{4} & \text{if } r > \frac{1+a+P+2\kappa}{2} \\ (r-P-\kappa)(1+a-r+\kappa) & \text{if } r \leq \frac{1+a+P+2\kappa}{2}. \end{cases}\end{aligned} \tag{19}$$

Since a fixed bribe κ should be paid to the intermediary, when $c_S = P$, the profit owned by S is $\overline{b}^{\kappa} - \kappa - P$, and the winning probability for S is $\Pr(c_W > \overline{b}^{\kappa} - \kappa)$. Making a comparison between the expected profits associated with the case when S refuses to bribe *and* agrees to bribe as stated in (18) and (19), respectively, we get:

Theorem 3. *When the bribe takes the form of a fixed amount κ, it is optimal for manufacturer S to refuse paying bribes at all times.*

From Theorem 3, we observe that irrespective of the amount of bribe κ, the reserve price r and S's realized production cost c_S, it is optimal for manufacturer S to refuse bribery at all times. Therefore, we conclude that the bribery decision under the fixed bribe case is consistent with that under the proportional bribe case as presented in Section 5.

9 Extension 2: Offering the Right of First Refusal Back and Forth Between Bidders When Both Bribe

In our base model as depicted in Figure 3, we assume that, when both manufacturers bribe the intermediary, the intermediary will withdraw the right of first refusal from both bidders and let S and W join a fair first-price auction. Now, we extend our analysis to the case where the intermediary will go between S and W back and forth, offering the right of first refusal (i.e., opportunities to match the prices) when both S and W offer bribes. Observe that this setting is equivalent to a second-price sealed bid auction when both manufacturer bribe. Knowing that S's expected profit when refusing to offer a bribe is shown in (11). Now, we just focus on the case when S agrees to offer a bribe and then join a second-price auction with W.

Under the second-price sealed bid auction (SPA), the winning bidder should provide δ portion of its revenue to the intermediary. In this case, it is well known that the optimal bidding strategy for each manufacturer is reporting its true effective cost, irrespective of the cost distribution of its competitor. That is to say, even though manufacturer W can infer a more precise cost distribution of S through S's bribery decision, this information advantage is useless and does not affect the optimal bidding strategy under the second-price auction. Regardless of S's bribery strategy, the optimal bids offered by S and W under the second-price auction are always $b_S^* = \frac{c_S}{1-\delta}$ and $b_W^* = \frac{c_W}{1-\delta}$. Based on the submitted quotes, we know that if $c_S \le \min\{c_W, r(1-\delta)\}$, S wins the retailer's contract and the retailer pays the second lowest quote, i.e., $\min\{\frac{c_W}{1-\delta}, r\}$, to S. Therefore, we can write down S's expected profit at cost c_S as follows:

(1) when $a < (1-\delta)r$,

$$\mathbb{E}(\pi_S^{Y:SPA}(c_S)) = \mathbb{E}_{c_W}\left\{\left[\min(\frac{c_W}{1-\delta}, r) \times (1-\delta) - c_S\right] \cdot \mathbb{1}_{c_S \le \min\{c_W, r(1-\delta)\}}\right\}$$
$$= \begin{cases} r(1-\delta) - \frac{((1-\delta)r-a)^2}{2} - c_S & 0 \le c_S \le a \\ (r(1-\delta) - c_S)(1 + a - \frac{c_S + (1-\delta)r}{2}) & a < c_S \le (1-\delta)r \\ 0 & (1-\delta)r < c_S \end{cases} \tag{20}$$

(2) when $a \ge (1-\delta)r$,

$$\mathbb{E}(\pi_S^{Y:SPA}(c_S)) = \begin{cases} r(1-\delta) - c_S & 0 \le c_S \le (1-\delta)r \\ 0 & (1-\delta)r < c_S. \end{cases} \tag{21}$$

Similar to the analysis in our base model, we derive S's optimal bribery decision through comparing S's expected profits associated with the cases of refusing and accepting to offer a bribe. Knowing that S's expected profit is shown (11) when refusing to bribe and S's expected profit is shown (20) and (21) when accepting to bribe, we derive the following Theorem.

Theorem 4. *If the intermediary goes between two bidders offering the opportunities to match prices when both manufacturers bribe, then S will refuse to offer a bribe when any one of the following conditions holds:*

(1) ***Region 1:*** $0 \le a \le r(1-\delta)$, $2r - \frac{1+a}{1-\delta} > 0$, *and* $\frac{(1+a)^2}{4(1-\delta)} + \frac{(r(1-\delta)-a)^2}{2} - r(1-\delta) \ge 0$;

(2) ***Region 2:*** $0 \le a \le r(1-\delta)$, $2r - \frac{1+a}{1-\delta} \le 0$, *and* $(a + r\delta)^2 - r(r-2\delta) \ge 0$;

(3) ***Region 3:*** $r(1-\delta) \le a \le 1$, $2r - \frac{1+a}{1-\delta} \le 0$.

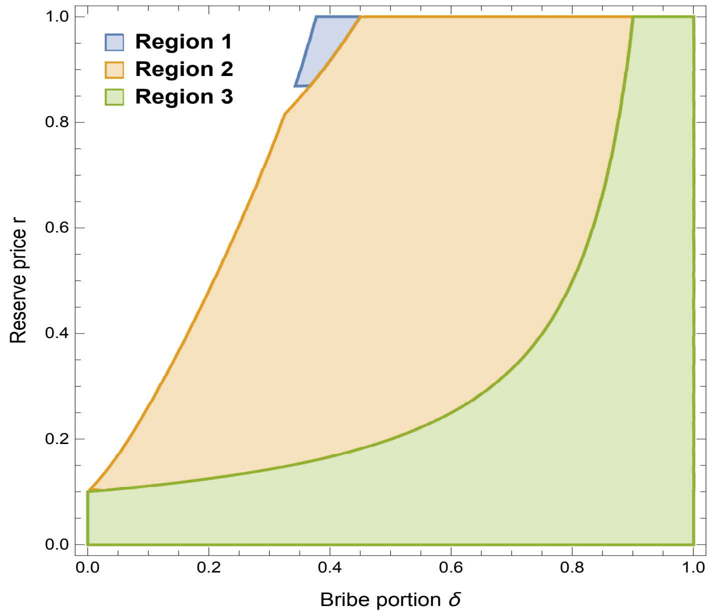

Figure 7: The region for S refusing to pay bribes (when $a = 0.1$)

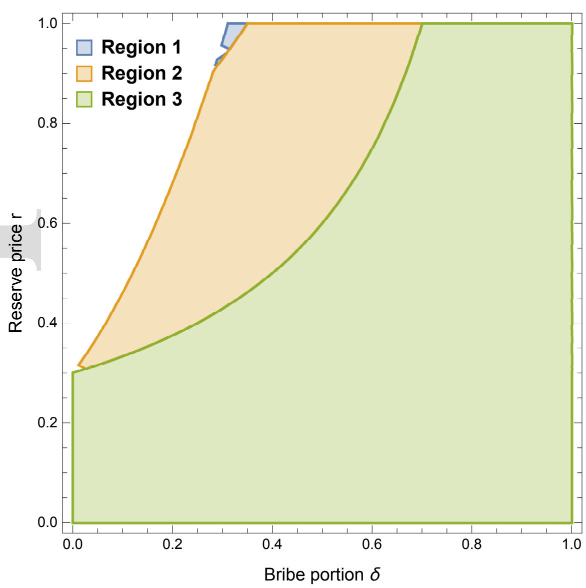

Figure 8: The region for S refusing to pay bribes (when $a = 0.3$)

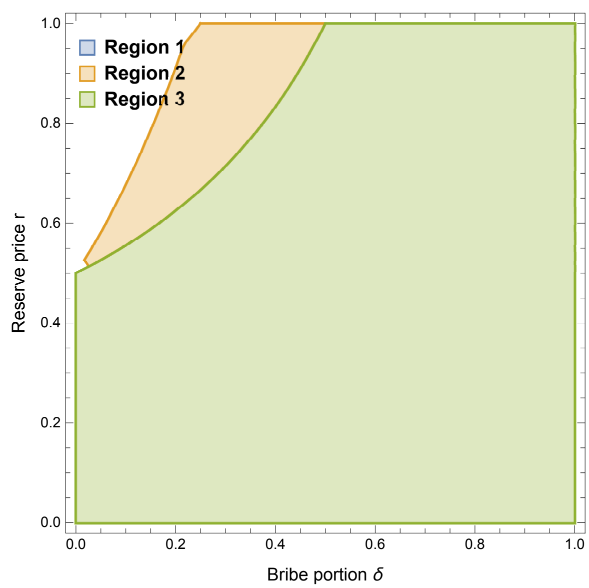

Figure 9: The region for S refusing to pay bribes (when $a = 0.5$)

To explicitly show the above theorem, we draw the region under which S's best strategy is refusing to offer a bribe in Figures 7, 8, and 9 when $a = 0.1$, $a = 0.3$ and $a = 0.5$, respectively. We observe that (1) as the production inefficiency a and the bribe portion δ increase, manufacturer S is inclined to refuse to offer a bribe; and (2) as the reserve price r increases, S is inclined to accept to offer a bribe. The intuition is as a increases, the probability for S to lose the contract decreases even though W has the right of first refusal, so S has less incentive to offer a bribe. Now, we focus on the relationship between S's bribery incentive and the bribe portion δ. On the one hand, as δ increases, the retained profit for S will be low if S offers a bribe. Hence, to maintain a high profit from the retailer's contract, S is inclined to refuse to offer a bribe. On the other hand, as δ increases, the probability that W is a qualified bidder (i.e., $c_W \leq r(1-\delta)$) decreases, which will benefit S when S does not offer a bribe. In terms of the reserve price, we know that as the increase of r, the probability for S to be a qualified bidder (i.e., $c_S \leq r(1-\delta)$) increases. Hence, S will be more willing to offer a bribe.

10 Discussion and Conclusion

Simultaneous bribery game when disparate corruption pressure is absent. Throughout the paper, we focus on the presence of disparate corruption pressure on W so that W is a committed briber and S is a non-committed briber who can strategically determine the bribery decision based on the realized production cost c_S. Hence, S's bribery decision can be regarded as a signal of S's cost c_S. We now discuss the case where there is no disparate corruption pressure and hence both W and S are not committed bribers. The key distinction from our base model is that, without disparate pressure, both bidders are non-committed bribers so that they can engage in a simultaneous bribery game (i.e., to bribe or not to bribe). That is to say, each bidder needs to decide whether or not to bribe before the realization of the private production cost c_i.[18] Unlike the sequential bribery game (i.e., our base model), there is "no information leakage" and "belief updating" under the simultaneous bribery game. In order to derive analytical results, we investigate the symmetric auction for the case when $a = 0$.

We use N to denote the decision that one bidder refuses to bribe and Y to denote that one bidder bribes the intermediary. To derive the equilibrium strategy, we investigate the following 4 strategies for S and W: $(N, N), (N, Y), (Y, N)$ and (Y, Y), where the first element represents S's decision and the second element is W's decision.[19]

[18]Another setting is both the bribery and bidding decisions are made *after* the production cost c is realized, but the analysis is complex due to the dynamics between the bribery decision and the bidding decision, and hence we leave it as future research.

[19]In (Y, Y) strategy, since we focus on the case where S and W have symmetric cost distribution, based on the revenue equivalence theorem, we know that bidders' ex-ante expected profits stay the same irrespective of the intermediary

Lemma 7. *Payoff of each bidder in a simultaneous bribery game can be summarized as follows:*

Table 1: The normal form of the simultaneous bribery game

S \ W	N	Y
N	$(\frac{r^2}{2}-\frac{r^3}{3},\frac{r^2}{2}-\frac{r^3}{3})$	(π_N,π_Y)
Y	(π_Y,π_N)	$(\frac{r^2(1-\delta)^2}{2}-\frac{r^3(1-\delta)^3}{3},\frac{r^2(1-\delta)^2}{2}-\frac{r^3(1-\delta)^3}{3})$

Here

$$\pi_N=\begin{cases}\frac{r^2(1-(1-\delta)r)}{2} & \text{if } r<\frac{1}{2(1-\delta)}\\ \frac{-1+6(1-\delta)r-6(1-\delta)^2r^2+2(1-\delta)^3r^3}{12(1-\delta)^2} & \text{if } r\geq\frac{1}{2(1-\delta)},\end{cases}$$

and

$$\pi_Y=\begin{cases}\frac{(1-\delta)^2r^2}{2} & \text{if } r<\frac{1}{2(1-\delta)}\\ \frac{1}{24(1-\delta)}[(2r-\frac{1}{1-\delta}+1)^3-1]+\frac{1}{2}(1-\delta)^2r^2 & \text{if } r\geq\frac{1}{2(1-\delta)}.\end{cases}$$

By comparing bidders' ex-ante expected profit under strategies $(N,N),(N,Y),(Y,N)$ and (Y,Y), we derive the equilibrium strategy for S and W:

Theorem 5. *Under the simultaneous bribery game, the equilibrium bribery strategy is characterized in Table 2.*

Table 2: (S,W)'s equilibrium Bribery Strategy in the Simultaneous Game

(S,W)**'s Equilibrium Strategy**	**Conditions**
(N,N)	$r<\min\{\frac{1}{2(1-\delta)},\frac{6\delta-3\delta^2}{2}\}$
	$r\geq\frac{1}{2(1-\delta)}$ and $f(\delta,r)\geq 0$
(N,Y) or (Y,N)	$\frac{6\delta-3\delta^2}{2}\leq r<\frac{1}{2(1-\delta)}$
	$r\geq\frac{1}{2(1-\delta)}$, $f(\delta,r)<0$ and $g(\delta,r)<0$
(Y,Y)	$r\geq\frac{1}{2(1-\delta)}$, $f(\delta,r)<0$ and $g(\delta,r)\geq 0$

Here $f(\delta,r)=2-16r^3(2-\delta)(1-\delta)^3-6\delta+6\delta^2-12r(1-\delta)\delta^2+r^2(1-\delta)^2(23-21\delta+21\delta^2+\delta^3)$ *and* $g(\delta,r)=1-2r(1-\delta)(3-r(1-\delta)(6-3(2-\delta)\delta-r(1-\delta)(3-2(2-\delta)\delta)))$.

offering or not offering the right of first refusal back and forth between bidders. Hence, in this section, we do not distinguish whether the intermediary offers the right of first refusal back and forth or not.

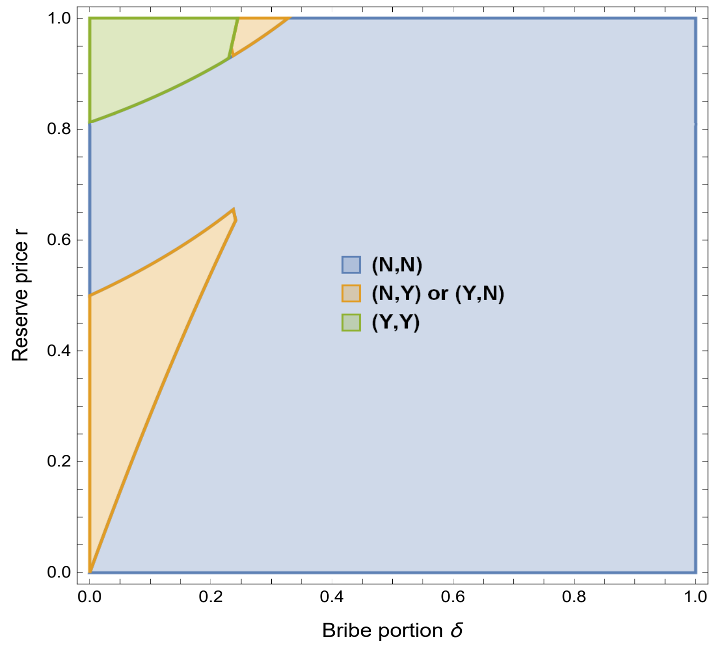

Figure 10: The equilibrium bribery strategy under the simultaneous game

Figure 10 illustrates the equilibrium bribery strategy. Observing from Figure 10 that, when the bribe portion δ is low and the reserve price r is high (see the green region in Figure 10), bidders fall into prisoners' dilemma in the following sense: both bidders will choose to bribe even though both bidders are better off from refusing to bribe (i.e., $\frac{r^2(1-\delta)^2}{2} - \frac{r^3(1-\delta)^3}{3} < \frac{r^2}{2} - \frac{r^3}{3}$). Also, all of the four strategies could be equilibrium under the different combination of parameters δ and r. This result is different from that we obtained from the sequential bribery game as follows. In the sequential bribery game, motivated by the case when W is a committed briber due to the disparate corruption pressure (Figure 3), S's best strategy is to refuse to bribe at all times in order to prevent from leaking S's cost information and intensifying the competition with W.

Conclusion. In this paper, we have analyzed manufacturer's bribery decision (i.e., to bribe or not to bribe) in a corrupt auction under disparate corruption pressure. While favoritism in auction has been well-developed in the auction literature, it is often assumed that bidders are homogeneous in bribery decision or their bribery decisions are exogenously determined. In this paper, to capture the phenomenon of "disparate corrupt pressure", we have examined a case in which the small manufacturer is a committed briber while the large one can determine its bribery decision endogenously.

Through our analysis, we have found that the large/strong manufacturer should refuse bribery at all times in order to avoid leaking its cost information to the weak manufacturer and to avoid intensifying the competition which drives profits down. Interestingly, we have identified a specific condition under which the “positive force” generated from the right of first refusal which enhances the large/strong manufacturer’s cost advantage outweighes the “negative force” which tilts the competition in favor of the small/weak manufacturer. Accordingly, the strong manufacturer can benefit from the corrupt auction even if being disadvantaged by the intermediary due to refusing to bribe. We concluded that, under such a sufficient condition, the strong manufacturer has no incentive to expose the illicit transaction. Such a “silence tactic” ironically promotes the prevalence and concealment of corrupt auctions. Also, we have formulated different forms of bribery (the bribe is based on a fixed value) in order to check for the robustness of the optimal bribery decision. We find that those results associated with the bribery decision under both proportional bribe case and fixed bribe case are consistent.

To our knowledge, our paper is the first to examine the disparate corruption pressures upon manufacturers with different market scales, and there are several directions for future research. For example, we have considered a simplified and polarized version of the disparate bribery pressure, where we assumed that the weak manufacturer is a committed briber and the strong one is a non-committed briber. Relaxing this assumption so that, the each manufacturer can determine its bribery decision endogenously with a certain probability. This extension is more complex and we defer it as further research.

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A Robustness check for $\delta_S = \delta_W$

In our base model, we assume that the bribery proportion δ is the same for both manufacturers. In this section, we check the robustness of our main result presented in Theorem 1 to this assumption by investigating the case where S and W pay differently, i.e., $\delta_S \neq \delta_W$.

Based on the analysis for the case when S refusing to offer bribe in Section 4.1, we can easily get S's expected profit as follows:

$$\begin{aligned}\mathbb{E}(\pi_S^N(c_S)) &= (b_S^N(c_S) - c_S)\Pr(\frac{c_W}{1-\delta_W} \geq b_S^N(c_S)) \\ &= \begin{cases} \frac{(1+a-(1-\delta_W)c_S)^2}{4(1-\delta_W)} & \text{if } 0 \leq c_S \leq (2r - \frac{1+a}{1-\delta_W})^+ \\ (r-c_S)(1+a-(1-\delta_W)r) & \text{if } (2r - \frac{1+a}{1-\delta_W})^+ < c_S \leq r \\ 0 & \text{if } c_S > r. \end{cases}\end{aligned} \tag{22}$$

Next, we still focus on the threshold structure where S agrees to offer bribes when $c_S \in [L, P]$. Similar to the analysis for Lemma 3, we get that the upper bound of manufacturers' bid $\bar{b}^Y$ is the solution of the following optimization problem:

$$\bar{b}^Y = \arg\max_{b \leq r}((1-\delta_S)b - P)(1+a-(1-\delta_W)b)$$

when $\delta_S \neq \delta_W$. Solving the above problem, we obtain that

Lemma 8. *The exact value of the upper bound $\bar{b}^Y$ can be described as follows:*

1. *If $r > \frac{1+a}{2(1-\delta_W)} + \frac{P}{2(1-\delta_S)}$, then $\bar{b}^Y = \frac{1+a}{2(1-\delta_W)} + \frac{P}{2(1-\delta_S)}$;*
2. *If $r \leq \frac{1+a}{2(1-\delta_W)} + \frac{P}{2(1-\delta_S)}$, then $\bar{b}^Y = r$,*

where P is the highest cost for c_S such that S agrees to bribe, and $\bar{b}^Y$ is the highest bid associated with the case when $c_S = P$.

With Lemma 8, we can calculate S's expected profit when S agrees to bribe and when $c_S = P$ as follows:

$$\begin{aligned}\mathbb{E}(\pi_S^Y(c_S = P)) &= ((1-\delta_S)\bar{b}^Y - P)\Pr(c_W > (1-\delta_W)\bar{b}^Y\}) \\ &= \begin{cases} \frac{((1-\delta_S)(1+a)-(1-\delta_W)P)^2}{4(1-\delta_S)(1-\delta_W)} & \text{if } r > \frac{1+a}{2(1-\delta_W)} + \frac{P}{2(1-\delta_S)} \\ (r(1-\delta_S) - P)(1+a-(1-\delta_W)r) & \text{if } r \leq \frac{1+a}{2(1-\delta_W)} + \frac{P}{2(1-\delta_S)}. \end{cases}\end{aligned} \tag{23}$$

Through comparing $\mathbb{E}(\pi_S^Y(c_S = P))$ and $\mathbb{E}(\pi_S^N(c_S = P))$, we observe the same result as the case when $\delta_S = \delta_W$. That is, for any feasible P, we get $\mathbb{E}(\pi_S^Y(c_S = P)) \leq \mathbb{E}(\pi_S^Y(c_S = P))$. Therefore, we derive the following theorem and validate our result in Theorem 1 is robust even when S and W pay differently.

Theorem 6. *Under the different bribery proportion case of $\delta_S \neq \delta_W$, it is still optimal for the manufacturer S to refuse bribery at all times.*

Proof of Theorem 6. Suppose P exists, a necessary existence condition for the threshold P is that when $c_S = P$, $E[\pi_S^Y]$ is equal to $E[\pi_S^N]$. Note that when $\delta_S = \delta_W$, we get $E[\pi_S^N(c_S = P)] > E[\pi_S^Y(c_S = P)]$ based on the analysis of our base model. Also, we observe that $E[\pi_S^Y(c_S = P)]$ is decreasing in δ_S. Hence, when $\delta_S > \delta_W$, we can straightly conclude that it is optimal for S to refuse to bribe at all times. Next, we focus on the scenario where $\delta_S < \delta_W$ and investigate three separate cases.

Case 1: $r > \frac{1+a}{2(1-\delta_W)} + \frac{P}{2(1-\delta_S)}$. When $r > \frac{1+a}{2(1-\delta_W)} + \frac{P}{2(1-\delta_S)}$, we need to find the P such that $E[\pi_S^Y(c_S = P)] = E[\pi_S^N(c_S = P)]$. When $P \leq 2r - \frac{1+a}{1-\delta_W}$, we get

$$T(\delta_S) = \frac{E[\pi_S^Y(c_S = P)]}{E[\pi_S^N(c_S = P)]} = \frac{[(1-\delta_S)(1+a) - (1-\delta_W)P]^2}{(1-\delta_S)(1+a-(1-\delta_W)P)^2}.$$

Since $sgn\{\frac{dT(\delta_S)}{d\delta_S}\} = sgn\{-(1+a)(1-\delta_S) - (1-\delta_W)P\} < 0$ and $T(\delta_S = 0) = 1$, we get that $T(\delta_S) \leq 1$ and hence it is optimal for S to refuse to bribe.

Secondly, we observe that the condition $r > \frac{1+a}{2(1-\delta_W)} + \frac{P}{2(1-\delta_S)}$ contradicts with $P > 2r - \frac{1+a}{1-\delta_W}$. Hence, we prove that when $r > \frac{1+a}{2(1-\delta_W)} + \frac{P}{2(1-\delta_S)}$, it is optimal for S to refuse to bribe.

Case 2: $\frac{1+a}{2(1-\delta_W)} < r \leq \frac{1+a}{2(1-\delta_W)} + \frac{P}{2(1-\delta_S)}$. When $\frac{1+a}{2(1-\delta_W)} < r \leq \frac{1+a}{2(1-\delta_W)} + \frac{P}{2(1-\delta_S)}$, we still need to find a P such that $E[\pi_S^Y(c_S = P)] = E[\pi_S^N(c_S = P)]$. Through comparing

$$E[\pi_S^Y(c_S = P)] = (r(1-\delta_S) - P)(1+a-(1-\delta_W)r)$$

and

$$E[\pi_S^N(c_S = P)] = \begin{cases} \frac{(1+a-(1-\delta_W)P)^2}{4(1-\delta_W)} & \text{if } 0 \leq c_S \leq (2r - \frac{1+a}{1-\delta_W})^+ \\ (r-P)(1+a-(1-\delta_W)r) & \text{if } (2r - \frac{1+a}{1-\delta_W})^+ < c_S \leq r \\ 0 & \text{if } c_S > r. \end{cases}$$

we get that when $P > 2r - \frac{1+a}{1-\delta_W}$, $E[\pi_S^Y(c_S = P)]$ is smaller than $E[\pi_S^N(c_S = P)]$ since $r - P > (1-\delta_S)r - P$. On the other hand, when $P \leq 2r - \frac{1+a}{1-\delta_W}$, we get that

$$\begin{aligned} &E[\pi_S^N(c_S = P)] - E[\pi_S^Y(c_S = P)]|_{\delta_S=0} \\ &\quad = [(1+a) - (1-\delta_W)P]^2 - 4(1-\delta_W)(r-P)(1+a-(1-\delta_W)r) > 0, \forall a \geq 0. \end{aligned}$$

Hence, for case 2, it is optimal for S to refuse to bribe.

Case 3: $r \leq \frac{1+a}{2(1-\delta_W)}$. Under the case of $r \leq \frac{1+a}{2(1-\delta_W)}$, we can easily get that $E[\pi_S^N(c_S = P)] = (r-P)(1+a-(1-\delta_W)r)$ which is larger than $E[\pi_S^Y(c_S = P)] = ((1-\delta_S)r - P)(1+a-(1-\delta_W)r)$. Therefore, the threshold P does not exist.

From the above analysis, we have proved that we cannot find an appropriate P as well as the equilibrium cost structure $[L, P]$. $\square$

B Coalition among Bidders and Intermediary (Where $a = 0$)

Because the bribe value is proportional to the winning bid, the intermediary has a strong incentive to find ways to raise the value of the winning bid. One way is for the intermediary to invite S to "join" a *coaltion* along with W and the intermediary so that they can manipulate and raise the winning bid to the reserve price r. In this section, we analyze a different form of bribery decision for S: to join the coalition or to refuse bribery. If S refuses to join, S's payoff is as presented in Section 4.1. If S joins the coalition, each party of the coalition will receive a certain proportion of the surplus. To maximize the total surplus of the coalition, the contract will be awarded to the more efficient manufacturer so that the effective cost is $\min\{c_S, c_W\}$.

To simplify our analysis, we consider a simple surplus allocation rule for the coalition so that the intermediary will get a bribe of value δr (where r is now the value of the winning bid associated with coalition), and both S and W will receive 50% of the remaining surplus so that:

$$\mathbb{E}(\pi_S) = \mathbb{E}(\pi_W) = \mathbb{E}[\frac{(r(1-\delta) - \min\{c_S, c_W\})^+}{2}]. \tag{24}$$

By noting S and W will carry out the procurer's contract only when $r(1-\delta) - \min\{c_S, c_W\} > 0$, the intermediary's expected payoff is:

$$\mathbb{E}(\pi_I) = \mathbb{E}[\delta r \cdot \mathbb{1}_{\min\{c_S, c_W\} < r(1-\delta)}],$$

By using the fact that S's belief on c_W is $U[0,1]$, we can apply (24) to show that S's expected payoff for joining the coalition is:

$$\mathbb{E}(\pi_S^Y) = \begin{cases} \frac{r(1-\delta)}{2} - \frac{1}{2}(c_S - \frac{c_S^2}{2}) & \text{if } c_S < (1-\delta)r \\ \frac{r^2(1-\delta)^2}{4} & \text{if } c_S \geq (1-\delta)r. \end{cases} \tag{25}$$

Also, S's expected payoff for refusing to bribery is given in (11). By comparing the expected payoffs in (25) and (11), we get:

Theorem 7. *In equilibrium, S's optimal bribery decision (to join the coalition or to refuse bribery) can be described as follows:*

1. *If the reserve price $r \in (0, \min\{\frac{4\delta}{1+2\delta-3\delta^2}, 1\}]$, S should refuse to bribe if $c_S \in [0, \tilde{c}]$. Otherwise, S should join the coalition and agree to bribe;*
2. *If the reserve price $r \in (\min\{\frac{4\delta}{1+2\delta-3\delta^2}, 1\}, \min\{\tilde{r}, 1\}]$, S should refuse to bribe if $c_S \in [0, \bar{c}]$. Otherwise, S should join the coalition and agree to bribe;*
3. *If the reserve price $r \in (\min\{\tilde{r}, 1\}, \min\{\frac{1}{2-4\delta+2\delta^2}, 1\}]$, S should refuse to bribe if $c_S \in [0, \sqrt{\frac{1-2r+4r\delta-2r\delta^2}{\delta-\delta^2}}]$. Otherwise, S should join the coalition and agree to bribe;*

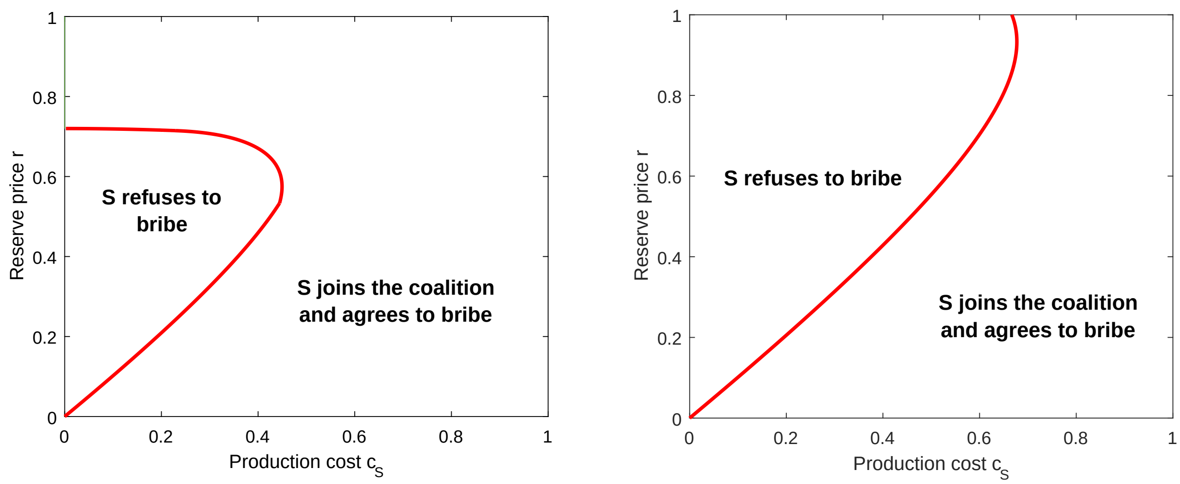

Figure 11: S's optimal bribery decision when $\delta = \frac{1}{6}$ (left) and when $\delta = \frac{1}{3}$ (right)

4. *If the reserve price* $r \in (\min\{\frac{1}{2-4\delta+2\delta^2}, 1\}, 1]$, *the optimal strategy is to join the coalition and agrees to bribe, where* $\widetilde{c} = \frac{4r-5r^2+6r^2\delta-r^2\delta^2}{4-4r+4r\delta}$, $\overline{c} = -1+2r-2r\delta+\sqrt{1-2r+6r\delta-4r^2\delta+4r^2\delta^2}$, $\widetilde{r} = \frac{1-4\delta+\delta^2}{-4(1-\delta)\delta} + \frac{1}{4}\sqrt{\frac{1-3\delta+7\delta^2-\delta^3}{(1-\delta)\delta^2}}$.

Theorem 7 reveals that, in the presence of a potential coalition, S's optimal bribery decision depends on r, δ and c_S. To elaborate, we illustrate S's optimal bribery decision for the case when $\delta = \frac{1}{6}$ and $\delta = \frac{1}{3}$ in Figure 11. From Figure 11, we observe that, given the bribe share δ and the reserve price r, there exists a cutoff structure when S's production cost is high (low), it is more likely that S accepts (refuses) to offer a bribe. When the production cost is significantly high i.e, $c_S \geq r$, S definitely cannot win the auction. However, joining the coalition will generate a positive expected payoff for S since it is possible that W has a low production cost (i.e., $c_W < (1-\delta)r$) and has the capability to implement the contract at the price r. Therefore, S is willing to join the coalition to be a free rider. When the production cost c_S is medium, if S refuses to bribe, S has to submit an aggressive bid to compete with W who has the right of first refusal and is favored by the intermediary. As a result, the expected payoff from competing with W may be lower than that secured from the coalition. Therefore, S is inclined to join the coalition and agree to pay bribes to the intermediary. However, when the production cost is small, it is highly likely that S can outbid W even though W can observe S's bid before submitting the bid b_W. Therefore, if choosing to refuse offering a bribe, S can solely enjoy the total surplus without sharing the surplus with the intermediary and the weak manufacturer, which offers sufficient incentive for S to refuse joining the coalition.

From Theorem 7, we also conclude that the impact of bribe share δ on S's optimal bribery

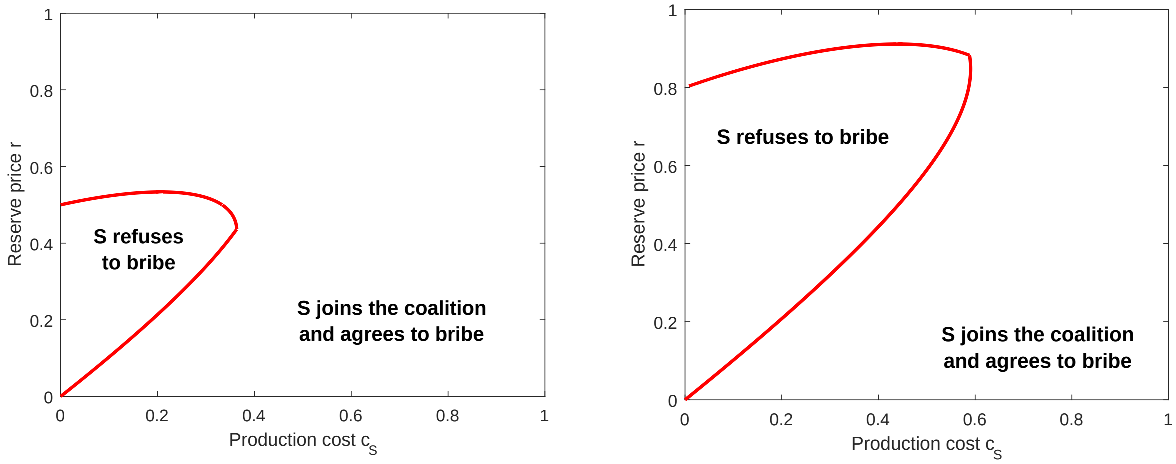

Figure 12: S's optimal bribery decision when $\delta = \frac{1}{6}$ (left) and when $\delta = \frac{1}{3}$ (right) under the split rule that S can enjoy $70\%(1-\delta)$ share of the total surplus

is monotone, that is, S is more inclined to refuse joining the coalition (i.e., refuse bribery) as the increase of commission rate δ. This is because when the bribe share δ is high, if choosing to join the coalition, the intermediary will extract a large share of the total surplus; however, if choosing to resist bribery, since W has to pay a high bribe share to the intermediary, S can take a cost advantage to compete with W and enjoy a satisfactory profit. Consequently, as the increase of the bribe share, S is more inclined to refuse bribery. Secondly, Theorem 7 indicates that the affect of reserve price r on S's optimal bribery decision is non-monotone, that is, S tends to refuse joining the coalition first and then tends to join it as the increase of the reserve price r. This novel phenomenon is generated by the trade-off between the enhanced total surplus (i.e., $E[(r - \min\{c_S, c_W\})^+]$) as the increase of the reserve price r if S chooses to join the coalition *and* the increased cost advantage due to a high bribe share (i.e., δr) if S chooses to refuse bribery and compete with W. When the reserve price is low, as the increase of the reserve price, the cost advantage for S outweighes the improved total surplus generated from the coalition, and therefore, S is inclined to refuse bribery. On the other hand, when the reserve price is high, as the increase of the reserve price, the improved total surplus dominates the cost advantage. As a result, S tends to join the coalition. A possible implication of Theorem 7 for the procurer is to set a reasonable reserve price to combat coalition among bidders and the intermediary.

We also extend our analysis to deal with a more general surplus allocation rule. Specifying the split rule as that the intermediary extracts δr from the coalition and S enjoys 70% share of the remaining surplus, we find a similar structure on S's bribery decision which is illustrated in Figure 12. Comparing Figures 11 and 12, we find that as an increase of the profit share for S, S is more inclined to join the coalition; and that when S can get a high share of profit from the coalition and when the reserve price is medium, S chooses to join the coalition not only when his production cost is high (in order to be a free rider) but also when his production cost is low (in order to grab more profits). Also, we observe that when the reserve price is high enough, S should agree to bribe and join the coalition at all times; and that when the reserve price is low, S should refuse to bribe and compete with W for the procurer's contract when his production cost is low but choose to join the coalition when his production cost is high. These two results are consistent under different revenue allocation rules.

C Proof

Proof of Lemma 2.

Consider the case where $c_S \sim U[0,1]$ and $c_W \sim U[a, 1+a]$. We can easily get that the distribution of the weak manufacturer W stochastically dominates that of the strong S, i.e.,

$$F_S(c) > F_W(c), \forall c \in [0, 1+a],$$

and also conditionally stochastically dominates (CSD) that of S, i.e.,

$$\frac{F_W'(c)}{F_W(c)} > \frac{F_S'(c)}{F_S(c)}, \forall c \in [0, 1+a].$$

Firstly, we prove that $F_S(b_S^{-1}(b)) > F_W(b_W^{-1}(b))$, for $b \in (\underline{b}, \bar{b})$.

Step 1: Based on the boundary condition shown in (2), we know that $b_S^{-1}(\underline{b}) = 0$ and $b_W^{-1}(\underline{b}) = a$. Hence, $F_S(b_S^{-1}(\underline{b})) = F_W(b_W^{-1}(\underline{b})) = 0$.

Step 2: Suppose for some $\hat{b} \in (\underline{b}, \bar{b})$, the equation $F_S(b_S^{-1}(\hat{b})) = F_W(b_W^{-1}(\hat{b}))$ holds. Since $F_S(c) > F_W(c), \forall c \in [0, 1+a]$, we can derive that $b_S^{-1}(\hat{b}) < b_W^{-1}(\hat{b})$. Next, based on the ODEs shown in (1) we can derive that

$$F_W'(b_W^{-1}(\hat{b})) = \frac{1 - F_W(b_W^{-1}(\hat{b}))}{b - b_S^{-1}(\hat{b})} < \frac{1 - F_S(b_S^{-1}(\hat{b}))}{b - b_W^{-1}(\hat{b})} = F_S'(b_S^{-1}(\hat{b})).$$

Therefore, we prove that for $b \in (\underline{b}, \bar{b})$, $F_S(b_S^{-1}(b)) > F_W(b_W^{-1}(b))$ always holds.

Secondly, we show that $b_W^{-1}(b) > b_S^{-1}(b)$, for $b \in (\underline{b}, \bar{b})$.

Step 1: Based on the boundary condition in (2), we know that $b_S^{-1}(\underline{b}) = 0$ and $b_W^{-1}(\underline{b}) = a$. Hence, $b_W^{-1}(\underline{b}) > b_S^{-1}(\underline{b})$.

Step 2: Suppose for some $\hat{b} \in (\underline{b}, \bar{b})$, the equation $b_W^{-1}(\hat{b}) = b_S^{-1}(\hat{b})$ holds. Since CSD indicates that $\frac{F_W'(c)}{F_W(c)} > \frac{F_S'(c)}{F_S(c)}, \forall c \in [0, 1+a]$, we can easily derive the following relationship, that is,

$$\frac{F_W'(c)}{1 - F_W(c)} < \frac{F_S'(c)}{1 - F_S(c)}, \forall c \in [0, 1+a]. \tag{26}$$

Next, based on the above equation (26) along with the ODEs shown in (1), we can show that

$$(b_W^{-1}(\hat{b}))' = \frac{1 - F_W(b_W^{-1}(\hat{b}))}{f_W(b_W^{-1}(\hat{b}))(b - b_S^{-1}(\hat{b}))} > \frac{1 - F_S(b_S^{-1}(\hat{b}))}{f_S(b_S^{-1}(\hat{b}))(b - b_W^{-1}(\hat{b}))} = (b_S^{-1}(\hat{b}))'.$$

Therefore, we prove that for $b \in (\underline{b}, \bar{b})$, the inequality $b_W^{-1}(b) > b_S^{-1}(b)$ holds.

We use $\overline{\mu}_W$ to denote the maximum bid of the symmetric auction with two W. To prove the third statement of Lemma 2, we should first establish the fact of $\bar{b} \leq r = \overline{\mu}_W$. (The proof for the fourth statement is similar as that of the third statement.)

Step 1: With an effective reserve price $r, r \in (a, 1+a)$, we know that the maximum bid $\overline{\mu}_W$ should be equal to the reserve price r. However, the maximum bid in the asymmetric auction with one S and one W will depend on the distribution asymmetry captured by a and the reserve price r, but we know that the maximum bid will not exceed the reserve price and hence $\bar{b} \leq r = \overline{\mu}_W$. Based on the fact $\bar{b} \leq r = \overline{\mu}_W$, we can get that $\bar{b} = b_W^{-1}(\bar{b}) > \gamma_W^{-1}(\bar{b})$.

Step 2: Suppose for some $\hat{b} \in (\underline{b}, \bar{b})$, the equation $b_W^{-1}(\hat{b}) = \gamma_W^{-1}(\hat{b})$ holds. Based on the second statement of Lemma 2, we know that $b_W^{-1}(\hat{b}) = \gamma_W^{-1}(\hat{b}) > b_S^{-1}(\hat{b})$. According to the system of ODEs presented in (1), we can get that

$$(b_W^{-1}(\hat{b}))' = \frac{1 - F_W(b_W^{-1}(\hat{b}))}{f_W(b_W^{-1}(\hat{b}))(b - b_S^{-1}(\hat{b}))} < \frac{1 - F_W(\gamma_W^{-1}(\hat{b}))}{f_W(\gamma_W^{-1}(\hat{b}))(b - \gamma_W^{-1}(\hat{b}))} = (\gamma_W^{-1}(\hat{b}))'.$$

Based on the above analysis, we show that $b_W^{-1}(b) > \gamma_W^{-1}(b)$, for $b \in (\bar{b}, \bar{b})$.

$\square$

Proof of Proposition 1.

When S refuses to bribe, the manufacturer S's optimal bidding strategy satisfies:

$$b_S^N(c_S) = \begin{cases} \min\{\frac{J^{-1}((1-\delta)c_S)}{1-\delta}, r\} & \text{if } c_S \leq r \\ \infty \text{ (i.e., do not bid)} & \text{if } c_S > r. \end{cases}$$

Plugging $F_W(x) = x - a$ and $f_W(x) = 1$ for $\forall x \in [0,1]$ into the above equation along with the fact that $J(x) = 2x - 1 - a$, we can simplify the optimal bidding strategies $b_S^N(c_S)$ when S refuses to bribe. Formally,

$$b_S^N(c_S) = \begin{cases} \frac{c_S}{2} + \frac{1+a}{2(1-\delta)} & \text{if } 0 < c_S \leq (2r - \frac{1+a}{1-\delta})^+ \\ r & \text{if } (2r - \frac{1+a}{1-\delta})^+ < c_S \leq r \\ \infty \text{(i.e., do not bid)} & \text{if } c_S > r. \end{cases}$$

$\square$

Proof of Lemma 3.

We first determine the optimal bidding strategy for S. Suppose W adopts a bidding strategy

$b_W(c_W)$. Then S's best response function $b_S(c_S)$ for the case when S agrees to bribe is the solution to the following problem:

$$\begin{aligned} \max_b \mathbb{E}(\pi_S(c_S)) &= ((1-\delta)b - c_S)\Pr(b < b_W(c_W)) = ((1-\delta)b - c_S)\Pr(b_W^{-1}(b) < c_W) \\ &= ((1-\delta)b - c_S)(1 + a - b_W^{-1}(b)), \end{aligned} \tag{27}$$

where $b_W^{-1}(b)$ is an "inverse" function of the bidding strategy $b_W(c_W)$ associated with W.

From (10), the boundary conditions can be rewritten as follows:

$$\begin{aligned} b_W^{-1}(\bar{b}^Y) &= \bar{b}(1-\delta), \\ b_S^{-1}(\bar{b}^Y) &= P. \end{aligned}$$

Through the boundary conditions, we know that $\bar{b} = \arg\max_b \mathbb{E}(\pi_S(c_S = P))$. Also, based on the fact that, $b \le r$, $b_W^{-1}(b) \le (1-\delta)b$, we can derive the following inequation:

$$((1-\delta)\bar{b}^Y - P)(1 + a - b_W^{-1}(\bar{b}^Y)) \ge ((1-\delta)b - P)(1 + a - b_W^{-1}(b)) \ge ((1-\delta)b - P)(1 + a - (1-\delta)b). \tag{28}$$

The first inequality is due to the optimality of $\bar{b}^Y$ when $c_S = P$ and the second inequality is due to the fact that $b_W^{-1}(b) \le (1-\delta)b$. From (28), we know that $\bar{b}^Y$ is the maximizer of the function $((1-\delta)b - P)(1 + a - (1-\delta)b)$. Therefore, $\bar{b}^Y = \min\{\frac{1+a+P}{2(1-\delta)}, r\}$. □

Proof of Theorem 1.

Suppose P exists, a necessary existence condition for the threshold P is that when $c_S = P$, $E[\pi_S^Y]$ is equal to $E[\pi_S^N]$.

When $r > \frac{1+a+P}{2(1-\delta)}$, we need to find the P such that $E[\pi_S^Y(c_S = P)] = E[\pi_S^N(c_S = P)]$. Through comparing $E[\pi_S^Y(c_S = P)]$ and $E[\pi_S^N(c_S = P)]$ (Plugging $c_S = P$ into (11)); i.e.,

$$E[\pi_S^Y(c_S = P)] = [(1-\delta)\bar{b} - P]\Pr(c_W > \bar{b}(1-\delta)) = (\frac{1+a+P}{2} - P)(1 + a - \frac{1+a+P}{2}) = \frac{(1+a-P)^2}{4}$$

and

$$E[\pi_S^N(c_S = P)] = \begin{cases} \frac{(1+a-(1-\delta)P)^2}{4(1-\delta)} & \text{if } P \le 2r - \frac{1+a}{1-\delta} \\ (r - P)(1 + a - (1-\delta)r) & \text{if } P > 2r - \frac{1+a}{1-\delta}, \end{cases}$$

we can get that when $P \le 2r - \frac{1+a}{1-\delta}$, $E[\pi_S^Y(c_S = P)]$ is always smaller than $E[\pi_S^N(c_S = P)]$ (since the FOC of $\frac{(1+a-(1-\delta)P)^2}{4(1-\delta)}$ on δ is positive); and when $P > 2r - \frac{1+a}{1-\delta}$, the conditions $P > 2r - \frac{1+a}{1-\delta}$ and $r > \frac{1+a+P}{2(1-\delta)}$ are contradicted with each other. Therefore, under the circumstance that $r > \frac{1+a+P}{2(1-\delta)}$, the threshold P does not exist.

When $\frac{1+a+P}{2(1-\delta)} \ge r > \frac{1+a}{2(1-\delta)}$, we need to find a P such that $E[\pi_S^Y(c_S = P)] = E[\pi_S^N(c_S = P)]$.

Through comparing $E[\pi_S^Y(c_S = P)]$ and $E[\pi_S^N(c_S = P)]$ (Plugging $c_S = P$ into (11)); i.e.,

$$E[\pi_S^Y(c_S = P)] = [(1-\delta)\bar{b} - P]\Pr(c_W > \bar{b}(1-\delta)) = (r(1-\delta) - P)(1 + a - r(1-\delta))$$

and

$$E[\pi_S^N(c_S = P)] = \begin{cases} \frac{(1+a-(1-\delta)P)^2}{4(1-\delta)} & \text{if } P \leq 2r - \frac{1+a}{1-\delta} \\ (r-P)(1+a-r(1-\delta)) & \text{if } P > 2r - \frac{1+a}{1-\delta}, \end{cases}$$

we get that when $P > 2r - \frac{1+a}{1-\delta}$ or when $P \leq 2r - \frac{1+a}{1-\delta}$, $E[\pi_S^Y(c_S = P)]$ is smaller than $E[\pi_S^N(c_S = P)]$. The former is trivial, since $r > (1-\delta)r$. For the latter, if we want to prove $E[\pi_S^Y(c_S = P)] < E[\pi_S^N(c_S = P)]$, we only need to prove that:

$$\begin{aligned} &4(1-\delta)((1-\delta)r - P)(1 + a - (1-\delta)r) < (1 + a - (1-\delta)P)^2 \\ \Leftrightarrow &0 < a^2 + 4(1-\delta)^3 r^2 - 4(1-\delta)^2 r(1+P) + 2a(1 - 2(1-\delta)^2 r + (1-\delta)P) + (1 + P(1-\delta))^2 \end{aligned} \tag{29}$$

Based on simple algebraic calculus, we know that for all P which is smaller than $2r - \frac{1+a}{1-\delta}$, the inequality (29) is established. Therefore the threshold P does not exist.

Finally, let us look at the case $r \leq \frac{1+a}{2(1-\delta)}$, then $E[\pi_S^N(c_S = P)] = (r-P)(1+a-(1-\delta)r)$ which is larger than $E[\pi_S^Y(c_S = P)] = ((1-\delta)r - P)(1+a-(1-\delta)r)$. Therefore, the threshold P does not exist.

From the above analysis, we have proved that we cannot find an appropriate P as well as the equilibrium cost structure $[L, P]$. □

Proof of Lemma 4.

The proof is similar as that in Maskin and Riley (2000) (the superscript SPA denotes the Second-Price Auction and the superscript FPA denotes the First-Price Auction). The difference is there is an effective reserve price r in our model.

Next, we first prove that for any realized cost c ($c \leq \min\{1, r\}$), S could get a higher profit from the second-price auction than that from the first-price auction. Reminder that we use $\gamma_W(c)$ to denote the equilibrium bidding strategy in the symmetric auction with two W, and $b_i(c_i), i \in \{S, W\}$ denote the equilibrium bidding strategies in the asymmetric auction with one S and one

W. The detailed proof is as follows:

$$
\begin{aligned}
E^{SPA}(\pi_S(c, F_S, F_W)) &= E^{SPA}(\pi_W(c, F_W, F_W)) \\
&= E^{FPA}(\pi_W(c, F_W, F_W)) && \text{(by the revenue equivalence theorem)} \\
&= \max_b (b - c)(1 + a - \gamma_W^{-1}(b)) \\
&> (b_S(c) - c)(1 + a - \gamma_W^{-1}(b_S(c))) \\
&> (b_S(c) - c)(1 + a - b_W^{-1}(b_S(c))) && \text{(by Lemma 2 that } b_W^{-1}(b) > \gamma_W^{-1}(b)) \\
&= E^{FPA}(\pi_S(c, F_S, F_W)).
\end{aligned}
$$

Secondly, when the cost realization $c_S > \min\{r, 1\}$, then we can easily get that $E^{SPA}(\pi_S(c, F_S, F_W)) = E^{FPA}(\pi_S(c, F_S, F_W)) = 0$. Hence, we conclude that the *ex ante* expected profit of S is higher under the second-price auction than that under the first-price auction. □

Proof of Lemma 5.

When $r > c_S > 2r - \frac{1}{1-\delta}$, it is trivial that $r > \frac{2r - r^2 - c_S^2}{2(1-c_S)}$ since S will not bid higher than the reserve price r in order to get a positive winning probability. When $c_S \leq (2r - \frac{1}{1-\delta})$, taking the derivative of $b_S^N(c_S) - b_S^0(c_S)$ on c_S, we get $\frac{db_S^N(c_S) - b_S^0(c_S)}{dc_S} = \frac{(1-r)^2}{2(1-c_S)^2} > 0$. Besides, knowing that $b_S^N(c_S) - b_S^0(c_S)|_{c_S=0} > 0$, we find that S's bid in the case of declining to bribe is higher than that in the auction without corruption (bribe). □

Proof of Corollary 2.

If $r < \frac{1}{2(1-\delta)}$, then manufacturer S's payoffs associated with the case of refusing to bribe and with the no bribery case are as follows:

$$
\begin{aligned}
E[\pi_S^N(c_S)] &= \int_0^r (b^N(c_S) - c_S) \Pr(c_W > (1-\delta)r) dc_S \\
&= \int_0^r (r - c_S)(1 - (1-\delta)r) dc_S = \frac{r^2(1 - (1-\delta)r)}{2},
\end{aligned}
$$

and

$$
\begin{aligned}
E[\pi_S^0(c_S)] &= \int_0^r (b^0(c_S) - c_S) \Pr(c_S \leq c_W) dc_S \\
&= \int_0^r \left(\frac{2r - r^2 - c_S^2}{2(1-c_S)} - c_S\right) \times (1 - c_S) dc_S = \frac{r^2}{2} - \frac{r^3}{3}.
\end{aligned} \tag{30}
$$

We derive that when the condition $\delta > \frac{1}{3}$ is satisfied, $E[\pi_S^N(c_S)] - E[\pi_S^0(c_S)] = \frac{-r^3}{6} + \frac{\delta r^3}{2} > 0$.

□

Proof of Lemma 6.

Given the bidding strategy $b_W(c_W)$ of W, S's best strategy $b_s(c_S)$ is derived through solving the following optimization problem:

$$\begin{aligned}\max_b \pi_S(c_S) &= (b - c_S - \kappa)\Pr(b < b_W(c_W)) \\ &= (b - c_S - \kappa)\Pr(b_W^{-1}(b) < c_W) \\ &= (b - c_S - \kappa)(1 + a - b_W^{-1}(b)).\end{aligned}$$

Therefore, the best response $b_S(c_S)$ should satisfy: $\frac{\partial \pi_S(c_S)}{\partial b} = (1 + a - b_W^{-1}(b)) - (b_W^{-1}(b))'(b - c_S - \kappa) = 0$. The boundary conditions are as follows:

$$\begin{aligned}b_W^{-1}(\overline{b}^\kappa) &= \overline{b}^\kappa - \kappa, \\ b_W^{-1}(\underline{b}^\kappa) &= a, \\ b_S^{-1}(\overline{b}^\kappa) &= P, \\ b_S^{-1}(\underline{b}^\kappa) &= L.\end{aligned}$$

Through the boundary condition, we know that when $c_S = P$, the optimal bid for S is $\overline{b}^\kappa \leq r$. Therefore, we can derive the following inequation:

$$(\overline{b}^\kappa - P - \kappa)(1 + a - b_W^{-1}(\overline{b}^\kappa)) \geq (b - P - \kappa)(1 + a - b_W^{-1}(b)) \geq (b - P - \kappa)(1 + a - (b - \kappa)). \quad (31)$$

The second inequality is due to the fact that $b_W^{-1}(b) \leq b - \kappa$. From (31), we know that $\overline{b}^\kappa$ is the maximizer of the function $(b - P - \kappa)(1 + a - (b - \kappa))$. Therefore, $\overline{b}^\kappa = \min\{\frac{1+a+P+2\kappa}{2}, r\}$.

Proof of Theorem 3.

Suppose P exists, a necessary existence condition for the threshold P is that when $c_S = P$, $E[\pi_S^Y]$ is equal to $E[\pi_S^N]$.

When $r > \frac{1+a+P+2\kappa}{2}$, $\overline{b}^\kappa = \frac{1+a+P+2\kappa}{2}$. Accordingly, $E[\pi_S^Y(c_S = P)]$ and $E[\pi_S^N(c_S = P)]$ are as follows:

$$E[\pi_S^Y(c_S = P)] = [\overline{b}^\kappa - \kappa - U]\Pr(c_W > \overline{b}^\kappa - \kappa) = (\frac{1 + a - P}{2})(1 + a - \frac{1 + a + P}{2}) = \frac{(1 + a - P)^2}{4}$$

and

$$E[\pi_S^N(c_S = P)] = \begin{cases} \frac{(1+a+\kappa-P)^2}{4} & \text{if } P \leq 2r - 1 - a - \kappa \\ (r - P)(1 + a - r + \kappa) & \text{if } P > 2r - 1 - a - \kappa. \end{cases} \quad (32)$$

Through comparing the above two equations, we can get that when $P \leq 2r - 1 - a - \kappa$, $E[\pi_S^Y(c_S = P)]$ is always smaller than $\pi_S^N(c_S = P)$; and when $P > 2r - 1 - a - \kappa$, the conditions $P > 2r - 1 -$

$a-\kappa$ and $r > \frac{1+a+P+2\kappa}{2}$ are contradicted with each other. Therefore, under the circumstance that $r > \frac{1+a+P+2\kappa}{2}$, the threshold P does not exist.

When $\frac{1+a+P+2\kappa}{2} \geq r > \frac{1+a+\kappa}{2}$, we need to find a P such that $E[\pi_S^Y(c_S = P)] = E[\pi_S^N(c_S = P)]$. Through comparing $E[\pi_S^Y(c_S = P)]$ in (19), i.e, $E[\pi_S^Y(c_S = P)] = (r-P-\kappa)(1+a-r+\kappa)$ and $E[\pi_S^N(c_S = P)]$ in (32), we get that when $P > 2r-1-a-\kappa$ or when $P \leq 2r-1-a-\kappa$, $E[\pi_S^Y(c_S = P)]$ is smaller than $E[\pi_S^N(c_S = P)]$. The former is trivial due to the fact of $r > r-\kappa$. For the latter, we find that

$$\begin{aligned} &\max_P\{(r-P-\kappa)(1+a-r+\kappa) - \frac{(1+a+\kappa-P)^2}{4}\} \\ &= (r-P-\kappa)(1+a-r+\kappa) - \frac{(1+a+\kappa-P)^2}{4}|_{P=-1-a-\kappa+2r} = \kappa(-1-a-\kappa+r) \leq 0. \end{aligned} \tag{33}$$

Therefore we prove that the threshold P does not exist.

Finally, let us look at the case $r \leq \frac{1+a+\kappa}{2}$, then $E[\pi_S^N(c_S = P)] = (r-P)(1+a-r+\kappa)$ which is larger than $E[\pi_S^Y(c_S = P)] = (r-P-\kappa)(1+a-r+\kappa)$. Therefore, the threshold P does not exist.

To conclude, we have proved that we cannot find an appropriate P as well as the cost structure $[L, P]$. □

Proof of Theorem 7.

When $c_S > r$, it is extremely trivial that S should choose to collude to be a free rider. To fully derive the optimal bribery decision for S, we need to consider 5 cases when $c_S \leq r$:

1. when $2r - \frac{1}{1-\delta} \leq 0$ and $c_S < (1-\delta)r$, $\mathbb{E}(\pi_S^Y-)\mathbb{E}(\pi_S^N) = \frac{r(1-\delta)}{2} - \frac{1}{2}(c_S - \frac{c_S^2}{2}) - (r-c_S)(1-(1-\delta)r)$;
2. when $2r - \frac{1}{1-\delta} \leq 0$ and $1-\delta)r \leq c_S \leq r$, $\mathbb{E}(\pi_S^Y-)\mathbb{E}(\pi_S^N) = \frac{r^2(1-\delta)^2}{4} - (r-c_S)(1-(1-\delta)r)$;
3. when $2r - \frac{1}{1-\delta} > 0$ and $c_S \leq 2r - \frac{1}{1-\delta}$, $\mathbb{E}(\pi_S^Y-)\mathbb{E}(\pi_S^N) = \frac{r(1-\delta)}{2} - \frac{1}{2}(c_S - \frac{c_S^2}{2}) - \frac{(1-(1-\delta)c_S)^2}{4(1-\delta)}$
4. when $2r - \frac{1}{1-\delta} > 0$ and $2r - \frac{1}{1-\delta} < c_S < (1-\delta)r$, $\mathbb{E}(\pi_S^Y-)\mathbb{E}(\pi_S^N) = \frac{r(1-\delta)}{2} - \frac{1}{2}(c_S - \frac{c_S^2}{2}) - (r-c_S)(1-(1-\delta)r)$
5. when $2r - \frac{1}{1-\delta} > 0$ and $(1-\delta)r \leq c_S \leq r$, $\mathbb{E}(\pi_S^Y) - \mathbb{E}(\pi_S^N) = \frac{r^2(1-\delta)^2}{4} - (r-c_S)(1-(1-\delta)r)$.

To simplify our expression, we just show how to derive the optimal bribery strategy for case 1). For the other cases, the same method is implemented.

Analyzing the function $\mathbb{E}(\pi_S^Y -)\mathbb{E}(\pi_S^N)$, we obtain that when

$$r \leq \frac{1-3\delta}{-4(1-\delta)\delta} + \frac{1}{4}\sqrt{\frac{1-2\delta+5\delta^2}{(1-\delta)^2\delta^2}} \tag{34}$$

and $c_S \leq \bar{r}$, the function $\mathbb{E}(\pi_S^Y) - \mathbb{E}(\pi_S^N)$ is non-positive. Furthermore, we find that $\frac{1-3\delta}{-4(1-\delta)\delta} + \frac{1}{4}\sqrt{\frac{1-2\delta+5\delta^2}{(1-\delta)^2\delta^2}} > \frac{1}{2(1-\delta)}$. Therefore, (34) can be transferred into $r \leq \frac{1}{2(1-\delta)}$. Now, we pay attention to the condition $c_S \leq \bar{c}$ where $\bar{c} = -1 + 2r - 2r\delta + \sqrt{1 - 2r + 6r\delta - 4r^2\delta + 4r^2\delta^2}$. We find that: 1) when $r \leq \frac{4\delta}{1+2\delta-3\delta^2}$, $\bar{c} \geq r(1-\delta)$; 2) when $r < \frac{4\delta}{1+2\delta-3\delta^2}$, $\bar{c} < r(1-\delta)$; 3) when $\delta \leq \frac{1}{5}$, $\frac{4\delta}{1+2\delta-3\delta^2} \leq \frac{1}{2(1-\delta)}$. Consequently, we can conclude that:

1. when $\delta \leq \frac{1}{5}$ and $r \leq \frac{4\delta}{1+2\delta-3\delta^2}$, the optimal strategy for S is refusing to offer a bribe;
2. when $\delta \leq \frac{1}{5}$ and $\frac{4\delta}{1+2\delta-3\delta^2} < r$, if $c_S \leq \bar{c}$, the optimal strategy for S is refusing to offer a bribe; and if $\bar{c} < c < (1-\delta)r$, the optimal strategy is to join the coalition;
3. when $\delta > \frac{1}{5}$ and when $r \leq \frac{1}{2(1-\delta)}$, if $c_S \leq \bar{c}$, the optimal strategy for S is refusing to offer a bribe.

Similarly, we can derive the optimal strategy for the other cases and eventually, we can get Theorem 7. $\square$