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A Parsimonious Model of Momentum and Reversals in Financial Markets

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Abstract

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We develop a model where overconfident investors overestimate their ability to produce information but are skeptical of others' ability. Skeptical traders that are yet to receive information believe that the informed have learned little. This leads to excess liquidity provision and hence, underreaction and momentum. Skepticism also causes prices to react to stale information, implying overreaction and long-term reversals. Hence, skepticism alone can generate both momentum and reversals; however, reversals are amplified because traders over-assess their signal quality. We explain how long-run reversals can disappear while shorter-term momentum prevails, provide empirical implications, and link momentum to liquidity and informational efficiency.

1 Introduction

Since Jegadeesh and Titman (1993) financial economists have puzzled over evidence that stocks that perform relatively well over a six to twelve month period tend to exhibit positive excess returns over the following 12 months. With some exceptions, this momentum phenomenon is observed in most stock markets around the world.[1] Jegadeesh and Titman (1993) show that part of the momentum effect reverses after one year, which is consistent with the longer term reversals originally documented in De Bondt and Thaler (1985). Jegadeesh and Titman (2001) show that while long-run reversals are present in their early sample period, they disappear in their later sample period. More recent evidence by Avramov, Cheng, and Hameed (2016) finds that momentum profits are positively related to aggregate market liquidity and Daniel and Moskowitz (2016) find that momentum strategies do not yield positive average returns in adverse states (i.e., recessions or down markets).

The issue of how the empirically observed return predictability can arise is important, and accordingly, an extensive literature models both return momentum and reversals. Some models generate underreaction and momentum, but not subsequent reversals (e.g., Grinblatt and Han (2005), and Da, Gurun, and Warachka (2014)). Underreaction also occurs in Eyster, Rabin, and Vayanos (2019) and Banerjee (2011) where traders underestimate the information content of prices.[2] The same phenomenon occurs in Odean (1998), if investors under-assess the precision of either their own or others' information signals. Other models generate momentum and reversals due to multiple cognitive biases (e.g., biased self-attribution and overconfidence drive the Daniel, Hirshleifer, and Subrahmanyam (1998) model; and Barberis, Shleifer, and Vishny (1998) use conservatism and representativeness).[3]

[1]The international evidence is described in Rouwenhorst (1998) and Rouwenhorst (1999), Griffin, Ji, and Martin (2005), and Chui, Titman, and Wei (2010). The notable exceptions include Japan and China. Asness, Moskowitz, and Pedersen (2013) show that momentum extends to other asset classes such as currencies and commodities. There is evidence of excess returns associated with time-series momentum, i.e., buying stocks when their returns are high relative to their past returns (Moskowitz, Ooi, and Pedersen (2012)), as well as cross-sectional momentum. Goyal and Jegadeesh (2017) argue that time-series momentum strategies based on exogenous benchmarks (such as buying stocks whose returns exceed zero) are not market-neutral (unlike standard cross-sectional momentum strategies). They show that accounting for market performance materially mitigates time-series momentum profits.

[2]See also Banerjee, Kaniel, and Kremer (2009), Banerjee and Kremer (2010), and Vives and Yang (2017).

[3]Cespa and Vives (2011) and Andrei and Cujean (2017) rely on rationality and risk aversion to gener-

In Hong and Stein (1999), momentum arises due to "news watchers" who do not condition on market price, and reversals occur because of unwinding of positions by "trend-chasers" (who trade based on direction of past price changes). These models have been quite influential in shaping thought on how financial markets can yield both momentum in the short-run and reversals in the long-run.[4]

While acknowledging the insights provided by previous work, we show that short-term momentum and long-term reversals arise in a simpler and more intuitive setting than the existing frameworks in the literature. Specifically, we show the emergence of these return patterns in a framework where overconfident investors overestimate their ability to produce information and underestimate the ability of others to do so.[5] In fact, we go one step further and demonstrate that the former aspect of overconfidence is not required for either continuations or reversals. Thus, both short-term momentum and long-term reversals arise when informed investors simply underestimate their competition. Our work thus adds parsimony and clarity to earlier work that links investors' biases to asset prices. We also contribute to the literature in other ways. First, we provide a framework for understanding conditions under which momentum and reversals might weaken with time, and why the reversals might weaken or even disappear without material changes in the momentum effect. Second, we address two other recent findings related to momentum. Specifically, we show that our model accords with the empirical findings of Avramov, Cheng, and Hameed (2016) on the link between liquidity and momentum, and under certain conditions, with momentum crashes (Daniel and Moskowitz (2016)). Finally, we provide several untested implications.

In our model, not all investors receive information at the same time; that is, some in-

ate momentum and reversals. Though these papers provide important economic insights, the Sharpe ratios achievable via momentum seem too large to be explained by rational models (Brennan, Chordia, and Subrahmanyam (1998)). Reversals at weekly and monthly horizons (Jegadeesh (1990), Lehmann (1990)) are usually attributed to illiquidity (see, for example, Nagel (2012)).

[4]Attesting to their influence, the models of Hong and Stein (1999), Daniel, Hirshleifer, and Subrahmanyam (1998), and Barberis, Shleifer, and Vishny (1998) have collectively achieved more than 15,000 Google Scholar citations to date.

[5]De Bondt and Thaler (1995) surmise that "perhaps the most robust finding in the psychology of judgment is that people are overconfident." See also Lichtenstein, Fischhoff, and Phillips (1982) for evidence on the pervasive nature of this bias. Odean (1998) and, more recently, Daniel and Hirshleifer (2015) provide excellent discussions of how overconfidence affects securities markets.

vestors become informed before others.[6] Further, while investors are non-myopic utility maximizers, they over-assess their own signals' quality and are skeptical about the quality of others' information signals. The latter facet of overconfidence has been in the literature since Odean (1998), and accords with Johnson and Fowler (2011), who state (p. 317) that "overconfidence amounts to an 'error' of judgement or decision-making, because it leads to overestimating one's capabilities and/or underestimating an opponent..." (see also Ando (2004)). Due to the latter notion of overconfidence (i.e., skepticism), as-yet uninformed investors assume that those with valid information have learned little of consequence.[7] Thus, the former investors provide too much liquidity to the latter ones, which means that informed trades move prices too little. If the mass of skeptical investors is not too small, there is short-run momentum in equilibrium.

Prices also exhibit long-run reversals in our setting. These reversals arise for two reasons. The first is the standard notion that the investors overestimate the quality of their own signal, which causes overreaction and a subsequent reversal to fundamentals. Second, informed investors discount the possibility that the information they receive is already incorporated in past prices, which causes the price to underreact to information in earlier rounds and then react to stale information in later rounds, before finally reversing on average.[8] The latter phenomenon implies that the skepticism aspect of overconfidence can generate momentum as well as subsequent reversals. This result adds new theoretical perspective to the intuitive notion that skepticism about information quality results in underreaction, and overassessing information quality implies overreaction. Note that our model does include overconfident investors as in Daniel, Hirshleifer, and Subrahmanyam (1998) and these investors underreact to information like the "news-watchers" in Hong and Stein (1999). However, in the former model, the self-attribution bias generates momentum, while in the latter, trend-chasing behavior generates reversals. Our parsimonious setting demonstrates that the secondary aspects of these models, namely the self-attribution bias and trend-chasers, are not essential

[6]See Froot, Scharfstein, and Stein (1992) and Hirshleifer, Subrahmanyam, and Titman (1994).

[7]For tractability, the signal of those receiving information later strictly dominates that of the those receiving information sooner, which precludes the early informed from being skeptical about the late informed signal. Relaxing this assumption does not alter the thrust of our economic intuition.

[8]See Huberman and Regev (2001) for evidence of prices moving in response to stale information.

attributes for the empirically observed momentum and reversal patterns.

After presenting our basic results, we extend the model to incorporate disclosures, which can be analysts' revisions, earnings releases, or other information obtained from public sources. Given that overconfident investors with their own signals would tend to be skeptical about the quality of information from these outside sources,[9] they underreact on average to such announcements. This accords with post-event drift following earnings releases and analysts' revisions (Bernard and Thomas (1989) and Womack (1996)).[10] We also consider how changes in the timing of disclosures affect momentum profits within our setting. For example, because of improvements in technology the speed of information flows has probably increased (Economides (2001)), which implies that information is publicly available earlier in time. We show that moderately accelerated disclosures can reduce long-term overreaction and thus reduce reversals without necessarily affecting the shorter-term momentum effect. The finding in Jegadeesh and Titman (2001), that momentum profits are similar in the pre- and post-1982 period even though reversals substantially decrease post-1982, is consistent with this observation.[11] Accelerating disclosures even further, of course, attenuates both momentum and reversals.

The model can also be used to explore the implications of the recent emergence of quantitative investors (Patterson (2010); Abis (2017)). To explore this, we extend the model to include rational uninformed investors, who represent these "quants." We show that these investors tend to mitigate momentum (Chordia, Subrahmanyam, and Tong (2014)), but

[9] An overconfident investor reasons as follows: "An analyst or a manager can teach me little about this stock that I don't already know."

[10] The form of overconfidence in Daniel, Hirshleifer, and Subrahmanyam (1998) (DHS) does not deliver drift following earnings announcements. This is because the overconfident investor over-assesses the noise variance of only the private signal. Conditional on public signals alone, returns are unpredictable since the noise variance of public signals is correctly assessed (see Part 1 of Proposition 4 in DHS). Our recognition that overconfidence not only leads to overestimation of the quality of one's own information, but also to underestimation of the quality of other information sources, naturally leads to drift following public signal releases.

[11] Conrad and Yavuz (2017) provide evidence suggesting that stocks that contribute most to momentum profits do not reverse. The evolution of momentum and reversal profits over time is not the focus of their paper, so they only consider aggregated results for the full (1963-2010) sample period. Their point estimates of the long-term reversal coefficients, however, are negative (with a maximum one-sided p-value of 9%) even for high momentum stocks (see Table III of the paper), suggesting statistical power issues in reliably detecting long-term reversals.

as long as they are risk averse, they do not completely eliminate the pattern. Further, our analysis provides insights about the relation between liquidity and momentum. Specifically, greater skepticism implies that investors without valid signals provide more liquidity to the informed investors, which makes markets more liquid, and also enhances momentum. This implies that if overconfidence and, consequently, skepticism, differ either across stocks (e.g., investors tend to be more skeptical about the quality of others' technology-related information) or over time, then we will observe a positive relation between momentum and liquidity. Consistent with this implication, Avramov, Cheng, and Hameed (2016) find that momentum profits are markedly larger when the market is more liquid. Our setting also implies a link between the variance of noise trading and momentum. Because other investors demand price discounts to accommodate noise trades, a large increase in the variance of noise trading implies an attenuation and even reversal of momentum profits. This implies that phenomena such as fire sales by mutual funds, and investors' sales for liquidity needs during economic downturns, will tend to reduce, and may even reverse, momentum (Daniel and Moskowitz (2016)).

Earlier literature (e.g., Odean (1998)) indicates that if investors overestimate the precision of their information (due to overconfidence), then prices deviate more from fundamentals. This occurs because investors put too much weight on the noisy signal. The other aspect of overconfidence, skepticism, mitigates this distortion in our setting, because skeptical investors underweight others' signals. Excessive skepticism "overcorrects" the distortion, however. The intuition is that as skepticism becomes large, prices respond so little to fundamentals that the gap between prices and fundamentals actually increases.

Our contribution is also related to Hong, Lim, and Stein (2000), who show that the momentum effect is stronger for stocks that are followed by fewer analysts. We provide two possible channels that can generate this result. The first channel, that arises because late informed investors provide too much liquidity to the early informed investors, is very much in the spirit of the Hong, Lim, and Stein (2000) idea that information tends to travel too slowly in stocks with limited analyst coverage, and analysts' disclosures speed up the incorporation of news into prices, thus reducing momentum. Our second channel proposes an alternative role for stock analysts in mitigating the effect of skepticism. As we show, momentum can arise because skeptical investors believe they are the first to uncover relevant information, and as a result, in the absence of analysts who publicly describe what already is known, they overreact to stale information. We show that analysts' disclosures counteract the late informed's skepticism by directly releasing the stale information in earlier rounds, and thus reduce momentum.

This paper is organized as follows. Section 2 presents the model. Section 3 extends the model, and examines how rational uninformed investors, news releases, and noise trades affect momentum. Section 4 studies other patterns such as price quality and liquidity. Section 5 shows that skepticism alone can lead to both momentum and reversals. Section 6 relates our analysis to the diminution of momentum and reversals over time, and also presents untested implications. Section 7 concludes. All proofs, unless otherwise stated, appear in the appendix.

2 The Model

There are $J + K$ stocks traded at Dates 0, 1, and 2. The liquidation value of the j'th ($j = 1, ..., J$) stock at Date 3 is given by

$$V_j = \bar{V}_j + \sum_{k=1}^{K} (\beta_{jk} f_k) + \theta_j, \tag{1}$$

where $\bar{V}_j$ is public knowledge and is the unconditional mean, and θ_j represents the firm-specific risk. The liquidation value of the $J + k$'th ($k = 1, ..., K$) stock at Date 3 equals the realization of the k'th factor, i.e.,

$$V_{J+k} = f_k.$$

Thus, this security is a portfolio mimicking the k'th factor. All f's and θ's follow independent normal distributions with mean zero.

There are two types of informed investors: a mass m of "early-informed" and a mass $1 - m$ of "late-informed" investors. For the j'th ($j = 1, ..., J$) stock, the early-informed observe a private signal of the firm-specific risk θ_j at Date 1, $s_j = \theta_j + \mu_j + \epsilon_j$. Late-informed investors observe a private signal at Date 2, $\gamma_j = \theta_j + \mu_j$. μ_j and ϵ_j are independent normal random variables with mean zero, and are independent of θ_j. Here, we let γ_j strictly dominate s_j. This assumption, which greatly simplifies our derivation, captures the idea that information received later contains more news and therefore tends to be more precise than early information. Throughout the paper, unless otherwise specified, random variables follow a normal distribution with zero mean. The variance of a random variable ζ is denoted by ν_ζ.

All investors observe publicly available signals f_k about factor k. Specifically, they observe $s_{J+k} = f_k + \mu_{J+k} + \epsilon_{J+k}$ at Date 1, and $\gamma_{J+k} = f_k + \mu_{J+k}$ at Date 2, where μ_{J+k} and ϵ_{J+k} are independent normal random variables with mean zero. We assume that investors assess these factor signals in an unbiased way. There is also a risk free asset, the price and return of which is normalized to be 1. For now, we fix the total supply of each stock, which is normalized to be zero. With fixed supply, the stock price reveals early- and late-informed investors' private information, and trade happens because the early- and late-informed use different values of the model parameters in their optimization problems. This framework allows us to obtain clear intuition about how beliefs affect stock prices. In Section 3.3, we examine how allowing noise trades affects the equilibrium.

The i'th early- or late-informed investor's utility function is the standard exponential:

$$U(W_{i3}) = -\exp(-AW_{i3}),$$

where W_{i3} is final wealth, and A is a positive constant representing the absolute risk aversion coefficient. The investor is endowed with $\bar{W}_{i0}$ units of the risk free asset. Note that we analyze a single cohort of investors trading prior to a news announcement at Date 3. The model can easily be extended to an infinite horizon setting with multiple periodic news announcements, however, under the assumption that each cohort of traders reverses its position after each announcement. Our results then readily carry over to this setting (see, for example, Holden and Subrahmanyam (2002), Section II).

2.1 Skepticism About Signal Quality

An overconfident (skeptical) belief about the variance of the random variable ζ is denoted ω_ζ (κ_ζ). Specifically, the early-informed overestimate the quality of their Date-1 information s_j in that they believe ϵ_j has a smaller variance (i.e., $\omega_{\epsilon_j} < \nu_{\epsilon_j}$). Late informed investors also overestimate the quality of their late Date-2 information γ_j in that they believe μ_j has a smaller variance (i.e., $\omega_{\mu_j} < \nu_{\mu_j}$). We assume that the late-informed are skeptical about the quality of early-informed investors' Date-1 information s_j in that they believe ϵ_j has a larger variance (i.e., $\kappa_{\epsilon_j} > \nu_{\epsilon_j}$). Note that our nested information structure where the early informed signal is late informed signal plus noise (assumed for tractability) rules out early informed being skeptical about the late informed signal. This is because such an assumption would mean that the early informed are skeptical about their own signal while also over-assessing its quality, which is impossible.[12]

2.2 Skepticism About Fundamentals

Late informed may also exhibit skepticism in another form; specifically, they may believe that the early informed have not observed a fundamental component of value when in fact the latter traders have. To accommodate this, we assume a revised signal structure that accords with Section 2.1 (as we will shortly show). Suppose that the firm specific variable θ_j can be decomposed into two components: $\theta_j \equiv \theta_{j1} + \theta_{j2}$, where θ_{j1} and θ_{j2} follow independent normal distributions with mean zero and variance $(1 - \Delta_j)\nu_{\theta_j}$ and $\Delta_j \nu_{\theta_j}$, respectively. $\Delta_j \in [0, 1)$ is a constant parameter. Further, μ_j can be decomposed into two components: $\mu_j \equiv \mu_{j1} + \mu_{j2}$, where μ_{j1} and μ_{j2} follow independent normal distributions with mean zero and variance $(1 - \Delta_j)\nu_{\mu_j}$ and $\Delta_j \nu_{\mu_j}$, respectively. Both early- and late-informed investors observe a signal $\gamma_{j2} = \theta_{j2} + \mu_{j2}$ at Date 2. Late informed believe that the early signal contains only information about θ_{j1}, that is, $s_j = \theta_{j1} + \mu_{j1} + \epsilon_j$ (accordingly, they believe that the late signal $\gamma_j = \theta_{j1} + \mu_{j1}$). In this specification, the proportion of information in θ_{2j} equals the proportion of noise in μ_{2j}. This implies that γ_j dominates γ_{j2} regarding θ_j; thus, γ_{j2} is stale

[12]It is possible to model non-nested information structures, but we lose closed-form solutions and have to analyze the model numerically. Though skepticism by the early informed about late signals at Date 2 tends to mitigate long-run reversals, the basic results continue to obtain under a wide range of parameter values.

information.

For now, we fix $\Delta_j \equiv 0$; thus, $\gamma_{j2} \equiv 0$ and $\theta_{j1} \equiv \theta_j$, which is equivalent to Section 2.1, and thus amounts to assuming that skepticism only involves underestimation of the early informed's signal quality. In Section 5, we allow $\Delta_j > 0$ to obtain additional insights on skepticism's effects on asset prices. The most general version of the model also has (i) a set of rational, risk averse traders who act as market makers and infer information from market prices, (ii) public signals that arrive either at Date 1 or at Date 2, and (iii) noise traders at either date. To demonstrate our main results transparently and tractably, we abstract from these features for now, and introduce them sequentially in Section 3.

2.3 The Equilibrium with Skepticism About Signal Quality

We can decompose the $J + K$ original stocks into the risk-free asset and what we refer to as the basic securities, which include the J firm-specific risks and the K factor-mimicking portfolios.[13] The j'th ($j = 1, ..., J$) basic security has a payoff θ_j. The $J+k$'th ($k = 1, ..., K$) basic security has a payoff f_k. Note that one unit of the j'th ($j = 1, ..., J$) original stock includes $\bar{V}_j$ units of the risk free asset, β_{jk} units of the k'th factor (f_k), and one unit of the j'th firm-specific risk (θ_j).

Lemma 1 *Trading the original stocks is equivalent to trading the basic securities in that the price of the j'th ($j = 1, ..., J$) non-factor original stock, denoted $P_t(V_j)$ ($t = 0,1,$ and 2), is a linear combination of the prices of the basic securities:*

$$P_t(V_j) = \bar{V}_j + \sum_{k=1}^{K}(\beta_{jk} P_{J+k,t}) + P_{j,t},$$

and the price of the $J + k$'th ($k = 1, ..., K$) original factor security is given by $P_t(V_{J+k}) = P_{J+k,t}$. $P_{j,t}$ and $P_{J+k,t}$ are the prices of the basic securities.

Conjecture that the equilibrium prices of the j'th ($j = 1, ..., J$) firm-specific risk (θ_j) at Dates 0, 1, and 2 take a linear form:

$$P_{j0} = 0, \quad P_{j1} = \alpha_{j1} s_j, \text{ and } P_{j2} = \alpha_{j2} \gamma_j, \tag{2}$$

[13]See, for example, Van Nieuwerburgh and Veldkamp (2010), Banerjee (2011), and Daniel, Hirshleifer, and Subrahmanyam (2001) for a similar exercise.

where the parameters α_{1j} and α_{2j} are to be determined. Further postulate that the prices of the k'th factor (f_k) at Dates 0, 1, and 2 are given by

$$P_{J+k,0} = 0, \quad P_{J+k,1} = \frac{\nu_{f_k}}{\nu_{s_{J+k}}} s_{J+k}, \text{ and } P_{J+k,2} = \frac{\nu_{f_k}}{\nu_{\gamma_{J+k}}} \gamma_{J+k}. \tag{3}$$

The early-informed's total variance of their signal s_j is denoted $\omega_{s_j} = \nu_{\theta_j} + \nu_{\mu_j} + \omega_{\epsilon_j}$. Similarly, late informed assume that $\text{var}(\gamma_j) = \omega_{\gamma_j} = \nu_{\theta_j} + \omega_{\mu_j}$. Their belief about the total variance of s_j is denoted $\kappa_{s_j} = \omega_{\gamma_j} + \kappa_{\epsilon_j}$. Define a function

$$H(x, y) \equiv \frac{y}{x(y - x)}, \tag{4}$$

to which we will frequently refer in what follows. All the $H(.)$'s can be interpreted as certain conditional precisions based on early- or late-informed investors' biased or rational beliefs. For example, $H(\nu_{\gamma_j}, \omega_{s_j})$ is the precision of γ_j conditional on s_j based on early-informed investors' belief of the variances of ν_{γ_j} and ω_{s_j}. Denote

$$\begin{aligned}
n_{j\eta} &= m \left[H(\nu_{\gamma_j}, \omega_{s_j}) + H(\nu_{\theta_j}, \nu_{\gamma_j}) \left(\frac{\nu_{\theta_j}}{\nu_{\gamma_j}} - \alpha_{j2} \right)^2 \right], \\
n_{j\ell} &= (1 - m) \left[H(\omega_{\gamma_j}, \kappa_{s_j}) + H(\nu_{\theta_j}, \omega_{\gamma_j}) \left(\frac{\nu_{\theta_j}}{\omega_{\gamma_j}} - \alpha_{j2} \right)^2 \right], \text{ and} \\
N_j &= n_{j\eta} + n_{j\ell}.
\end{aligned}$$

We also use the subscript-η (ℓ) to refer to early- (late-) informed investors.

The proposition below verifies the conjectured price functions in Eqs. (2) and (3), and presents the parameters α_{1j} and α_{2j}.

Proposition 1 *The parameters in the equilibrium prices of the j'th ($j = 1, ..., J$) firm-specific risk (θ_j) in Eq. (2), α_{1j} and α_{2j}, are specified by*

$$\begin{aligned}
\alpha_{j2} &= \frac{mH(\nu_{\theta_j}, \nu_{\gamma_j}) \nu_{\gamma_j}{}^{-1} \nu_{\theta_j} + (1 - m) H(\nu_{\theta_j}, \omega_{\gamma_j}) \omega_{\gamma_j}{}^{-1} \nu_{\theta_j}}{mH(\nu_{\theta_j}, \nu_{\gamma_j}) + (1 - m) H(\nu_{\theta_j}, \omega_{\gamma_j})}, \text{ and} \\
\alpha_{j1} &= \frac{\alpha_{j2}}{N_j} \left[mH(\nu_{\gamma_j}, \omega_{s_j}) \frac{\nu_{\gamma_j}}{\omega_{s_j}} + (1 - m) H(\omega_{\gamma_j}, \kappa_{s_j}) \frac{\omega_{\gamma_j}}{\kappa_{s_j}} \right].
\end{aligned}$$

As can be seen, the prices of the firm-specific risks are linear in the signals at each date, and the parameters α_{j1} and α_{j2} depend on both the extent to which investors overestimate (underestimate) the quality of their own (others') information.

In contrast, the prices of the factor securities are linear in the signals at each date, but the linear parameters do not depend on biased beliefs because there are no biases regarding factor information. In particular, it is easy to show that

$$\mathrm{Cov}(P_{J+k,1} - P_{J+k,0}, P_{J+k,2} - P_{J+k,1}) = \mathrm{Cov}(P_{J+k,2} - P_{J+k,1}, f_k - P_{J+k,2}) = \mathrm{Cov}(P_{J+k,1} - P_{J+k,0}, f_k - P_{J+k,2}) = 0.$$

Now consider two short-run momentum investments using the J original stocks. First, at Date 1, buy $P_1(V_j) - P_0(V_j)$ shares of the j'th original stock if $P_1(V_j) > P_0(V_j)$ and sell $P_0(V_j) - P_1(V_j)$ shares if $P_1(V_j) \leq P_0(V_j)$, and hold this position until Date 2. Second, at Date 2, buy $P_2(V_j) - P_1(V_j)$ shares if $P_2(V_j) > P_1(V_j)$ and sell $P_1(V_j) - P_2(V_j)$ shares if $P_2(V_j) \leq P_1(V_j)$, and hold this position until Date 3. From Eq. (1), Lemma 1, and Proposition 1, the expected profits of the two short-run momentum investments can be expressed as

$$\begin{aligned} E\left[\sum_{j=1}^{J}[(P_1(V_j) - P_0(V_j))(P_2(V_j) - P_1(V_j))]\right] &= \sum_{j=1}^{J}\mathrm{Cov}(P_{j1} - P_{j0}, P_{j2} - P_{j1}), \text{ and} \\ E\left[\sum_{j=1}^{J}[(P_2(V_j) - P_1(V_j))(V_j - P_2(V_j))]\right] &= \sum_{j=1}^{J}\mathrm{Cov}(P_{j2} - P_{j1}, \theta_j - P_{j2}). \end{aligned}$$

It also follows from Proposition 1 that (for convenience, we suppress the index for stock j here)

$$\mathrm{Cov}(P_1 - P_0, P_2 - P_1) = \alpha_1(\alpha_2\nu_\gamma - \alpha_1\nu_s), \tag{5}$$

and

$$\mathrm{Cov}(P_2 - P_1, \theta - P_2) = (\alpha_2 - \alpha_1)(\nu_\theta - \alpha_2\nu_\gamma). \tag{6}$$

Now, if the following expression

$$\overline{MOM} \equiv [\mathrm{Cov}(P_1 - P_0, P_2 - P_1) + \mathrm{Cov}(P_2 - P_1, \theta - P_2)]/2 \tag{7}$$

is positive (negative), then this stock will contribute a momentum profit (loss). Henceforth, we occasionally refer to $\overline{MOM}$ as the "momentum parameter."

We follow Jegadeesh and Titman (2001) to measure the long-run performance of the short-run momentum investment using

$$E\left[\sum_{j=1}^{J}[(P_1(V_j)-P_0(V_j))(V_j-P_2(V_j))]\right]=\sum_{j=1}^{J}\mathrm{Cov}(P_{j1}-P_{j0},\theta_j-P_{j2}).$$

We can use a similar analysis as above to show that the j'th stock's contribution to this performance is indicated by (for convenience, we suppress the index for stock j again here)

$$\mathrm{Cov}(P_1-P_0,\theta-P_2)=\alpha_1(\nu_\theta-\alpha_2\nu_\gamma). \tag{8}$$

The above expression is the quantity that captures long-run reversals in our model (and is consistent with Jegadeesh and Titman (2001)).[14]

We now analyze conditions under which we obtain short-run momentum and long-term reversals.

The following proposition provides comparative statics associated with the parameter α_2 in the Date-2 price, which influences momentum and reversals.

Proposition 2 *The sensitivity of the Date 2 price to the information signal γ, i.e., $\alpha_2 \in [\nu_\theta/\nu_\gamma, \nu_\theta/\omega_\gamma]$ (see Proposition 1), is higher if there is a bigger mass of late-informed investors (high $1-m$), and if they overestimate the quality of their information γ to a greater extent (low ω_μ).*

If there is a big mass of late-informed investors, since they overestimate the precision of their signal γ, the Date-2 price P_2 overreacts to their information and increases the sensitivity of P_2 to γ (high α_2). This overreaction also leads to long-run reversals:

Corollary 1 *Stock returns reverse in the long-run; i.e.,* $\mathrm{Cov}(P_1-P_0,\theta-P_2)<0$.

Figure 1 plots the long-term reversal parameter $\mathrm{Cov}(P_1-P_0,\theta-P_2)$, conditional on late-informed investors' skepticism about the quality of the early Date-1 information (i.e., $\kappa_\epsilon > \nu_\epsilon$), and overestimation of the quality of their late Date-2 information (i.e., $\omega_\mu < \nu_\mu$).

[14] Instead defining reversals as $\mathrm{cov}(\theta-P_2,P_2-P_0)$ (i.e., capturing future performance based on the past "long-run" from time 2 to time 0) leads to similar results.

We assume the parameter values $m = 0.1$, $A = 1$, $\nu_\theta = 1$, $\nu_\mu = 1$, and $\nu_\epsilon = 1$. We let $\omega_\epsilon = \nu_\epsilon$ so that early-informed investors have rational beliefs about their information's quality. There are three notable observations in Figure 1. First, if late-informed investors are rational about their own information quality (i.e., $\omega_\mu = \nu_\mu = 1$), there is no long-term reversal (or momentum). Second, if the late-informed overestimate the quality of their own information (i.e., $\omega_\mu < \nu_\mu = 1$), we do obtain long-run reversal. This result is consistent with Corollary 1. Third, if late-informed investors underestimate the quality of the early information to a greater extent (i.e., $\kappa_\epsilon \gg \nu_\epsilon = 1$), the long-run reversal is smaller in magnitude (less negative). The reason is that as we show below, late-informed investors' skepticism suppresses the Date-1 stock price reaction to information (i.e., α_1 in $P_1 = \alpha_1 s$; see Proposition 1). This tends to lower the dependence between $P_1 - P_0$ and the later price movement, $\theta - P_2$.

Next, the corollary below describes short-term return patterns.

Corollary 2 *In equilibrium,*

(i) $\mathrm{Cov}(P_2 - P_1, \theta - P_2) < 0$, *and*

(ii) there exists a constant parameter $m^* \in (0, 1)$ *such that if* $m < m^*$, $\mathrm{Cov}(P_1 - P_0, P_2 - P_1) > 0$.

Part (i) of this corollary is consistent with Corollary 1. The price overreacts to new information at Date 2 because late-informed investors overestimate the quality of the information. It reverts subsequently as the fundamental θ is revealed. The intuition for Part (ii) of this corollary is as follows. Late-informed investors are skeptical about the quality of the early Date-1 information. A substantial mass of such investors (high $1-m$) allows for the provision of "too much" liquidity, that is, the absorption of early-informed trades at prices excessively favorable to these investors. This means that on a positive information signal, for example, the price does not rise sufficiently at Date 1. In turn, this leads to momentum.

From Corollary 2, since $\mathrm{Cov}(P_2-P_1, \theta-P_2)$ is negative and $\mathrm{Cov}(P_1-P_0, P_2-P_1)$ is positive if m is small, a big mass of late-informed investors, $1 - m$, is a necessary condition for the momentum parameter, $\overline{MOM}$, to be positive; however, it is not a sufficient condition. To see why, let m be very small (i.e., $m \to 0$). In this case, only a few investors are early-informed, but they still trade to reveal their early Date-1 information s. Most investors are late-informed; they effectively set the price. Suppose that late-informed investors overestimate the quality of their Date-2 information γ to a significant extent (i.e., $\omega_\mu < \nu_\mu$). It follows from Proposition 1 that in the price functions $P_1 = \alpha_1 s$ and $P_2 = \alpha_2 \gamma$, $\alpha_1 = \nu_\theta/\kappa_s$ and $\alpha_2 = \nu_\theta/\omega_\gamma$; thus, from Eqs. (5) and (6):

$$\begin{aligned}\overline{MOM} &= \left[\alpha_1(\alpha_2\nu_\gamma - \alpha_1\nu_s) + (\alpha_2 - \alpha_1)(\nu_\theta - \alpha_2\nu_\gamma)\right]/2 \\ &= \left[\frac{1}{\kappa_s}\left(\frac{\nu_\gamma}{\omega_\gamma} - \frac{\nu_s}{\kappa_s}\right) + \left(\frac{1}{\omega_\gamma} - \frac{1}{\kappa_s}\right)\frac{\omega_\mu - \nu_\mu}{\omega_\gamma}\right]\nu_\theta^2/2.\end{aligned}$$

An immediate observation is that if $\kappa_\epsilon \to \infty$ (so that late-informed investors are very skeptical about the quality of the early Date-1 information s), then $\kappa_s \to \infty$ and the first item in the bracket (which corresponds to $\mathrm{Cov}(P_1 - P_0, P_2 - P_1)$) converges to zero. The intuition for this is that the late-informed ignore the Date-1 information s completely. As the Date-1 price does not react to s (i.e., $\alpha_1 = 0$ in $P_1 = \alpha_1 s$), P_1 becomes non-stochastic. This causes $\mathrm{Cov}(P_1 - P_0, P_2 - P_1)$ to be vanishingly small. The second item in the bracket corresponding to $\mathrm{Cov}(P_2 - P_1, \theta - P_2)$ is still negative because late-informed investors continue to overestimate the quality of their Date-2 information γ (i.e., $\omega_\mu < \nu_\mu$), resulting in an overreaction and reversal.

A further set of conditions for $\overline{MOM}$ to be positive is:

$$\frac{1}{\kappa_s} > \frac{1}{\omega_\gamma} - \frac{1}{\kappa_s}, \text{ and } \frac{\nu_\gamma}{\omega_\gamma} - \frac{\nu_s}{\kappa_s} > \frac{\omega_\mu - \nu_\mu}{\omega_\gamma};$$

or equivalently,

$$\nu_\theta + \omega_\mu > \kappa_\epsilon > \nu_\epsilon + (\nu_\mu - \omega_\mu). \tag{9}$$

This requires that late-informed investors be sufficiently skeptical (i.e., that κ_ϵ be sufficiently high) to cause an underreaction at Date 1, and thus price continuation from Date 1 to Date 2. But they cannot be very skeptical (i.e., κ_ϵ cannot be very high) to avoid a situation where P_1 is virtually non-stochastic.

Figure 2 plots $\overline{MOM}$ as functions of the late-informed's skepticism (κ_ϵ) and the overestimation of their own signal quality (ω_μ). We assume the parameter values $m = 0.1$, $A = 1$,

$\nu_\theta = 1$, $\nu_\mu = 1$, and $\nu_\epsilon = 1$. To simplify the presentation, we let $\omega_\epsilon = \nu_\epsilon$, so that early-informed investors have rational beliefs about the quality of their information. There are three notable observations in Figure 2. First, if late-informed investors are rational about their own and other investors' information quality (i.e., $\omega_\mu = \nu_\mu = 1$ and $\kappa_\epsilon = \nu_\epsilon = 1$), $\overline{MOM}$ equals zero. Second, if late-informed investors do not overestimate their own information quality by much (i.e., $\omega_\mu \to \nu_\mu = 1$), we obtain momentum. The intuition is similar to that provided above; specifically, skeptical late-informed investors provide too much liquidity at Date 1, causing underreaction of prices to Date-1 information which is then followed by a price continuation when late-informed investors observe their Date-2 information. Third, if late-informed investors overestimate their own information quality to a greater extent (i.e., $\omega_\mu \ll \nu_\mu = 1$), $\overline{MOM}$ turns negative. Here, the Date-2 price overreacts to information and then reverts at Date 3.

2.4 A Simple Case: When All Biased Beliefs are about the Quality of Early Information

We next consider a scenario in which investors are biased only about the precision of the Date-1 signal. Specifically, early-informed investors overestimate the quality of their signal, s (i.e., $\omega_\epsilon < \nu_\epsilon$). Late-informed investors are rational about the quality of the late Date-2 information γ (i.e., $\omega_\mu = \nu_\mu$), but underestimate the quality of the early-informed signal s (i.e., $\kappa_\epsilon > \nu_\epsilon$). These assumptions allow us to bring out clear intuition behind how the price reacts to the Date-1 information, and shows how underreaction due to skepticism leads to momentum. First, due to increased tractability, the following result can be proved analytically.

Proposition 3 *In the simplified case where biases prevail at Date 1 but not at Date 2 (i.e., $\omega_\nu = \nu_\mu$), the following results hold:*

(i) The sensitivity of the Date 1 price to the information signal s, i.e., $\alpha_1 \in [\nu_\theta/\kappa_s, \nu_\theta/\omega_s]$ (see Proposition 1), is lower if there is a bigger mass of late-informed investors (high $1-m$), if they are more skeptical (high κ_ϵ), and if early-informed investors overestimate the quality of s to a lesser extent (high ω_ϵ).

(ii) The sensitivity of the Date 2 price to the information signal γ, i.e., α_2, equals ν_θ/ν_γ.

If there is a big mass of late-informed investors and if they are skeptical about the quality of the Date-1 information s, they trade heavily against the early-informed and provide too much liquidity. This decreases the sensitivity of the price P_1 to the information s. If early-informed investors overestimate the quality of their information s to a lesser extent, they do not trade very heavily. This also lowers the responsiveness of P_1 to s. Since there are no biased beliefs at Date 2, as indicated in Part (ii) of Proposition 3, the price P_2 reacts properly to the Date 2 information γ. An immediate consequence of this is that there is no further price buildup or reversal from Dates 2 to 3. Therefore, both $\mathrm{Cov}(P_2 - P_1, \theta - P_2)$ (see Eq. (6)) and $\mathrm{Cov}(P_1 - P_0, \theta - P_2)$ (see Eq. (8)) equal zero.

The corollary below derives results on momentum for the simplified case of this section.

Corollary 3 *Under the conditions of Proposition 3, the following results hold:*

(i) There exists a constant parameter $m^{**} \in (0,1)$ such that iff. $m < m^{**}$, we obtain short-run momentum, i.e., $\overline{MOM} > 0$.

(ii) m^{**} is higher, and thus momentum arises under a larger parameter space, if the late-informed are more skeptical (high κ_ϵ).

Part (i) of this corollary indicates that a big mass of late-informed investors (high $1 - m$) is both a necessary and sufficient condition for momentum. The intuition is mostly consistent with that for Part (i) of Proposition 3. A large mass of skeptical late-informed investors provides excessive liquidity at Date 1. This causes P_1 to underreact to the early Date-1 information, which implies a price continuation, i.e., momentum, in the subsequent period. It follows naturally from this intuition that if the late-informed are more skeptical, then even a small mass of such investors can cause momentum. Therefore, momentum is more likely to arise.

Although late-informed investors' skepticism leads to momentum in equilibrium, it does not necessarily increase its scale (i.e., lead to a bigger momentum profit). The corollary below presents this result formally.

Corollary 4 *In the special case of Proposition 3,*

(i) if the skepticism of late-informed investors is small (i.e., κ_ϵ is low such that $\alpha_1 > 0.5\ \nu_\theta/\nu_s$), then an increase in skepticism enhances $\overline{MOM}$, and

(ii) if late-informed investors are very skeptical (i.e., $\alpha_1 < 0.5\ \nu_\theta/\nu_s$), then an increase in skepticism reduces $\overline{MOM}$.

We show in the proof of Corollary 4 that the second covariance in Eq. (7) drops out when the Date-2 price is unbiased, and

$$\overline{MOM} = \alpha_1(\nu_\theta - \alpha_1\nu_s)/2. \tag{10}$$

Note from Proposition 3 that in the Date-1 price of the firm-specific risk $P_1 = \alpha_1 s$, α_1 decreases in the late-informed's skepticism. Thus, an increase in skepticism has two effects on $\overline{MOM}$. First, by lowering α_1, it increases the price continuation from Dates 1 to 2, $P_2 - P_1$. This tends to increase $\overline{MOM}$. Second, by lowering α_1, it attenuates the price change from Dates 0 to 1, $P_1 - P_0$. This tends to lower $\overline{MOM}$. Corollary 4 provides parameter values under which the first effect dominates, or is dominated by, the second effect.

Figure 3 plots $\overline{MOM}$, as a function of late-informed investors' skepticism about the quality of the early Date-1 information (κ_ϵ). We set the other parameter values as follows: $m = 0.05$, $A = 1$, $\nu_\theta = 1$, $\nu_\mu = 1$, $\nu_\epsilon = 1$, $\omega_\epsilon = 0.5$, and $\omega_\mu = 1$. Consistent with Corollary 4, as κ_ϵ increases, $\overline{MOM}$ initially increases and then declines. Taken together, our analysis here indicates that while momentum arises due to skepticism, its scale does not necessarily increase in skepticism.

3 Extensions

In this section, we pursue three extensions to the general setting of our model: First, we consider the impact of rational, risk averse market makers; second, we model public disclosures (or news releases) to which overconfident investors react rationally; and third, we introduce noise trading into our setting.

3.1 Rational Market Makers, Risk Aversion, and Momentum

Up to this point, we have assumed that all investors in the model have biased beliefs. We now introduce a mass λ of rational uninformed investors, who serve as the market making sector. This allows us to investigate the impact of uninformed but unbiased investors on the equilibrium. The i'th market maker's utility function is the standard exponential:

$$U_N(W_{i3}) = -\exp(-A_N W_{i3}),$$

where W_{i3} is the final wealth, and A_N is a positive constant representing the absolute risk aversion coefficient. The market maker is endowed with $\bar{W}_{i0}$ units of the risk free asset.

The risk averse market makers play the role of arbitrageurs in our setting. If they have a small mass (low λ), and have a high risk aversion (high A_N), then they are not able to completely arbitrage momentum and reversals away, owing to limited risk bearing capacity. As λ becomes large, and/or A_N becomes small, however, arbitrage becomes perfect and the scale of momentum and reversals is mitigated. The proposition below presents this intuition formally. Here, we use λ/A_N to measure the risk bearing capacity of the market making sector.

Proposition 4 *Short-run momentum and long-run reversals only obtain when the risk bearing capacity of the market making sector, λ/A_N, is not unboundedly large. More specifically, as $\lambda/A_N \to \infty$, $\overline{MOM}$ and the long-run reversal parameter* $\mathrm{Cov}(P_1 - P_0, \theta - P_2)$ *converge to zero.*

Figure 4 plots $\overline{MOM}$ as a function of the risk bearing capacity of the market making sector, λ/A_N. For clarity of intuition, we let $\omega_\mu = \nu_\mu$ so late-informed investors are rational about the quality of their late Date-2 information γ. Other parameter values are presented in the caption of the figure. As shown in the figure, as λ/A_N increases, $\overline{MOM}$ initially rises and then declines. The reason is that in this simple case with $\omega_\mu = \nu_\mu$, $\overline{MOM}$ is given by Eq. (10) (see the analysis in Section 2.4). With a higher risk bearing capacity, rational and risk-averse market makers bring the stock price reaction to information (i.e., α_1 in $P_1 = \alpha_1 s$) closer to the rational level (i.e., ν_θ/ν_s). If the price reaction to information and thus $\overline{MOM}$

are very low to begin with, increasing the reaction of stock price to information causes an increase in $\overline{MOM}$. A further increase in the reaction, however, reduces the price continuation between Dates 1 and 2. This causes a decline in $\overline{MOM}$. As λ/A_N increases further, $\overline{MOM}$ converges to zero. This link between market making capacity and momentum can be used to address the substantial reduction in momentum profits in recent years (see, e.g., Chordia, Subrahmanyam, and Tong (2014)). As quantitative investing has become more prevalent (Patterson (2010); Abis (2017)), the risk-bearing capacity of the market making sector has likely increased. In our model, this phenomenon leads to attenuated momentum.

3.2 Public Information Disclosure, Momentum, and Drift

We now consider the nature of the equilibrium in the model of Section 3.1 where firm-specific public information, such as a news release, becomes available. The public information signal is denoted as $t = \theta + \xi$. This signal may be an analyst disclosure, an announcement by a firm's manager, or another source of information flows. We consider the notion that an overconfident investor would tend to under-assess the quality of information sources other than one's own signal. Accordingly, we allow early and late-informed investors to be skeptical about the quality of t about θ; they believe that the variance of ξ equals $\kappa\nu_\xi$ where $\kappa \geq 1$. For simplicity, let ξ be independent of θ, μ, and ϵ. The uninformed market makers hold unbiased beliefs about the quality of t (i.e., the variance of ξ, ν_ξ).

The public information signal can reach investors at either of Dates 1, 2, or 3. If it arrives at Date 3 (or after trade at Date 2), then it has no impact on the equilibrium; however, if the signal arrives at Dates 1 or 2, it does affect prices and trades. Now, if the public signal is not precise (i.e., high ν_ξ), then the investors will not put much weight on it and phenomena identical to those described in Section 3.1 (see Proposition 4) will obtain. If the public information is precise (i.e., low ν_ξ), then the investors rely on it heavily, which attenuates momentum and reversals. The proposition below presents the effect of disclosure on price patterns.

Proposition 5 *(i) Suppose that the public signal arrives at Date 2. If the signal is very precise, then long-run reversals go to zero but short-run momentum still prevails.*

(ii) If the public signal reaches investors at Date 1, then, as the precision of the signal increases, both long-run reversals and short-run momentum go to zero.

If the public information that arrives at Date 2 is precise, the magnitude of mispricing at Date 2 reduces. In contrast, momentum can still prevail because late-informed investors, who are skeptical about the quality of the early Date-1 information, continue to underreact, which causes momentum between Dates 1 and 2. On the other hand, as Part (ii) indicates, a very precise disclosure at Date 1 tends to reduce mispricing at both Dates 1 and 2, and therefore attenuate momentum and reversals. In terms of the overall intuition, progressively accelerating disclosures to earlier in time first reduces long-term, then short-term predictability. This is because a disclosures generally offset misreactions due to overconfidence. A disclosure at Date 2 offsets the overreaction at this date, mitigating long-run reversals, whereas the same at Date 1 mitigates the underreaction at this date, reducing short-run momentum.

If the information releases at each date can be interpreted as analysts' disclosures, our analysis above indicates that such signals close to major news dates (interpreted as Date 3) reduce momentum.[15] Further, in recent years, new technology such as internet has caused a speedier flow of information (Economides (2001)). The public signal can also be interpreted as an accelerated flow of information (at Dates 1 or 2, as opposed to Date 3). Thus, our analysis suggests that long-run reversals, or even short-run momentum, should weaken in recent years. Our analysis is thus consistent with disappearing long-run reversals (Jegadeesh and Titman (2001)), and the substantial reduction in momentum profits in recent years (see, e.g., Chordia, Subrahmanyam, and Tong (2014)).

The proposition below shows that the stock's return is predictable from the public signal.

Proposition 6 *Provided that $\kappa > 1$, there is post-public-announcement drift in equilibrium; that is* $\mathrm{Cov}(\theta - P_2, t) > 0$.

Since investors are skeptical about the quality of the public information, the stock price underreacts to the public information. The above result is consistent with drift follow-

[15]Since speedier disclosures are more likely with greater analyst coverage, our model is consistent with Hong, Lim, and Stein (2000), which shows that momentum is weaker for stocks with greater analyst coverage.

ing analysts' revisions and earnings surprises (Bernard and Thomas (1989) and Womack (1996)).[16] As with momentum and reversals we would expect drifts to also decline with higher quality disclosures in recent years; which accords with Chordia, Subrahmanyam, and Tong (2014).[17] Also, it is straightforward to show that $\mathrm{Cov}(\theta - P_2, t) \to 0$ as κ approaches unity from above. This implies that less investor overconfidence implies smaller post-public announcement drift. Under the plausible assumption that retail investors are more likely to be overconfident (Barber, Lee, Liu, and Odean (2008)), we would expect greater drift when such investors are the predominant market participants.

3.3 Noise Trading and Momentum

In the model up to now, we have assumed that prices are fully revealing, and trading occurs because overconfident investors agree to disagree about the distribution of information signals they possess. We now consider a more complicated version of Section 3.1 in which information is only partly revealed because of the presence of noise trading at Date 1.[18] We prove that because of the risk premia required to absorb the noise trades, prices tend to exhibit increased reversals, which attenuates and may even reverse momentum.

Suppose that noise trading causes the supply of the risky stock at Date 1 to be a random quantity z. [Specifically, we assume that at Date 1, the supply of the j'th ($j = 1, ..., J$) original risky security is z_j, and the supply of the $J + k$'th ($k = 1, ..., K$) original risky security is $-\sum_{j=1}^{J}(\beta_{jk} z_j)$. This implies that the supply of the j'th basic security (θ_j) equals z_j, and the supply of the k'th basic security (f_k) equals zero. We continue to ignore the index j for notational convenience.] The supply is normally distributed with mean zero and the variance ν_z.

[16]Returns in the model of Daniel, Hirshleifer, and Subrahmanyam (1998) are predictable only from managerial actions that condition on misvaluation (e.g., new issues during periods of overvaluation) and not from public announcements per se since overconfident investors assess public signals unbiasedly. Frazzini (2006) and George, Hwang, and Li (2015) provide explanations of earnings drift based on the disposition effect and the anchoring bias, respectively.

[17]Note here that as ν_ξ goes to zero (as in Proposition 5), so does $\kappa\nu_\xi$, and hence post-announcement drift becomes vanishingly small.

[18]To preserve analytical solutions, here and beyond, we abstract from Date 2 noise trading as well as public signals at either date. Adding these features leaves the central results qualitatively unaltered, however. Details appear in the Internet Appendix.

In this setting, if the scale of noise trade is small (i.e., low ν_z), then phenomena identical to those described in Section 3.1 (see Proposition 4) obtain. A high ν_z, however, can cause $\overline{MOM}$ to become negative. The proposition below presents this intuition formally.

Proposition 7 *If noise trades are sufficiently volatile (i.e., $\nu_z \to \infty$), then we obtain short-run reversals, i.e., $\overline{MOM} \to -\infty$.*

We show in the proof of this proposition that as ν_z becomes large, $\text{Cov}(P_1 - P_0, P_2 - P_1) \to -\infty$, while $\text{Cov}(P_2 - P_1, \theta - P_2)$ is bounded. The reason is that risk averse informed investors are not able to fully absorb the noise trade without a substantial price discount, which results in reversals.[19]

Our analysis here suggests that excessive noise trading causes a reversal of momentum profits. Daniel and Moskowitz (2016) find that momentum strategies experience negative returns in adverse states (i.e., recessions or down markets). Assuming that panic-induced noise trades are more likely to arise during such periods (Næs, Skjeltorp, and Ødegaard (2011)), our model accords with their finding. A further implication of our analysis is that assets subject to fire sales (a form of noise trading - viz. Coval and Stafford (2007)), should experience weaker momentum.

4 Other Stock Price Patterns

The principal focus of this section is to study the effect of skepticism on stock prices beyond momentum and reversals.[20] Our analysis here is based on the setting in Section 3.3. We obtain the prices of the j'th ($j = 1, ..., J$) original stock from Lemma 1, Proposition 1, and

[19]Hirshleifer, Subrahmanyam, and Titman (2006) consider a model where noise trades are autocorrelated. They have a risk-neutral market making sector, however, which prevents prices from exhibiting serial dependence. It is likely that an extension to risk averse market makers would produce short-run momentum due to autocorrelation in risk premia.

[20]The other side of overconfidence, overestimation of one's own information quality, has been extensively studied elsewhere in the literature. Interested readers are referred to Odean (1998), Daniel, Hirshleifer, and Subrahmanyam (1998), and Ko and Huang (2007).

the proof of Proposition 7:

$$P_0(V) = \bar{V}, \tag{11}$$

$$P_1(V) = \bar{V} + \sum_{k=1}^{K} \left(\beta_k \frac{\nu_{f_k}}{\nu_{s_{J+k}}} s_{J+k} \right) + \alpha_\tau \tau, \tag{12}$$

and

$$P_2(V) = \bar{V} + \sum_{k=1}^{K} \left(\beta_k \frac{\nu_{f_k}}{\nu_{\gamma_{J+k}}} \gamma_{J+k} \right) + \alpha_2 \gamma, \tag{13}$$

where $\tau = s - \delta z$. The parameters α_2, δ, and α_τ are given in Eqs. (29), (42), and (43) in the appendix (within the proof of Proposition 7). It is notable that late-informed investors' skepticism (i.e., $\kappa_\epsilon > \nu_\epsilon$) affects the price $P_1(V)$ only through α_τ.

4.1 Liquidity and the Late-Informed

In the setting of Section 3.3, one unit of a liquidity sale lowers the Date-1 stock price in Eq. (12) by

$$-\frac{dP_1(V)}{dz} = -\alpha_\tau \frac{d\tau}{dz} = \alpha_\tau \delta$$

units. Therefore, we can measure liquidity by $\alpha_\tau \delta$ (a low level indicates high liquidity). The following proposition describes the effect of skepticism on liquidity.[21]

Proposition 8 *Liquidity increases in the degree of skepticism (κ_ϵ).*

If the late-informed are more skeptical about the quality of early-informed investors' private information, they are less concerned about trading against other investors with superior information. Therefore, they provide "too much" liquidity.

Figure 5 plots the illiquidity measure, $\alpha_\tau \delta$, as functions of the late-informed investors' skepticism about the quality of the early Date-1 information s (i.e., $\kappa_\epsilon > \nu_\epsilon$), and the parameter that represents overestimation of the quality of their late Date-2 information γ (i.e.,

[21]In the Internet Appendix, we use numerical analyses to confirm that similar results obtain when there is noise trading at both dates.

$\omega_\mu < \nu_\mu$). For clarity of intuition, we let $\omega_\epsilon = \nu_\epsilon$ so early-informed investors have rational beliefs about the quality of their information. Other parameter values are presented in the caption of the figure (the results are not particularly sensitive to the chosen values). Consistent with Proposition 8, if late-informed are more skeptical (higher κ_ϵ), liquidity is higher (i.e., $\alpha_\tau\delta$ is lower). Figure 5 also indicates that as late-informed investors overestimate the quality of their late information γ to a lesser extent (i.e., as ω_μ increases), liquidity increases (i.e., $\alpha_\tau\delta$ decreases). The reason is that in this case, the Date-2 price, which becomes less sensitive to the late information, is more predictable. This implies that at Date 1, late-informed investors tend to be more aggressive in providing liquidity to early-informed investors. We use Figure 6 to verify that, consistent with our earlier analysis in Section 2.3, the momentum parameter, $\overline{MOM}$, increases as late informed investors underestimate the quality of early information s to a greater extent (i.e., as κ_ϵ increases), and as they overestimate the quality of their late information γ to a lesser extent (i.e., as ω_μ increases).

Overall, in Figures 5 and 6, parameter configurations that favor liquidity also tend to promote momentum, simply because conditions that promote the aggressiveness of the late-informed at Date 1 increase both momentum and liquidity. This observation relates to Avramov, Cheng, and Hameed (2016), who find that momentum profits are markedly larger in liquid market states. They argue that this evidence is surprising because it is inconsistent with the basic intuition that arbitrage is easier when markets are most liquid. This evidence, however, is consistent with our analysis.[22] There is one point worth noting here. Specifically, observe that as overconfidence increases we would expect κ_ϵ to increase, but ω_μ to decrease, which moves liquidity and momentum in opposite directions, as Figures 5 and 6 demonstrate. Thus, whether increasing the overconfidence of the late-informed promotes liquidity and momentum depends on the sensitivity of these phenomena to κ_ϵ and ω_μ. If overconfidence primarily operates through skepticism, however, then increasing overconfidence enhances both momentum and liquidity. Thus, Figure 7 plots the momentum parameter, $\overline{MOM}$, and

[22]The consistency between our result and that of Avramov, Cheng, and Hameed (2016) applies if liquidity variations are primarily due to variations in the parameters within Figures 5 and 6. Liquidity can also increase due to an increase in the variance of completely uninformed noise trading; it is easy to show that $-dP_1(V)/dz$ decreases in ν_z. As Section 3.3 demonstrates, in the limit, when ν_z becomes unboundedly large, momentum profits become negative.

the liquidity measure, $\alpha_\tau\delta$, as functions of a summary measure of late-informed investors' overconfidence, $OC \equiv (\kappa_\epsilon - \nu_\epsilon) + (\nu_\mu - \omega_\mu)$. We assume that $\nu_\mu - \omega_\mu = 0.2(\kappa_\epsilon - \nu_\epsilon)$ and vary κ_ϵ while simultaneously changing ω_μ. Other parameter values are presented in the caption of the figure. It can be seen that in this case increasing overconfidence promotes both momentum and liquidity, consistent with Avramov, Cheng, and Hameed (2016). As we observed in the discussion following Proposition 6, it is plausible that overconfidence is greater when retail investors (who are more likely to be biased) are primary participants in markets. Our analysis then suggests that during periods/markets with greater retail participation, momentum and liquidity will both be higher.

4.2 Price Quality

As in Odean (1998) and Ko and Huang (2007), we measure price quality using the mean squared error between a stock's payoff and its price (denoted as MSE; a lower MSE indicates a better price quality). Note from Eqs. (1), (12), and (13) that the pricing errors at Dates 1 and 2 can be expressed as:

$$\begin{aligned} V - P_1(V) &= \sum_{k=1}^{K}\left[\beta_k\left(f_k - \frac{\nu_{f_k}}{\nu_{s_{J+k}}}s_{J+k}\right)\right] + \theta - \alpha_\tau\tau, \quad \text{and} \\ V - P_2(V) &= \sum_{k=1}^{K}\left[\beta_k\left(f_k - \frac{\nu_{f_k}}{\nu_{\gamma_{J+k}}}\gamma_{J+k}\right)\right] + \theta - \alpha_2\gamma. \end{aligned}$$

Thus, the MSE's are given by

$$\text{MSE}_1 = E\left[(V - P_1(V))^2\right] = \sum_{k=1}^{K}\left[\beta_k^2 \text{Var}\left(f_k - \frac{\nu_{f_k}}{\nu_{s_{J+k}}}s_{J+k}\right)\right] + E\left[(\theta - \alpha_\tau\tau)^2\right] \tag{14}$$

and

$$\text{MSE}_2 = E\left[(V - P_2(V))^2\right] = \sum_{k=1}^{K}\left[\beta_k^2 \text{Var}\left(f_k - \frac{\nu_{f_k}}{\nu_{\gamma_{J+k}}}\gamma_{J+k}\right)\right] + E\left[(\theta - \alpha_2\gamma)^2\right]. \tag{15}$$

The proposition below describes the effect of skepticism on the MSEs.

Proposition 9 *(i) If late-informed investors are not too skeptical (i.e., if $\alpha_\tau > \nu_\theta/\nu_\tau$), then an increase in skepticism (κ_ϵ) lowers* MSE_1*, enhancing the quality of the Date-1 stock price P_1.*

(ii) When skepticism is high (i.e., if $\alpha_\tau < \nu_\theta/\nu_\tau$), then an increase in skepticism increases MSE_1*, worsening the quality of the Date-1 stock price* P_1*.*

(iii) The quality of the Date-2 stock price P_2 (MSE_2) *is not affected by late-informed investors' skepticism.*

At Date 1, early-informed investors overestimate the quality of their Date-1 information s. They trade too aggressively based on this information, causing the stock price to overreact to s. Late-informed investors learn $\tau = s - \delta z$ from the Date-1 price P_1. They are skeptical about the quality of s and therefore τ, and provide liquidity to early-informed investors. A modest level of their skepticism tends to mitigate the overreaction, improving price quality. If their skepticism is excessive, however, then there will be too much liquidity provision. This can over-correct the overreaction, and worsen price quality. Part (iii) of this proposition is consistent with the above analysis, that is, the influence of late-informed investors' skepticism is limited to the contemporary Date-1 stock price.

5 Skepticism About Fundamentals and Reversals

In this section, we show that skepticism about fundamentals (alone) can generate both long-run reversals and momentum. Our analysis is thus in contrast to the earlier literature that only illustrates how skepticism can lead to underreaction. We extend the model of the previous section by allowing the Δ parameter introduced in Section 2.2 to be positive (dropping the j subscript for convenience). As discussed in that section, this is equivalent to late-informed assuming that other investors have a less complete view of the fundamental than they actually do. To reiterate briefly, θ is the sum of two random components θ_1 and θ_2; μ is the sum of two random components μ_1 and μ_2. Both early- and late-informed observe $\gamma_2 = \theta_2 + \mu_2$ at Date 2, and late informed believe that the early informed's signal is $\theta_1 + \mu_1 + \epsilon$ (and that their own Date-2 signal is $\theta_1 + \mu_1$).[23] In this scenario, late informed react to information that is already in earlier prices due to trades by the early-informed.

[23]For notational simplicity, we abstract from public signals and assume noise trades only occur at Date 1. The Internet Appendix presents a more general model with noise trading at both dates as well as public signals, and shows that the results of this section continue to obtain.

The appendix shows that in the extended model, the equilibrium prices take the following form:

$$P_1 = B\tau, \tag{16}$$

and

$$P_2 = C(\gamma + a\gamma_2) + D\tau, \tag{17}$$

where $\tau \equiv s - \delta z$, and B, δ, C, a, and D are constants. θ, γ, γ_2, and τ follow a multi-variate normal distribution with mean zeros and variance-covariance matrix

$$\begin{pmatrix} \nu_\theta & \nu_\theta & \Delta\nu_\theta & \nu_\theta \\ \nu_\theta & \nu_\gamma & \Delta\nu_\gamma & \nu_\gamma \\ \Delta\nu_\theta & \Delta\nu_\gamma & \Delta\nu_\gamma & \Delta\nu_\gamma \\ \nu_\theta & \nu_\gamma & \Delta\nu_\gamma & \nu_\tau \end{pmatrix}.$$

We can use Eqs. (16)-(17) to compute the long-run reversal parameter

$$\mathrm{Cov}(P_1 - P_0, \theta - P_2),$$

and the short-run momentum parameter

$$\overline{MOM} = \left[\mathrm{Cov}(P_1 - P_0, P_2 - P_1) + \mathrm{Cov}(P_2 - P_1, \theta - P_2)\right]/2.$$

This setting generally precludes solving for B, δ, C, a, and D in closed form, so numerical analysis is necessary for the general version of the framework.

5.1 A Special Case

To build intuition, we first consider an analytical solution for a special case. Specifically, suppose that all the biased beliefs are about the fundamental information structure (i.e., $\omega_\epsilon = \kappa_\epsilon = \nu_\epsilon$ and $\omega_\mu = \nu_\mu$, but $\Delta > 0$). Further, let $\nu_\epsilon = 0$ and assume that there is no noise trading ($\nu_z = 0$). This implies that $s = \gamma$, so there is no new information at Date 2, and γ_2 is redundant information. Finally, let the risk-bearing capacity of the uninformed rational investors be negligible (i.e., $\lambda/A_N \to 0$). Denote

$$\Upsilon = \frac{m(1-m)(1-\Delta)^{-1}}{\Delta^{-1} + m(1-\Delta)^{-1} + m(1-m)\nu_\theta(\nu_\gamma - \nu_\theta)^{-1}}.$$

We can follow the derivation in the proof of Eqs. (16) and (17) to show that the equilibrium prices of the firm-specific risk (θ) are given by

$$P_1 = \frac{\nu_\theta}{\nu_\gamma}(1+\Upsilon)\gamma, \ \ \text{and} P_2 = \frac{\nu_\theta}{\nu_\gamma}(\gamma + (1-m)\gamma_2). \tag{18}$$

As the late informed believe that the early informed have not observed a fundamental component of the value (i.e., γ contains only $\theta_1 + \mu_1$ and does not contain $\gamma_2 = \theta_2 + \mu_2$), they condition on both γ and γ_2 when they trade at Date 2. Thus, the Date-2 price reacts not only to γ_2 through γ, but also to γ_2 directly. We refer to this as a "double-counting" effect. When early informed trade at Date 1, they take the double-counting effect into consideration. Thus, the information γ causes an extra movement in the Date-1 price, which is captured by the term Υ in P_1.

Proposition 10 *In the special case in which all the biased beliefs are about the fundamental information structure (i.e., $\omega_\epsilon = \kappa_\epsilon = \nu_\epsilon$ and $\omega_\mu = \nu_\mu$, but $\Delta > 0$; and further, $\nu_\epsilon = \nu_z = 0$ with $\lambda/A_N \to 0$), the following results hold*

(i) Stock returns reverse in the long-run; i.e., $\text{Cov}(P_1 - P_0, \theta - P_2) < 0$.

(ii) There is drift around Date 1; i.e., $\text{Cov}(P_1 - P_0, P_2 - P_1) > 0$.

(iii) There is a reversal around Date 2; i.e., $\text{Cov}(P_2 - P_1, \theta - P_2) < 0$.

(iv) On average, there is short-run momentum; i.e., $\overline{MOM} > 0$.

The drift occurs because the late informed provide too much liquidity, causing the price to underreact to Date-1 information. The reversals arise because of the double-counting effect. Across Cases (ii) and (iii), short-run momentum dominates. Thus, the setting with skepticism about fundamentals alone accounts for short-term momentum and long-run reversals.

5.2 Numerical Analysis

We now study the general case. We use the parameter values $m = 0.1$, $\lambda = 0.1$, $A = 1$, $A_N = 1$, $\nu_\theta = 1$, $\nu_z = 0.01$, and $\Delta = 0.5$. To isolate the effect of skepticism about fundamentals, we assume in the numerical analysis that information quality is estimated correctly (i.e., $\omega_\epsilon = \nu_\epsilon$, $\omega_\mu = \nu_\mu$, and $\kappa_\epsilon = \nu_\epsilon$).[24] Figure 8 plots $\mathrm{Cov}(P_1 - P_0, \theta - P_2)$ as functions of the noise variances in the early Date-1 information (ν_ϵ) and the late Date-2 information (ν_μ). The figure reveals three notable observations. First, the price reverses in long run. The reason for the reversals is that when late-informed investors receive their information at Date 2, they mistakenly believe that this information has not been revealed in past prices. This leads to a "double-counting" effect: they continue to react to this information, which causes the price to overreact and subsequently reverse. Second, if the Date-2 information is precise (i.e., ν_μ is low), reversals are more extreme. The reason is that in this case, late-informed investors trade more aggressively on the Date-2 information, which increases the degree of overreaction. Third, the strength of reversals increases in the Date-1 signal's quality becomes (i.e., reversals decrease in ν_ϵ). The reason is that in this case, investors are less conservative in reacting to the Date-1 information, which also enhances long-run reversals.

Figure 9 plots $\overline{MOM}$ as a function of the signal noise variances ν_ϵ and ν_μ. First, the momentum effect (i.e., $\overline{MOM} > 0$) also arises here. The reason is that as late-informed investors believe that the early Date-1 signal contains only limited information, they provide too much liquidity to the early-informed investors. This causes an underreaction at Date 1, and thus a continuation across Dates 1 and 2. Second, the momentum effect tends to obtain when the Date-1 information is precise (i.e., with small ν_ϵ). The reason is that in this case, late-informed investors are less conservative in providing liquidity, which causes a stronger price underreaction at Date 1. Third, the momentum effect reverses for a sufficiently high precision of the Date-2 signal (i.e., low ν_μ). The reason is that in this case, as Figure 8 indicates, there is a stronger reversal around Date 2. This more than offsets the momentum around Date 1.

We now explore the role of analysts' reports in the model of this section. Suppose a public signal, $\gamma_2 + \xi$, is released at Date 1. We let all investors have unbiased beliefs about

[24] Generally similar patterns obtain for other parameter values, with the exception that as suggested by our earlier analysis, the masses of the early-informed and market makers (m and λ, respectively) and the variance of noise trading have to be sufficiently small to generate momentum.

the quality of this signal. The signal precision (quality) can be viewed as decreasing in the mass of analysts following the stock. Thus, if each analyst releases a signal that equals γ_2 plus i.i.d. noise, then the average signal is a sufficient statistic for the stock of analysts' information whose precision increases in mass of analysts releasing the reports on the stock. As all investors condition their trades on this signal at Date 1, and rational uninformed investors continue to condition their trades on this signal at Date 2, the stock prices, P_1 and P_2, are also linear in this signal. (This is a simple variant of the general setting presented in the Internet Appendix. The technical details are straightforward and available upon request from the authors.)

The public signal essentially allays the skepticism of the late-informed. Specifically, by directly revealing γ_2, albeit noisily, it counteracts the late informed's belief that the early informed have not observed γ_2. Figure 10 plots the momentum parameter $\overline{MOM}$ as a function of the signal noise variance, ν_ξ. As the public signal becomes very precise (i.e., as analyst following increases so that ν_ξ converges to zero), the momentum parameter $\overline{MOM}$ declines sharply. Since the late informed condition their trades on the signal $\gamma_2 + \xi$ at Date 1, the Date-1 price P_1 responds to γ_2. This limits the additional price reaction to γ_2 at Date 2, and therefore the continuation of the price from Date 1 to Date 2. Thus, investors are prone to trading on stale information at Date 2 because of skepticism, and analysts mitigate this phenomenon by directly providing that information via their Date 1 reports.

6 Applications

6.1 Disappearing Long-Run Reversals and Short-Run Momentum

Jegadeesh and Titman (2001) find that the long-run reversals that accompanied shorter-run momentum were mostly significant in early years (specifically, 1965-1981; see their Table VI, p. 77), but disappeared in their later sample period. Chordia, Subrahmanyam, and Tong (2014) find a substantial reduction in momentum profits during recent years. These patterns can be explained in the context of our model as follows. Before the 1990s, companies relied on slow traditional media, implying sequential receipt of private information, and, in consequence, momentum and reversals. In recent years, new technology such as internet has caused an ever speedier flow of information (Economides (2001)). Our analysis in Section 3.2 (see Proposition 5) indicates that over time, long-run reversals should first attenuate, followed by weakening of short-run momentum, which accords with Jegadeesh and Titman (2001) and Chordia, Subrahmanyam, and Tong (2014).[25]

6.2 Empirical Implications

Our analysis suggests the following empirical implications. For these, we presume that the parameter governing skepticism (κ_ϵ) is in the range where marginal increases in the parameter enhance momentum (see Condition (9) and Proposition 3).[26] We precede each implication below with the supporting result(s) or model features.

- [Corollary 3(i)] The work of Chui, Titman, and Wei (2010) and Markus and Kitayama (1991) suggests that collectivist cultures discourage skepticism about others and overestimation of own ability by encouraging conformity, whereas individualistic cultures do the opposite. Hence we predict the following: (i) As foreign equity ownership from individualist cultures increases in a collectivist country, momentum and reversals should strengthen in that market. (ii) Collectivist countries without extensive foreign ownership from individualistic countries will exhibit less momentum and reversals than other collectivist countries. (iii) Under limited arbitrage (e.g., Shleifer and Vishny (1997)), cross-listed stocks across countries with different cultures will exhibit more momentum and reversals in the individualistic country.

- [Proposition 4] Restrictions on market making capacity (such as leverage constraints imposed after the financial crisis - viz Acharya, Philippon, and Richardson (2009)) should enhance momentum.

[25]Other regulations like RegFD (Gintschel and Markov (2004)), and tightened insider trading regulations (Bettis, Duncan, and Harmon (2011)) should also reduce the tendency for tradeable information to arrive sequentially (first to insiders, then to others), thus attenuating momentum. Further, in the U.S., "quant investing," a form of *de facto* market making, has increased in recent years (Patterson (2010); Abis (2017)), enhancing the risk-bearing capacity of uninformed investors. Our analysis in Section 3.1 suggests that this also weakens momentum and reversals.

[26]Essentially, in this range κ_ϵ must be non-zero but cannot be too high, and a modest level of bias is a reasonable parameter restriction.

- [Proposition 5] More informative analysts' disclosures should be linked to less momentum and reversals.

- [Proposition 6] Markets and periods in which retail investors are more active should exhibit stronger evidence of drift following earnings releases and analysts' revisions.

- [Proposition 7] Assets experiencing high levels of noise trading such as fire sales (Coval and Stafford (2007)), should experience weaker momentum.

- [Proposition 8 and Figure 7] Time-periods with greater retail participation in equity markets imply stronger momentum and more liquidity.

- [Sequential receipt of information] Since, in our setting, momentum relies on investors receiving information sequentially, we propose that in economies where some investors are more likely to receive information before others, e.g., because of lax enforcement of insider trading (Bhattacharya and Daouk (2002)), there will be more momentum.

7 Conclusions

Consider a setting where traders receive informational signals at different times, and are overconfident, in that they overestimate their own talent at generating information but are skeptical of others' ability to do so. We show that because of this skepticism, some traders (who get informed later) provide excessive liquidity to others (who get informed earlier), which leads to underreaction, and hence, momentum. The notion that all investors overassess their own information quality, of course, causes overreaction and hence a long-run reversal. We go one step further and show that skepticism alone can cause short-term momentum as well as long-term reversals. This is because trade can occur based on the assumption that received information is new and was not traded upon at earlier dates by other investors. While such skepticism causes underreaction in earlier rounds, it also causes prices to react to stale information and subsequently reverse. Our finding adds perspective to the notion that under-assessing others' information quality should lead to underreaction. We show that analysts' disclosures mitigate the late informed's tendency to overreact to stale information due to skepticism, and thus reduce momentum.

We address several other aspects of momentum and reversals. Accelerating public information releases first mitigates long-run reversals and further speeding up disclosures reduces short-run momentum as well. Our model thus forms an initial attempt to address the patterns of disappearing long-run reversals and weakening short-run momentum over time, given recent technological innovations that have likely speeded up information disclosures. We also show that noise trades attenuate momentum, by creating a tendency for reversals via premia required to bear demand shocks. Rational and risk averse market makers reduce, but do not eliminate return predictability. We predict that the rise of quantitative investing, that *de facto* increases risk bearing capacity, should reduce the cross-sectional dependence of equity returns on past returns. We show that since skeptical investors provide excessive liquidity to informed investors, which gives rise to momentum, high liquidity is associated with high momentum. Further, skepticism about the quality of disclosures by other information producers, namely, firms and analysts, leads to drift following earnings surprises and analysts' revisions. Our analysis implies that greater levels of overconfidence, more likely when retail participation is higher, lead to greater drift following earnings releases and analyst revisions. Across economies, we predict that economies where news diffuses slowly (as measured by pre-event price runups), or where insider trading is prevalent, will exhibit more momentum.

Appendix

Proof of Lemma 1: We use two steps to prove this lemma.

Step 1: We focus on Date 2 in this step.

The $J+k$'th ($k = 1, ..., K$) original stock has a Date-3 payoff $V_{J+k} = f_k$. Write its Date-2 price as $P_2(V_{J+k}) = P_{J+k,2}$. From Eq. (1), write the j'th ($j = 1, ..., J$) original stock's payoff and Date-2 price as

$$V_j = \bar{V}_j + \sum_{k=1}^{K}(\beta_{jk} f_k) + \theta_j, \text{ and } P_2(V_j) = \bar{V}_j + \sum_{k=1}^{K}(\beta_{jk} P_{J+k,2}) + P_{j,2},$$

where $P_{J+k,2}$ and $P_{j,2}$ are to be determined.

Denote the Date-2 demands of the i'th early- or late-informed investor for the original stocks Y_{ij2} ($j = 1, ..., J$) and $Y_{i,J+k,2}$ ($k = 1, ..., K$). The Date-3 wealth can be expressed as

$$\begin{aligned} W_{i3} &= W_{i2} + \sum_{j=1}^{J}\left[Y_{ij2}(V_j - P_2(V_j))\right] + \sum_{k=1}^{K}\left[Y_{i,J+k,2}(f_k - P_2(V_{J+k}))\right] && (19) \\ &= W_{i2} + \sum_{j=1}^{J}\left[X_{ij2}(\theta_j - P_{j,2})\right] + \sum_{k=1}^{K}\left[X_{i,J+k,2}(f_k - P_{J+k,2})\right], && (20) \end{aligned}$$

where we use the notation that

$$X_{ij2} \equiv Y_{ij2}, \text{ and } X_{i,J+k,2} \equiv Y_{i,J+k,2} + \sum_{j=1}^{J}(\beta_{jk} Y_{ij2}).$$

The expression in Eq. (19) indicates that the investor can choose the demands for the original stocks indicated by Y_{ij2} and $Y_{i,J+k,2}$, conditional on the prices of the original stocks $P_2(V_j)$ and $P_2(V_{J+k})$. The expression in Eq. (20) indicates that the investor can equivalently choose the demands for the basic securities indicated by X_{ij2} and $X_{i,J+k,2}$, conditional on the prices of the basic securities $P_{j,2}$ and $P_{J+k,2}$.

The market clearing condition for the original stocks,

$$\sum_i Y_{ij2} = 0, \text{ and } \sum_i Y_{i,J+k,2} = 0,$$

implies that for the basic securities, i.e.,

$$\sum_i X_{ij2} = \sum_i Y_{ij2} = 0, \text{ and } \sum_i X_{i,J+k,2} = \sum_i \left[Y_{i,J+k,2} + \sum_{j=1}^{J}(\beta_{jk} Y_{ij2})\right] = 0.$$

Step 2: We can use a similar analysis as in the above Step 1 to show that the Date-1 and Date-0 prices of the original stocks take the linear form as specified in Lemma 1. □

Proof of Proposition 1: In the multi-asset setup with basic securities, the payoffs of these securities take the independent-normal structure. This implies that an investor's expected (negative exponential) utility takes a multiplicative form, in which each multiplicative component represents the expected utility obtained from trading a specific basic security. The optimization problem regarding a specific basic security is independent of the optimization problem in the other basic securities. Therefore, in the following derivation and all the other derivations for the multi-asset setup, we restrict our attention to the optimization problem in only one basic security. This is for notational convenience. It is straightforward to extend the analysis to include the optimization for other basic securities. We solve for the equilibrium using backward induction.

Date 2: Let us focus on the j'th ($j = 1, ..., J$) firm-specific risk, θ_j, for the moment (in what follows, we suppress the index for the basic security j for convenience). The i'th early-informed investor learns γ from the price P_2, which is fully revealing. The investor believes that γ is a sufficient statistic and that

$$\theta|\gamma \sim N\left(\frac{\nu_\theta}{\nu_\gamma}\gamma, H(\nu_\theta, \nu_\gamma)^{-1}\right),$$

where the function $H(.)$ is defined in Eq. (4).

Write the investor's wealth at Date 3 as $W_{i3} = W_{i2} + X_{i2}(\theta - P_2)$. The demand X_{i2} maximizes

$$\begin{aligned} E_\eta\left[U_\eta(W_{i3})|\gamma\right] &= E_\eta\left[-\exp\left[-AW_{i2} - AX_{i2}(\theta - P_2)\right]|\gamma\right] \\ &= -\exp\left[-AW_{i2} - AX_{i2}(\frac{\nu_\theta}{\nu_\gamma}\gamma - P_2) + 0.5A^2X_{i2}^2 H(\nu_\theta, \nu_\gamma)^{-1}\right], \end{aligned} \quad (21)$$

where the second equality is based on the normality assumption. The first order condition (f.o.c.) with respect to (w.r.t.) X_{i2} implies that the demand can be expressed as:

$$X_{\eta 2}(\gamma, P_2) = A^{-1}H(\nu_\theta, \nu_\gamma)\left(\frac{\nu_\theta}{\nu_\gamma}\gamma - P_2\right). \quad (22)$$

The second order condition holds obviously in the above case, and all other cases below, so that we omit referencing it in the rest of the proofs. We can use a similar analysis to show that the demand of the i'th late-informed investor who observes γ can be expressed as:

$$X_{\ell 2}(\gamma, P_2) = A^{-1} H(\nu_\theta, \omega_\gamma) \left(\frac{\nu_\theta}{\omega_\gamma} \gamma - P_2 \right). \tag{23}$$

From Eqs. (22) and (23), the market clearing condition, $0 = m X_{\eta 2}(\gamma, P_2) + (1-m) X_{\ell 2}(\gamma, P_2)$, implies $P_2 = \alpha_2 \gamma$, where α_2 is as specified in Proposition 1.

Consider the i'th early-informed investor's expected utility in Eq. (21). Substituting for the optimal demand from Eq. (22) and the above derived $P_2 = \alpha_2 \gamma$,, we can write the expected utility at Date 2 as

$$E_\eta \left[U_\eta(W_{i3}) | \gamma \right] \;\; = \;\; -\exp \left[-AW_{i2} - 0.5 H(\nu_\theta, \nu_\gamma) (\frac{\nu_\theta}{\nu_\gamma} - \alpha_2)^2 \gamma^2 \right]. \tag{24}$$

Date 1: At Date 1, the i'th early-informed investor observes s. The investor believes that

$$\gamma | s \sim N \left(\frac{\nu_\gamma}{\omega_s} s, H(\nu_\gamma, \omega_s)^{-1} \right).$$

Consider the investor's expected utility in Eq. (24). Write the wealth at Date 2 as $W_{i2} = W_{i1} + X_{i1}(P_2 - P_1)$, where the price $P_2 = \alpha_2 \gamma$ is as derived above. It follows that the expected utility at Date 1 can be expressed as

$$\begin{aligned} & E_\eta \left[U_\eta(W_{i3}) | s \right] \\ = \;\; & E_\eta \left[-\exp \left[-AW_{i1} - AX_{i1}(\alpha_2 \gamma - P_1) - 0.5 H(\nu_\theta, \nu_\gamma) (\frac{\nu_\theta}{\nu_\gamma} - \alpha_2)^2 \gamma^2 \right] | s \right] \\ \propto \;\; & -\exp \Bigg[-AW_{i1} - AX_{i1}(-P_1) + 0.5 \frac{(AX_{i1}\alpha_2 - H(\nu_\gamma, \omega_s) {\omega_s}^{-1} \nu_\gamma s)^2}{H(\nu_\gamma, \omega_s) + H(\nu_\theta, \nu_\gamma)({\nu_\gamma}^{-1}\nu_\theta - \alpha_2)^2} \\ & -0.5 H(\nu_\gamma, \omega_s) (\frac{\nu_\gamma}{\omega_s} s)^2 \Bigg]. \end{aligned} \tag{25}$$

Here, we used the fact in Footnote 27 below.[27] The f.o.c. w.r.t. X_{i1} implies that the demand can be expressed as

$$X_{\eta 1}(s, P_1) = \frac{-P_1}{A \alpha_2^2} \left[H(\nu_\gamma, \omega_s) + H(\nu_\theta, \nu_\gamma) (\frac{\nu_\theta}{\nu_\gamma} - \alpha_2)^2 \right] + \frac{1}{A \alpha_2} H(\nu_\gamma, \omega_s) \frac{\nu_\gamma}{\omega_s} s. \tag{26}$$

[27] If a vector $x \sim N(\bar{x}, \nu)$, then

$$E \left[\exp(\Lambda^T x - 0.5 x^T \Sigma x) \right] = \frac{|\nu^{-1} + \Sigma|^{-1/2}}{|\nu|^{1/2}} \exp \left[0.5(\Lambda + \nu^{-1}\bar{x})^T (\nu^{-1} + \Sigma)^{-1} (\Lambda + \nu^{-1}\bar{x}) - 0.5 \bar{x}^T \nu^{-1} \bar{x} \right].$$

If a scalar $x \sim N(\bar{x}, \nu)$, then

$$E \left[\exp(\Lambda x - 0.5 \Sigma x^2) \right] = \sqrt{\frac{1}{1 + \nu \Sigma}} \exp \left[0.5 (\Lambda + \frac{\bar{x}}{\nu})^2 (\frac{1}{\nu} + \Sigma)^{-1} - 0.5 \frac{\bar{x}^2}{\nu} \right].$$

We can use a similar analysis to show that the demand of the i'th late-informed investor who learns s from the price P_1 can be expressed as:

$$X_{\ell 1}(s, P_1) = \frac{-P_1}{A\alpha_2^2}\left[H(\omega_\gamma, \kappa_s) + H(\nu_\theta, \omega_\gamma)(\frac{\nu_\theta}{\omega_\gamma} - \alpha_2)^2\right] + \frac{1}{A\alpha_2}H(\omega_\gamma, \kappa_s)\frac{\omega_\gamma}{\kappa_s}s. \tag{27}$$

From Eqs. (26) and (27), the market clearing condition, $0 = mX_{\eta 1}(s, P_1) + (1-m)X_{\ell 1}(s, P_1)$, implies $P_1 = \alpha_1 s$, where α_1 is as specified in Proposition 1.

Date 0: Consider the i'th early-informed investor's expected utility in Eq. (25). Substitute for the optimal demand from Eq. (26) and the above derived $P_1 = \alpha_1 s$, and write the wealth at Date 1 as $W_{i1} = \bar{W}_{i0} + X_{i0}(P_1 - P_0)$. Then, we have

$$\begin{aligned} E_\eta\left[U_\eta(W_{i3})|s\right] &\propto -\exp\left[-AW_{i1} - 0.5\Sigma s^2\right] \\ &= -\exp\left[-A\bar{W}_{i0} - AX_{i0}(\alpha_1 s - P_0) - 0.5\Sigma s^2\right], \end{aligned}$$

where Σ is a positive constant which is a function of exogenous parameters. The investor's belief is $s \sim N(0, \omega_s)$. The choice of the demand X_{i0} maximizes

$$\begin{aligned} E_\eta\left[U_\eta(W_{i3})\right] &\propto E_\eta\left[-\exp\left[-A\bar{W}_{i0} - AX_{i0}(\alpha_1 s - P_0) - 0.5\Sigma s^2\right]\right] \\ &\propto -\exp\left[-A\bar{W}_{i0} - AX_{i0}(-P_0) + 0.5(AX_{i0}\alpha_1)^2(1/\omega_s + \Sigma)^{-1}\right]. \end{aligned}$$

Here, we again used the fact in Footnote 27. The f.o.c. w.r.t. X_{i0} implies that the demand is proportional to $-P_0$, i.e., $X_{\eta 0}(P_0) \propto -P_0$. We can use a similar derivation to show that the i'th late-informed investor's demand is also proportional to $-P_0$, i.e., $X_{\ell 0}(P_0) \propto -P_0$. The market clearing requirement, $0 = mX_{\eta 0}(P_0) + (1-m)X_{\ell 0}(P_0)$, implies $P_0 = 0$.

Finally, we can use a similar analysis as above to show that the prices of the k'th factor take the form given in Eq. (3). The presentation of these prices is much simpler because there are no biased beliefs for the factors. □

Proof of Proposition 2: Consider the expression for α_2 in Proposition 1. It follows from $\omega_\gamma < \nu_\gamma$ that $\alpha_2 \in [\nu_\theta/\nu_\gamma, \nu_\theta/\omega_\gamma]$. It is straightforward to show after taking derivatives that α_2 decreases in m and ω_γ (and thus in ω_μ). □

Proof of Corollary 1: Using Eq. (8), we have that $\mathrm{Cov}(P_1 - P_0, \theta - P_2) = \alpha_1(\nu_\theta - \alpha_2\nu_\gamma) < 0$, where the inequality obtains from $\alpha_2 > \nu_\theta/\nu_\gamma$ (see Proposition 2). □

Proof of Corollary 2: (i) Note from Propositions 1 and 2 that $\alpha_1 < \alpha_2$ and $\alpha_2 > \nu_\theta/\nu_\gamma$. It follows from Eq. (6) that $\mathrm{Cov}(P_2 - P_1, \theta - P_2) = (\alpha_2 - \alpha_1)(\nu_\theta - \alpha_2\nu_\gamma) < 0$.
(ii) Eq. (5) yields that for $\mathrm{Cov}(P_1 - P_0, P_2 - P_1) = \alpha_1(\alpha_2\nu_\gamma - \alpha_1\nu_s) > 0$, it suffices that $\alpha_1/\alpha_2 < \nu_\gamma/\nu_s$. Note that if $m = 1$, then $\alpha_1/\alpha_2 = \nu_\gamma/\omega_s > \nu_\gamma/\nu_s$. In this case, $\mathrm{Cov}(P_1 - P_0, P_2 - P_1) < 0$.

Let m be small. Then, from Proposition 1,

$$\alpha_1/\alpha_2 \quad \leq \quad \frac{mH(\nu_\gamma,\omega_s){\omega_s}^{-1}\nu_\gamma + (1-m)H(\omega_\gamma,\kappa_s){\kappa_s}^{-1}\omega_\gamma}{mH(\nu_\gamma,\omega_s) + (1-m)H(\omega_\gamma,\kappa_s)}.$$

If $m \to 0$, then the right hand side of the above inequality converges to ω_γ/κ_s. It is straightforward to show that because $\omega_\mu < \nu_\mu$ and $\kappa_\epsilon > \nu_\epsilon$,

$$\frac{\omega_\gamma}{\kappa_s} - \frac{\nu_\gamma}{\nu_s} = \frac{\nu_\theta + \omega_\mu}{\nu_\theta + \omega_\mu + \kappa_\epsilon} - \frac{\nu_\theta + \nu_\mu}{\nu_\theta + \nu_\mu + \nu_\epsilon} \propto \nu_\epsilon(\nu_\theta + \omega_\mu) - \kappa_\epsilon(\nu_\theta + \nu_\mu) < 0.$$

Therefore, for sufficiently small m, $\alpha_1/\alpha_2 < \nu_\gamma/\nu_s$ and thus $\mathrm{Cov}(P_1 - P_0, P_2 - P_1) > 0$. $\square$

Proof of Proposition 3: It follows from Proposition 1 that if $\omega_\mu = \nu_\mu$, then $\alpha_2 = \nu_\theta/\nu_\gamma$. It is straightforward to show after taking derivatives that α_1 increases in m and decreases in κ_s (and thus in κ_ϵ), and ω_s (and so in ω_ϵ). $\square$

Proof of Corollary 3: From Proposition 3, $\alpha_2 = \nu_\theta/\nu_\gamma$ and that as m increases from 0 to 1, α_1 increases from ν_θ/κ_s to ν_θ/ω_s. Then, it follows from Eqs. (5) and (7) that there exists a unique $m^{**} \in (0, 1)$ so that

$$\overline{MOM} \propto \mathrm{Cov}(P_1 - P_0, P_2 - P_1) \propto \nu_\theta - \alpha_1\nu_s = 0.$$

Part (i) of this corollary follows immediately from α_1 increasing in m.
(ii) Note from the above analysis that m^{**} is determined by $\nu_\theta - \alpha_1\nu_s = 0$, where α_1 increases in m and decreases in κ_ϵ (see Proposition 3). It follows after taking implicit derivatives that m^{**} increases in κ_ϵ. $\square$

Proof of Corollary 4: From Proposition 3, $\alpha_2 = \nu_\theta/\nu_\gamma$. It then follows from Eqs. (5) and (6) that

$$\overline{MOM} = \mathrm{Cov}(P_1 - P_0, P_2 - P_1)/2 = \alpha_1(\nu_\theta - \alpha_1\nu_s)/2.$$

Note that $d\alpha_1/d\kappa_\epsilon < 0$ from Proposition 3. It follows that

$$\frac{d\overline{MOM}}{d\kappa_\epsilon} = (\nu_\theta - 2\alpha_1\nu_s)\frac{d\alpha_1}{d\kappa_\epsilon} \propto -(\nu_\theta - 2\alpha_1\nu_s).$$

This corollary follows immediately. $\square$.

Proof of Proposition 4: We use two steps to prove this proposition.

Step 1: We solve for the equilibrium prices using backward induction.

Date 2: We can use the same derivation as in the proof of Proposition 1 to show that at Date 2, the i'th early-informed investor's demand, $X_{\eta 2}(\gamma, P_2)$, is given in Eq. (22), and that the i'th late-informed investor's demand, $X_{\ell 2}(\gamma, P_2)$, is given by Eq. (23).

The i'th uninformed investor learns γ from the price P_2, which is fully revealing. Like $X_{\eta 2}(\gamma, P_2)$ in Eq. (22), the demand (note that the belief is indicated by ν_γ, and risk aversion is A_N) can be expressed as:

$$X_{N2}(\gamma, P_2) = {A_N}^{-1} H(\nu_\theta, \nu_\gamma)\left(\frac{\nu_\theta}{\nu_\gamma}\gamma - P_2\right). \tag{28}$$

From Eqs. (22), (23), and (28), the market clearing condition

$$0 \;=\; mX_{\eta 2}(\gamma, P_2) + (1-m)X_{\ell 2}(\gamma, P_2) + \lambda X_{N2}(\gamma, P_2)$$

implies $P_2 = \alpha_2\gamma$, where

$$\alpha_2 = \frac{mH(\nu_\theta,\nu_\gamma){\nu_\gamma}^{-1}\nu_\theta + (1-m)H(\nu_\theta,\omega_\gamma){\omega_\gamma}^{-1}\nu_\theta + \lambda({A_N}^{-1}A)H(\nu_\theta,\nu_\gamma){\nu_\gamma}^{-1}\nu_\theta}{mH(\nu_\theta,\nu_\gamma) + (1-m)H(\nu_\theta,\omega_\gamma) + \lambda({A_N}^{-1}A)H(\nu_\theta,\nu_\gamma)}. \tag{29}$$

Date 1: We can use the same derivation as in the proof of Proposition 1 to show that at Date 1, the i'th early-informed investor's demand, $X_{\eta 1}(s, P_1)$, is given in Eq. (26), and that the i'th late-informed investor's demand, $X_{\ell 1}(s, P_1)$, is given in Eq. (27).

The i'th uninformed investor learns s from the price P_1, which is fully revealing. Like $X_{\eta 1}(s, P_1)$ in Eq. (26), the demand (the beliefs are indicated by ν_s and ν_γ) can be expressed as:

$$X_{N1}(s, P_1) = \frac{-P_1}{A_N\alpha_2^2}\left[H(\nu_\gamma,\nu_s) + H(\nu_\theta,\nu_\gamma)(\frac{\nu_\theta}{\nu_\gamma} - \alpha_2)^2\right] + \frac{1}{A_N\alpha_2}H(\nu_\gamma,\nu_s)\frac{\nu_\gamma}{\nu_s}s. \tag{30}$$

From Eqs. (26), (27), and (30), the market clearing condition

$$0 = mX_{\eta 1}(s, P_1) + (1-m)X_{\ell 1}(s, P_1) + \lambda X_{N1}(s, P_1)$$

implies $P_1 = \alpha_1 s$. The parameter is given by

$$\alpha_1 = \frac{\alpha_2}{Q}\left[mH(\nu_\gamma, \omega_s)\frac{\nu_\gamma}{\omega_s} + (1-m)H(\omega_\gamma, \kappa_s)\frac{\omega_\gamma}{\kappa_s} + \lambda(A/A_N)H(\nu_\gamma, \nu_s)\frac{\nu_\gamma}{\nu_s}\right], \quad (31)$$

where α_2 is given in Eq. (29), and

$$\begin{aligned} q_\eta &= m\left[H(\nu_\gamma, \omega_s) + H(\nu_\theta, \nu_\gamma)\left(\frac{\nu_\theta}{\nu_\gamma} - \alpha_2\right)^2\right], \\ q_\ell &= (1-m)\left[H(\omega_\gamma, \kappa_s) + H(\nu_\theta, \omega_\gamma)\left(\frac{\nu_\theta}{\omega_\gamma} - \alpha_2\right)^2\right], \\ q &= \lambda(A/A_N)\left[H(\nu_\gamma, \nu_s) + H(\nu_\theta, \nu_\gamma)\left(\frac{\nu_\theta}{\nu_\gamma} - \alpha_2\right)^2\right], \text{ with} \\ Q &= q_\eta + q_\ell + q. \end{aligned}$$

Date 0: We can use a similar derivation as for the proof of Proposition 1 to show that $P_0 = 0$.

Step 2: If $\lambda/A_N \to 0$, then it is straightforward to show that the equilibrium derived in Step 1 is identical to that described in Proposition 1. If $\lambda/A_N \to \infty$, then it follows that $\alpha_2 = \nu_\theta/\nu_\gamma$ and $\alpha_1 = \nu_\theta/\nu_s$. From Eqs. (5) and (6), we have that

$$\overline{MOM} = \left[\mathrm{Cov}(P_1 - P_0, P_2 - P_1) + \mathrm{Cov}(P_2 - P_1, \theta - P_2)\right]/2 \to 0.$$

It follows from Eq. (8) that $\mathrm{Cov}(P_1 - P_0, \theta - P_2) \to 0$. □

Proof of Proposition 5: (i) Suppose that the public information t reaches investors at Date 2. We first solve for the equilibrium prices using backward induction. Denote

$$\Gamma_\eta \equiv \frac{\gamma\nu_\mu^{-1} + t(\kappa\nu_\xi)^{-1}}{\nu_\mu^{-1} + (\kappa\nu_\xi)^{-1}}, \quad \Gamma_\ell \equiv \frac{\gamma\omega_\mu^{-1} + t(\kappa\nu_\xi)^{-1}}{\omega_\mu^{-1} + (\kappa\nu_\xi)^{-1}}, \text{ and } \Gamma \equiv \frac{\gamma\nu_\mu^{-1} + t{\nu_\xi}^{-1}}{\nu_\mu^{-1} + {\nu_\xi}^{-1}}.$$

At Date 2, early-informed (late-informed) (rational uninformed) investors believe that Γ_η (Γ_ℓ) (Γ) dominates the late Date-2 information γ, and condition their Date-2 trades on their own Γ. Based on the normality assumption, they believe that Γ_η (Γ_ℓ) (Γ) has a variance $\kappa_\Gamma = \nu_\theta + (1/\nu_\mu + 1/(\kappa\nu_\xi))^{-1}$, ($\omega_\Gamma = \nu_\theta + (1/\omega_\mu + 1/(\kappa\nu_\xi))^{-1}$) ($\nu_\Gamma = \nu_\theta + (1/\nu_\mu + 1/\nu_\xi)^{-1}$).

The i'th early-informed investor believes

$$\theta|\Gamma_\eta \sim N\left(\frac{\nu_\theta}{\kappa_\Gamma}\Gamma_\eta, H(\nu_\theta,\kappa_\Gamma)^{-1}\right).$$

Write the wealth at Date 3 as $W_{i3} = W_{i2} + X_{i2}(\theta - P_2)$. He needs to choose X_{i2} to maximize

$$\begin{aligned} E_\eta\left[U_\eta(W_{i3})|\Gamma_\eta\right] &= E_\eta\left[-\exp\left[-AW_{i2} - AX_{i2}(\theta - P_2)\right]|\Gamma_\eta\right] \\ &= -\exp\left[-AW_{i2} - AX_{i2}(\frac{\nu_\theta}{\kappa_\Gamma}\Gamma_\eta - P_2) + 0.5A^2X_{i2}^2H(\nu_\theta,\kappa_\Gamma)^{-1}\right]. \quad (32)\end{aligned}$$

The f.o.c. w.r.t. X_{i2} implies that the demand can be expressed as:

$$X_{\eta 2}(\Gamma_\eta, P_2) = A^{-1}H(\nu_\theta,\kappa_\Gamma)\left(\frac{\nu_\theta}{\kappa_\Gamma}\Gamma_\eta - P_2\right). \quad (33)$$

We can use a similar analysis to show that the demand of the i'th late-informed investor (whose beliefs are indicated by Γ_ℓ and ω_Γ) can be expressed as:

$$X_{\ell 2}(\Gamma_\ell, P_2) = A^{-1}H(\nu_\theta,\omega_\Gamma)\left(\frac{\nu_\theta}{\omega_\Gamma}\Gamma_\ell - P_2\right). \quad (34)$$

The demand of the i'th rational uninformed investor (whose belief is indicated by Γ and ν_Γ, and whose risk aversion is indicated by A_N) can be expressed as:

$$X_{N2}(\Gamma, P_2) = A_N^{-1}H(\nu_\theta,\nu_\Gamma)\left(\frac{\nu_\theta}{\nu_\Gamma}\Gamma - P_2\right). \quad (35)$$

From Eqs. (33), (34), and (35), the market clearing condition, $0 = mX_{\eta 2}(\Gamma_\eta, P_2) + (1 - m)X_{\ell 2}(\Gamma_\ell, P_2) + \lambda X_{N2}(\Gamma, P_2)$, implies

$$P_2 = \frac{mH(\nu_\theta,\kappa_\Gamma){\kappa_\Gamma}^{-1}\nu_\theta\Gamma_\eta + (1-m)H(\nu_\theta,\omega_\Gamma){\omega_\Gamma}^{-1}\nu_\theta\Gamma_\ell + \lambda({A_N}^{-1}A)H(\nu_\theta,\nu_\Gamma){\nu_\Gamma}^{-1}\nu_\theta\Gamma}{mH(\nu_\theta,\kappa_\Gamma) + (1-m)H(\nu_\theta,\omega_\Gamma) + \lambda({A_N}^{-1}A)H(\nu_\theta,\nu_\Gamma)}. \quad (36)$$

Date 1: Write an early-informed investor's wealth at Date 2 as $W_{i2} = W_{i1} + X_{i1}(P_2 - P_1)$. It follows from Eqs. (32) and (33) that the Date-2 expected utility can be expressed as:

$$E_\eta\left[U_\eta(W_{i3})|\Gamma_\eta\right] = -\exp\left[-AW_{i1} - AX_{i1}(P_2 - P_1) - 0.5H(\nu_\theta,\kappa_\Gamma)(\frac{\nu_\theta}{\kappa_\Gamma}\Gamma_\eta - P_2)^2\right].$$

Note that P_2 and Γ_η are linear in γ and t. Further, the belief is

$$\begin{pmatrix}\gamma \\ t\end{pmatrix}|s \sim N(\rho_\eta s, V_\eta),$$

where ρ_η and V_η are constant parameters based on the belief regarding the variance-covariance matrix of γ, t, and s. We can use the fact in Footnote 27 to show that

$$\begin{aligned} & E_\eta \left[U_\eta(W_{i3})|s\right] = E_\eta \left[E_\eta \left[U_\eta(W_{i3})|\Gamma_\eta\right]|s\right] \\ \propto \quad & -\exp\left[-AW_{i1} - AX_{i1}(-P_1) + 0.5(AX_{i1}D_P - V_\eta^{-1}\rho_\eta s)^T \Sigma_\eta^{-1}(AX_{i1}D_P - V_\eta^{-1}\rho_\eta s)\right], \end{aligned}$$

where D_P and Σ_η are constant parameters. The f.o.c. w.r.t. X_{i1} implies that the optimal demand is given by

$$X_{\eta 1}(s, P_1) \;=\; \phi_\eta s - \phi_{\eta P} P_1,$$

where ϕ_η and $\phi_{\eta P}$ are constant parameters. We can use a similar analysis to show that a late-informed or rational uninformed investor's optimal demand is given by

$$\begin{aligned} X_{\ell 1}(s, P_1) &= \phi_\ell s - \phi_{\ell P} P_1, \\ X_{N1}(s, P_1) &= \phi_N s - \phi_{NP} P_1, \end{aligned}$$

where ϕ_ℓ, $\phi_{\ell P}$, ϕ_N, and ϕ_{NP} are constant parameters. The market clearing condition, $0 = mX_{\eta 1}(s, P_1) + (1-m)X_{\ell 1}(s, P_1) + \lambda X_{N1}(s, P_1)$, implies

$$P_1 = \frac{m\phi_\eta + (1-m)\phi_\ell + \lambda\phi_N}{m\phi_{\eta P} + (1-m)\phi_{\ell P} + \lambda\phi_{NP}} s.$$

Date 0: We can use a similar derivation as for the proof of Proposition 1 to show $P_0 = 0$.

Now consider different levels of ν_ξ. If $\nu_\xi \to \infty$, then it is straightforward to show that $\Gamma_\eta, \Gamma_\ell, \Gamma \to \gamma$. The equilibrium is identical to that described in the proof of Proposition 4. Momentum and reversals arise under certain parameter values. If $\nu_\xi \to 0$, then it is straightforward to show that $\Gamma_\eta, \Gamma_\ell, \Gamma \to t \to \theta$. It follows from the above analysis that $P_2 \to \theta$, and

$$P_1 \to \frac{mH(\nu_\theta, \omega_s)\omega_s^{-1}\nu_\theta + (1-m)H(\nu_\theta, \kappa_s)\kappa_s^{-1}\nu_\theta + \lambda(A_N^{-1}A)H(\nu_\theta, \nu_s)\nu_s^{-1}\nu_\theta}{mH(\nu_\theta, \omega_s) + (1-m)H(\nu_\theta, \kappa_s) + \lambda(A_N^{-1}A)H(\nu_\theta, \nu_s)} s.$$

It follows that $\mathrm{Cov}(P_1 - P_0, \theta - P_2) \to 0$. Further, because $\mathrm{Cov}(P_2 - P_1, \theta - P_2) \to 0$, $\overline{MOM} \to \mathrm{Cov}(P_1 - P_0, P_2 - P_1)/2$. We can use a similar analysis as in the proof of Corollary 3 to show that under certain parameter values, $\overline{MOM} > 0$.

(ii) Suppose that the public information t reaches investors at Date 1. The above analysis for Date 2 still applies. Thus, P_2 takes the form as in Eq. (36). Note that at Date 1, investors know s and t. We can use a similar analysis as above to show that the Date-1 price P_1 will, instead, take a linear form of s and t; the Date-0 price is still $P_0 = 0$. Now consider different levels of ν_ξ. If $\nu_\xi \to \infty$, then it is straightforward to show that investors will ignore the public information t. The equilibrium is identical to that described in the proof of Proposition 4. Momentum and reversals arise under certain parameter values. If $\nu_\xi \to 0$, then it is straightforward to show that investors will condition their trades only on $t \to \theta$, and $P_1 = P_2 = \theta$. In this case, all returns equal zero so that momentum and reversals do not obtain. $\square$

Proof of Proposition 6: Eq. (36) in the proof of Proposition 5 indicates that P_2 is a weighted average of $\kappa_\Gamma{}^{-1}\nu_\theta\Gamma_\eta$, $\omega_\Gamma{}^{-1}\nu_\theta\Gamma_\ell$, and $\nu_\Gamma{}^{-1}\nu_\theta\Gamma$. Thus, for $\text{Cov}(\theta - P_2, t) > 0$, it suffices to show that

$$\text{Cov}\left(\theta - \kappa_\Gamma{}^{-1}\nu_\theta\Gamma_\eta, t\right) > 0, \;\; \text{Cov}(\theta - \omega_\Gamma{}^{-1}\nu_\theta\Gamma_\ell, t) > 0, \text{ and } \text{Cov}(\theta - \nu_\Gamma{}^{-1}\nu_\theta\Gamma, t) = 0.$$

It follows from the proof of Proposition 5 that

$$\begin{aligned}
\text{Cov}(\theta - \kappa_\Gamma{}^{-1}\nu_\theta\Gamma_\eta, t) &= \text{Cov}(\theta - \frac{\nu_\theta}{\nu_\theta + (\nu_\mu{}^{-1} + \kappa\nu_\xi^{-1})^{-1}} \frac{\gamma\nu_\mu^{-1} + t(\kappa\nu_\xi)^{-1}}{\nu_\mu^{-1} + (\kappa\nu_\xi)^{-1}}, t) \\
&= \nu_\theta - \frac{\nu_\theta}{\nu_\theta + (\nu_\mu{}^{-1} + \kappa\nu_\xi^{-1})^{-1}} \frac{\nu_\theta\nu_\mu^{-1} + (\nu_\theta + \nu_\xi)(\kappa\nu_\xi)^{-1}}{\nu_\mu^{-1} + (\kappa\nu_\xi)^{-1}} \\
&> 0,
\end{aligned}$$

where the inequality follows from $\kappa > 1$. We can use a similar analysis to show that $\text{Cov}(\theta - \omega_\Gamma{}^{-1}\nu_\theta\Gamma_\ell, t) > 0$, and $\text{Cov}(\theta - \nu_\Gamma{}^{-1}\nu_\theta\Gamma, t) = 0$. $\square$

Proof of Proposition 7: We use two steps to prove this proposition.

Step 1: We solve for the equilibrium prices using backward induction.

Date 2: Note that at this date, γ is a sufficient statistic for θ. We can use the same derivation as for the proofs of Propositions 1 and 4 (Date 2) to show that the price at this date is given by $P_2 = \alpha_2\gamma$, where α_2 is given in Eq. (29). Like Eq. (24), we can show that the expected utilities of the i'th late-informed investor at Date 2 can be expressed as:

$$E_\ell\left[U_\ell(W_{i3})|\gamma\right] = -\exp\left[-AW_{i2} - 0.5H(\nu_\theta, \omega_\gamma)(\frac{\nu_\theta}{\omega_\gamma} - \alpha_2)^2\gamma^2\right]. \tag{37}$$

Date 1: We can use the same derivation as for the proof of Proposition 1 (Date 1) to show that the i'th early-informed investor's demand can be expressed as

$$X_{\eta 1}(s, P_1) = \frac{-P_1}{A\alpha_2^2}\left[H(\nu_\gamma, \omega_s) + H(\nu_\theta, \nu_\gamma)(\frac{\nu_\theta}{\nu_\gamma} - \alpha_2)^2\right] + \frac{1}{A\alpha_2}H(\nu_\gamma, \omega_s)\frac{\nu_\gamma}{\omega_s}s. \tag{38}$$

Conjecture that the price at Date 1 takes a linear form given by

$$P_1 = \alpha_\tau \tau, \tag{39}$$

where $\tau = s - \delta z$, and α_τ and δ are constant parameters to be determined. The i'th late-informed investor learns τ from the price P_1, and believes that

$$\gamma|\tau \sim N\left(\frac{\omega_\gamma}{\kappa_\tau}\tau, H(\omega_\gamma, \kappa_\tau)^{-1}\right),$$

where $\kappa_\tau = \kappa_s + \delta^2\nu_z$, and $H(\omega_\gamma, \kappa_\tau)^{-1} = \omega_\gamma(1 - \omega_\gamma/\kappa_\tau)$. Consider the investor's expected utility in Eq. (37) and write the wealth at Date 2 as $W_{i2} = W_{i1} + X_{i1}(P_2 - P_1)$, where the price $P_2 = \alpha_2\gamma$ is as derived above. It follows that the expected utility at Date 1 can be expressed as

$$\begin{aligned} E_\ell\left[U_\ell(W_{i3})|\tau\right] &= E_\ell\left[-\exp\left[-AW_{i1} - AX_{i1}(\alpha_2\gamma - P_1) - 0.5H(\nu_\theta, \omega_\gamma)\left(\nu_\theta\omega_\gamma^{-1} - \alpha_2\right)^2\gamma^2\right]|\tau\right] \\ &\propto -\exp\left[-AW_{i1} - AX_{i1}(-P_1) + 0.5\frac{(AX_{i1}\alpha_2 - H(\omega_\gamma, \kappa_\tau)\kappa_\tau{}^{-1}\omega_\gamma\tau)^2}{H(\omega_\gamma, \kappa_\tau) + H(\nu_\theta, \omega_\gamma)(\omega_\gamma{}^{-1}\nu_\theta - \alpha_2)^2}\right. \\ &\quad \left. -0.5H(\omega_\gamma, \kappa_\tau)\left(\omega_\gamma\kappa_\tau^{-1}\tau\right)^2\right]. \end{aligned}$$

Here, we used the fact in Footnote 27. The f.o.c. w.r.t. X_{i1} implies that the demand can be expressed as

$$X_{\ell 1}(\tau, P_1) = \frac{-P_1}{A\alpha_2^2}\left[H(\omega_\gamma, \kappa_\tau) + H(\nu_\theta, \omega_\gamma)\left(\frac{\nu_\theta}{\omega_\gamma} - \alpha_2\right)^2\right] + \frac{1}{A\alpha_2}H(\omega_\gamma, \kappa_\tau)\frac{\omega_\gamma}{\kappa_\tau}\tau. \tag{40}$$

We can use a similar derivation to show that the i'th uninformed investor's demand (his belief is indicated by ν_τ and ν_γ) can be expressed as

$$X_{N1}(\tau, P_1) = \frac{-P_1}{A_N\alpha_2^2}\left[H(\nu_\gamma, \nu_\tau) + H(\nu_\theta, \nu_\gamma)\left(\frac{\nu_\theta}{\nu_\gamma} - \alpha_2\right)^2\right] + \frac{1}{A_N\alpha_2}H(\nu_\gamma, \nu_\tau)\frac{\nu_\gamma}{\nu_\tau}\tau. \tag{41}$$

From Eqs. (38), (40), and (41), the market clearing condition

$$z = mX_{\eta 1}(s, P_1) + (1-m)X_{\ell 1}(\tau, P_1) + \lambda X_{N1}(\tau, P_1)$$

implies that the parameters in the price P_1 in Eq. (39), δ and α_τ, are given by

$$\delta = m^{-1} A \alpha_2 (\omega_s - \nu_\gamma), \tag{42}$$

$$\alpha_\tau = \frac{\alpha_2}{\Psi}\left[mH(\nu_\gamma, \omega_s)\frac{\nu_\gamma}{\omega_s} + (1-m)H(\omega_\gamma, \kappa_\tau)\frac{\omega_\gamma}{\kappa_\tau} + \lambda(A/A_N)H(\nu_\gamma, \nu_\tau)\frac{\nu_\gamma}{\nu_\tau}\right], \tag{43}$$

where $\kappa_\tau = \kappa_s + \delta^2\nu_z$, $\nu_\tau = \nu_s + \delta^2 \nu_z$, and

$$\begin{aligned}
\psi_\eta &= m\left[H(\nu_\gamma, \omega_s) + H(\nu_\theta, \nu_\gamma)\left(\frac{\nu_\theta}{\nu_\gamma} - \alpha_2\right)^2\right], \\
\psi_\ell &= (1-m)\left[H(\omega_\gamma, \kappa_\tau) + H(\nu_\theta, \omega_\gamma)\left(\frac{\nu_\theta}{\omega_\gamma} - \alpha_2\right)^2\right], \\
\psi_\lambda &= \lambda(A/A_N)\left[H(\nu_\gamma, \nu_\tau) + H(\nu_\theta, \nu_\gamma)\left(\frac{\nu_\theta}{\nu_\gamma} - \alpha_2\right)^2\right], \text{with} \\
\Psi &= \psi_\eta + \psi_\ell + \psi_\lambda.
\end{aligned}$$

Date 0: We can use a similar derivation as for the proof of Proposition 1 to show that $P_0 = 0$.

Step 2: If $\nu_z \to \infty$, then it follows from the above analysis that $\kappa_\tau = \kappa_s + \delta^2\nu_z \to \infty$, $\nu_\tau = \nu_s + \delta^2\nu_z \to \infty$. Further, $\alpha_2, \alpha_\tau > 0$ are bounded from above. It follows that

$$\begin{aligned}
\mathrm{Cov}(P_1 - P_0, P_2 - P_1) &= \mathrm{Cov}(\alpha_\tau(s - \delta z), \alpha_2\gamma - \alpha_\tau(s - \delta z)) \\
&= \mathrm{Cov}(\alpha_\tau s, \alpha_2\gamma - \alpha_\tau s) - \alpha_\tau^2\delta^2\nu_z \\
&\to -\infty,
\end{aligned}$$

where the $\to$ obtains because $\nu_z \to \infty$ and other items are bounded;

$$\begin{aligned}
\mathrm{Cov}(P_2 - P_1, \theta - P_2) &= \mathrm{Cov}(\alpha_2\gamma - \alpha_\tau(s - \delta z), \theta - \alpha_2\gamma) \\
&= \mathrm{Cov}(\alpha_2\gamma - \alpha_\tau s, \theta - \alpha_2\gamma) \\
&= (\alpha_2 - \alpha_\tau)(\nu_\theta - \alpha_2\nu_\gamma)
\end{aligned}$$

is bounded. This proposition obtains immediately. □

Proof of Proposition 8: Note that α_2 and δ given in Eqs. (29) and (42) do not depend on κ_τ and thus κ_s and κ_ϵ. From Eq. (43), it is straightforward to show that α_τ decreases in κ_τ and therefore in κ_s and κ_ϵ. This proposition immediately obtains. □

Proof of Proposition 9: (i) and (ii) The difference between α_τ and ν_θ/ν_τ is a consequence of biased beliefs at Date 1. To see this, let $\omega_\mu \to \nu_\mu$ (so that there is no biased belief about the Date-2 information; thus, $\omega_\gamma \to \nu_\gamma$), and $\nu_z \to 0$ (so that the price P_1 fully reveals s; thus, $\nu_\tau = \nu_s$ and $\kappa_\tau = \kappa_s$). It follows from the proof of Proposition 7 that $\alpha_2 = \nu_\theta/\nu_\gamma$ and

$$\alpha_\tau \;=\; \frac{\nu_\theta}{\nu_\gamma} \times \frac{mH(\nu_\gamma,\omega_s){\omega_s}^{-1}\nu_\gamma + (1-m)H(\nu_\gamma,\kappa_s){\kappa_s}^{-1}\nu_\gamma + \lambda({A_N}^{-1}A)H(\nu_\gamma,\nu_s){\nu_s}^{-1}\nu_\gamma}{mH(\nu_\gamma,\omega_s) + (1-m)H(\nu_\gamma,\kappa_s) + \lambda({A_N}^{-1}A)H(\nu_\gamma,\nu_s)}.$$

Note that $\omega_s < \nu_s < \kappa_s$. If late-informed investors are not very skeptical about the quality of the early Date-1 information s (i.e., $\kappa_\epsilon \to \nu_\epsilon$ and therefore $\kappa_s \to \nu_s$), then $\alpha_\tau > \nu_\theta/\nu_\tau = \nu_\theta/\nu_s$. If late-informed investors are very skeptical about the quality of the early Date-1 information s (i.e., $\kappa_\epsilon \gg \nu_\epsilon$ and therefore $\kappa_s \gg \nu_s$), then it can be that $\alpha_\tau < \nu_\theta/\nu_\tau = \nu_\theta/\nu_s$.

From the proof of Proposition 8, α_τ decreases in κ_τ and therefore κ_s. Write Eq. (14) as

$$\mathrm{MSE}_1 = \sum_{k=1}^{K}\left[\beta_k^2 \mathrm{Var}(f_k - \frac{\nu_{f_k}}{\nu_{s_{J+k}}} s_{J+k})\right] + \mathrm{Var}\left(\theta - \frac{\nu_\theta}{\nu_\tau}\tau\right) + \left(\alpha_\tau - \frac{\nu_\theta}{\nu_\tau}\right)^2 \nu_\tau.$$

It follows that

$$\frac{d\mathrm{MSE}_1}{d\kappa_s} \;\propto\; \left(\alpha_\tau - \frac{\nu_\theta}{\nu_\tau}\right)\frac{d\alpha_\tau}{d\kappa_s} \propto -\left(\alpha_\tau - \frac{\nu_\theta}{\nu_\tau}\right).$$

If $\alpha_\tau - \nu_\theta/\nu_\tau$ is positive (negative), then MSE_1 decreases (increases) in κ_s and thus κ_ϵ.

From the proof of Proposition 8, α_2 does not depend on κ_ϵ and thus κ_s. Part (iii) of this proposition obtains from the expression of MSE_2 in Eq. (15). □

Proof of Eqs. (16)-(17): Conjecture that the prices take the forms in Eqs. (16)-(17). In what follows, we verify these conjectures and solve for the pricing parameters. We use backward induction.

Date 2: First, we fix δ and solve for the Date-2 price. At Date 2, the i'th early-informed investor learns γ (from P_2) and γ_2, and trades conditional on the information set, $I_{\eta 2} = \{\gamma, \gamma_2\}$. The investor believes that γ dominates γ_2, and that θ and γ follow a multi-variate normal distribution with a mean vector of zero and variance-covariance matrix

$$\begin{pmatrix} \nu_\theta & \nu_\theta \\ \nu_\theta & \nu_\gamma \end{pmatrix}.$$

Write the wealth at Date 3 as $W_{i3} = W_{i2} + X_{i2}(\theta - P_2)$. The investor chooses X_{i2} to maximize

$$\begin{aligned} & E_\eta \left[U_\eta(W_{i3}) | I_{\eta 2} \right] \\ = \; & -\exp\left[-AW_{i2} - AX_{i2}(E_\eta(\theta | I_{\eta 2}) - P_2) + 0.5A^2 X_{i2}^2 \mathrm{Var}_\eta(\theta | I_{\eta 2}) \right]. \end{aligned} \tag{44}$$

The f.o.c. w.r.t. X_{i2} implies that the demand can be expressed as:

$$X_{\eta 2} = \frac{E_\eta(\theta | I_{\eta 2}) - P_2}{A\mathrm{Var}_\eta(\theta | I_{\eta 2})} = \frac{c_\eta \gamma - P_2}{A\nu_{\eta 2}}, \tag{45}$$

where c_η and $\nu_{\eta 2}$ are constant parameters determined by the above variance-covariance matrix based on early-informed investors' beliefs.

At Date 2, a late-informed investor learns $I_{\ell 2} \equiv \{\gamma, \gamma_2\}$. From the investor's standpoint, θ and $I_{\ell 2}$ follow a multi-variate normal distribution with zero mean and variance-covariance matrix (note that from the investor's standpoint $\gamma = \theta_1 + \mu_1$)

$$\begin{pmatrix} \nu_\theta & (1-\Delta)\nu_\theta & \Delta\nu_\theta \\ (1-\Delta)\nu_\theta & (1-\Delta)\omega_\gamma & 0 \\ \Delta\nu_\theta & 0 & \Delta\omega_\gamma \end{pmatrix}.$$

From a similar analysis as above, the optimal demand can be expressed as:

$$X_{\ell 2} = \frac{E_\ell(\theta | I_{\ell 2}) - P_2}{A\mathrm{Var}_\ell(\theta | I_{\ell 2})} = \frac{c_\ell \gamma + a_\ell \gamma_2 - P_2}{A\nu_{\ell 2}},$$

where c_ℓ, a_ℓ, and $\nu_{\ell 2}$ are constant parameters determined by the above variance-covariance matrix based on late-informed investors' beliefs.

At Date 2, an uninformed rational investor learns $\gamma + a\gamma_2$ and $\tau = s - \delta z$, and trades conditional on the information set, $I_{N2} \equiv \{\gamma + a\gamma_2, \tau\}$. In this case, θ and I_{N2} follow a multi-variate normal distribution with a mean vector of zero and variance-covariance matrix

$$\begin{pmatrix} \nu_\theta & \nu_\theta + a\Delta\nu_\theta & \nu_\theta \\ \nu_\theta + a\Delta\nu_\theta & (1-\Delta)\nu_\gamma + (1+a)^2\Delta\nu_\gamma & \nu_\gamma + a\Delta\nu_\gamma \\ \nu_\theta & \nu_\gamma + a\Delta\nu_\gamma & \nu_\tau \end{pmatrix}.$$

The optimal demand can be expressed as:

$$X_{N2} = \frac{E_N(\theta | I_{N2}) - P_2}{A_N \mathrm{Var}_N(\theta | I_{N2})} = \frac{c_N(\gamma + a\gamma_2) + d_N\tau - P_2}{A_N \nu_{N2}},$$

where c_N, d_N, and ν_{N2} are constants again determined by the late-informed investors' beliefs about variances and covariances.

The market clearing condition, $0 = mX_{\eta 2} + (1-m)X_{\ell 2} + \lambda X_{N2}$, implies that P_2 takes the form in Eq. (17). Particularly, the parameters

$$\begin{aligned}
a &= \frac{(1-m)\nu_{\ell 2}^{-1} a_\ell}{m\nu_{\eta 2}^{-1} c_\eta + (1-m)\nu_{\ell 2}^{-1} c_\ell}, \\
C &= \frac{m\nu_{\eta 2}^{-1} c_\eta + (1-m)\nu_{\ell 2}^{-1} c_\ell + (\lambda A_N^{-1} A)\nu_{N2}^{-1} c_N}{m\nu_{\eta 2}^{-1} + (1-m)\nu_{\ell 2}^{-1} + (\lambda A_N^{-1} A)\nu_{N2}^{-1}}, \text{ and} \\
D &= \frac{(\lambda A_N^{-1} A)\nu_{N2}^{-1} d_N}{m\nu_{\eta 2}^{-1} + (1-m)\nu_{\ell 2}^{-1} + (\lambda A_N^{-1} A)\nu_{N2}^{-1}}.
\end{aligned}$$

Date 1: Write an early-informed investor's wealth at Date 2 as $W_{i2} = W_{i1} + X_{i1}(P_2 - P_1)$. It follows from Eqs. (44) and (45) that the Date-2 expected utility can be expressed as

$$\begin{aligned}
E_\eta\left[U_\eta(W_{i3})|I_{\eta 2}\right] &= -\exp\left[-AW_{i1} - AX_{i1}(P_2 - P_1) - 0.5\nu_{\eta 2}^{-1}(c_\eta \gamma - P_2)^2\right] \\
&= -\exp[-AW_{i1} - AX_{i1}(D\tau + F^T Y - P_1) - 0.5\nu_{\eta 2}^{-1}(R_\eta^T Y - D\tau)^2],
\end{aligned}$$

where

$$Y = \begin{pmatrix} \gamma \\ \gamma_2 \end{pmatrix}, F = \begin{pmatrix} C \\ Ca \end{pmatrix}, \text{ and } R_\eta = \begin{pmatrix} c_\eta \\ 0 \end{pmatrix} - F.$$

The investor learns s at Date 1, and believes that Y and s follow a multi-variate normal distribution with mean zero and variance-covariance matrix

$$\begin{pmatrix} \nu_\gamma & \Delta\nu_\gamma & \nu_\gamma \\ \Delta\nu_\gamma & \Delta\nu_\gamma & \Delta\nu_\gamma \\ \nu_\gamma & \Delta\nu_\gamma & \omega_s \end{pmatrix};$$

and thus, $Y|s \sim N(\iota_\eta s, \nu_{\eta 1})$, where ι_η and $\nu_{\eta 1}$ are constant parameters that depend on the above variance-covariance matrix. It follows that the investor needs to choose X_{i1} to maximize

$$\begin{aligned}
& E_\eta\left[U_\eta(W_{i3})|s\right] = E_\eta\left[E_\eta\left[U_\eta(W_{i3})|I_{\eta 2}\right]|s\right] \\
\propto\ & -\exp\Big[-AW_{i1} - AX_{i1}(D\tau - P_1) + 0.5(AX_{i1}F - \nu_{\eta 2}^{-1}R_\eta D\tau - \nu_{\eta 1}^{-1}\iota_\eta s)^T \\
& (\nu_{\eta 2}^{-1}R_\eta R_\eta^T + \nu_{\eta 1}^{-1})^{-1}(AX_{i1}F - \nu_{\eta 2}^{-1}R_\eta D\tau - \nu_{\eta 1}^{-1}\iota_\eta s)\Big].
\end{aligned}$$

The f.o.c. w.r.t. X_{i1} implies that the optimal demand is given by

$$\begin{aligned} X_{\eta 1} &= \left[AF^T(\nu_{\eta 2}^{-1}R_\eta R_\eta^T + \nu_{\eta 1}^{-1})^{-1}F\right]^{-1} \\ & \quad \left[D\tau - P_1 + F^T(\nu_{\eta 2}^{-1}R_\eta R_\eta^T + \nu_{\eta 1}^{-1})^{-1}(\nu_{\eta 2}^{-1}R_\eta D\tau + \nu_{\eta 1}^{-1}\iota_\eta s)\right] \\ &= \pi_{\eta s}s + \pi_{\eta\tau}\tau - \pi_{\eta P}P_1, \end{aligned}$$

where $\pi_{\eta s}$, $\pi_{\eta\tau}$, and $\pi_{\eta P}$ are constants.

Consider a late-informed investor. We can use a similar analysis as above to show that the Date-2 expected utility can be expressed as:

$$\begin{aligned} E_\ell\left[U_\ell(W_{i3})|I_{\ell 2}\right] &= -\exp\left[-AW_{i1} - AX_{i1}(P_2 - P_1) - 0.5\nu_{\ell 2}^{-1}(c_\ell\gamma + a_\ell\gamma_2 - P_2)^2\right] \\ &= -\exp\left[-AW_{i1} - AX_{i1}(D\tau + F^TY - P_1) - 0.5\nu_{\ell 2}^{-1}(R_\ell^TY - D\tau)^2\right], \end{aligned}$$

where

$$R_\ell = \begin{pmatrix} c_\ell \\ a_\ell \end{pmatrix} - F.$$

The investor learns $\tau = s - \delta z$ from P_1 at Date 1, and believes that Y and τ follow a multivariate normal distribution with zero mean and variance-covariance matrix (note that from the investor's standpoint $\gamma = \theta_1 + \mu_1$, and $s = \theta_1 + \mu_1 + \epsilon$)

$$\begin{pmatrix} (1-\Delta)\omega_\gamma & 0 & (1-\Delta)\omega_\gamma \\ 0 & \Delta\omega_\gamma & 0 \\ (1-\Delta)\omega_\gamma & 0 & (1-\Delta)\omega_\gamma + \kappa_\epsilon + \delta^2\nu_z \end{pmatrix},$$

and thus, $Y|\tau \sim N(\iota_\ell\tau, \nu_{\ell 1})$, where ι_ℓ and $\nu_{\ell 1}$ are constant parameters that depend on the above variance-covariance matrix. It follows that the investor needs to choose X_{i1} to maximize

$$\begin{aligned} & E_\ell\left[U_\ell(W_{i3})|\tau\right] = E_\ell\left[E_\ell\left[U_\ell(W_{i3})|I_{\ell 2}\right]|\tau\right] \\ \propto \; & -\exp\Big[-AW_{i1} - AX_{i1}(D\tau - P_1) + 0.5(AX_{i1}F - \nu_{\ell 2}^{-1}R_\ell D\tau - \nu_{\ell 1}^{-1}\iota_\ell\tau)^T \\ & (\nu_{\ell 2}^{-1}R_\ell R_\ell^T + \nu_{\ell 1}^{-1})^{-1}(AX_{i1}F - \nu_{\ell 2}^{-1}R_\ell D\tau - \nu_{\ell 1}^{-1}\iota_\ell\tau)\Big]. \end{aligned}$$

The f.o.c. w.r.t. X_{i1} implies that the optimal demand is given by

$$\begin{aligned} X_{\ell 1} &= \left[AF^T(\nu_{\ell 2}^{-1}R_\ell R_\ell^T + \nu_{\ell 1}^{-1})^{-1}F\right]^{-1} \\ & \quad \left[D\tau - P_1 + F^T(\nu_{\ell 2}^{-1}R_\ell R_\ell^T + \nu_{\ell 1}^{-1})^{-1}(\nu_{\ell 2}^{-1}R_\ell D\tau + \nu_{\ell 1}^{-1}\iota_\ell\tau)\right] \\ &= \pi_{\ell\tau}\tau - \pi_{\ell P}P_1, \end{aligned}$$

with $\pi_{\ell\tau}$ and $\pi_{\ell P}$ being constants that depend on exogenous parameters.

Consider a rational uninformed investor. We can use a similar analysis as above to show that the Date-2 expected utility can be expressed as:

$$\begin{aligned} & E_N\left[U_N(W_{i3})|I_{N2}\right] \\ = \; & -\exp\left[-A_N W_{i1} - A_N X_{i1}(D\tau + F^T Y - P_1) - 0.5\nu_{N2}^{-1}(R_N^T Y - (D-d_N)\tau)^2\right], \end{aligned}$$

where

$$R_N = \begin{pmatrix} c_N \\ c_N a \end{pmatrix} - F.$$

The investor learns $\tau = s - \delta z$ from P_1 at Date 1, and believes that Y and τ follow a multi-variate normal distribution with mean zeros and variance-covariance matrix

$$\begin{pmatrix} \nu_\gamma & \Delta\nu_\gamma & \nu_\gamma \\ \Delta\nu_\gamma & \Delta\nu_\gamma & \Delta\nu_\gamma \\ \nu_\gamma & \Delta\nu_\gamma & \nu_\tau \end{pmatrix},$$

and thus, $Y|\tau \sim N(\iota_N\tau, \nu_{N1})$, where ι_N and ν_{N1} are constant parameters which depend on the above variance-covariance matrix. It follows that the investor needs to choose X_{i1} to maximize

$$\begin{aligned} & E_N\left[U_N(W_{i3})|\tau\right] = E_N\left[E_N\left[U_N(W_{i3})|I_{N2}\right]|\tau\right] \\ \propto \; & -\exp\Big[-A_N W_{i1} - A_N X_{i1}(D\tau - P_1) + 0.5(A_N X_{i1} F - \nu_{N2}^{-1} R_N (D-d_N)\tau - \nu_{N1}^{-1}\iota_N\tau)^T \\ & (\nu_{N2}^{-1} R_N R_N^T + \nu_{N1}^{-1})^{-1}(A_N X_{i1} F - \nu_{N2}^{-1} R_N (D-d_N)\tau - \nu_{N1}^{-1}\iota_N\tau)\Big]. \end{aligned}$$

The f.o.c. w.r.t. X_{i1} implies that the optimal demand is given by

$$\begin{aligned} X_{N1} &= \left[A_N F^T(\nu_{N2}^{-1} R_N R_N^T + \nu_{N1}^{-1})^{-1} F\right]^{-1} \\ & \quad \left[D\tau - P_1 + F^T(\nu_{N2}^{-1} R_N R_N^T + \nu_{N1}^{-1})^{-1}(\nu_{N2}^{-1} R_N (D-d_N)\tau + \nu_{N1}^{-1}\iota_N\tau)\right] \\ &= \pi_{N\tau}\tau - \pi_{NP} P_1, \end{aligned}$$

where $\pi_{N\tau}$ and π_{NP} are again constants.

The market clearing condition, $z = mX_{\eta 1} + (1-m)X_{\ell 1} + \lambda X_{N1}$, implies that P_1 takes the form in Eq. (16). Particularly, the parameters

$$B = \frac{m(\pi_{\eta s} + \pi_{\eta\tau}) + (1-m)\pi_{\ell\tau} + \lambda\pi_{N\tau}}{m\pi_{\eta P} + (1-m)\pi_{\ell P} + \lambda\pi_{NP}}.$$

and

$$\delta = 1/(m\pi_{\eta s}) \tag{46}$$

The right hand side of Eq. (46) is also a function of δ; therefore, we can use Eq. (46) to solve for δ numerically. Using δ and the above derivation, we can solve for the other parameters in Eqs. (16) and (17).

Date 0: We can use a similar derivation as for the proof of Proposition 1 to show that $P_0 = 0$.

□

Proof of Proposition 10: It follows from the equilibrium prices in Eq. (18) that

$$\begin{aligned}
\mathrm{Cov}(P_1 - P_0, \theta - P_2) &= \mathrm{Cov}(\frac{\nu_\theta}{\nu_\gamma}(1+\alpha)\gamma, \theta - \frac{\nu_\theta}{\nu_\gamma}(\gamma + (1-m)\gamma_2)) \\
&= -\frac{\nu_\theta^2}{\nu_\gamma}(1+\alpha)(1-m)\Delta \\
&< 0,
\end{aligned}$$

where the second equality obtains from $E(\theta|\gamma) = \nu_\gamma^{-1}\nu_\theta\gamma$ and $\theta - E(\theta|\gamma) \perp \gamma$. From Eq. (18), we also have that

$$\begin{aligned}
&\mathrm{Cov}(P_1 - P_0, P_2 - P_1) \\
= \;& \mathrm{Cov}(\frac{\nu_\theta}{\nu_\gamma}(1+\Upsilon)\gamma, \frac{\nu_\theta}{\nu_\gamma}(\gamma + (1-m)\gamma_2) - \frac{\nu_\theta}{\nu_\gamma}(1+\Upsilon)\gamma) \\
= \;& \mathrm{Cov}(\frac{\nu_\theta}{\nu_\gamma}(1+\Upsilon)\gamma, \frac{\nu_\theta}{\nu_\gamma}((1-m)\gamma_2 - \Upsilon\gamma)) \\
= \;& \frac{\nu_\theta^2}{\nu_\gamma}(1+\Upsilon)((1-m)\Delta - \Upsilon) \\
> \;& 0,
\end{aligned}$$

where the last inequality holds because we can show $(1-m)\Delta > \Upsilon$.

$$\begin{aligned}
&\mathrm{Cov}(\theta - P_2, P_2 - P_1) \\
= \;& \mathrm{Cov}(\theta - \frac{\nu_\theta}{\nu_\gamma}(\gamma + (1-m)\gamma_2), \frac{\nu_\theta}{\nu_\gamma}(\gamma + (1-m)\gamma_2) - \frac{\nu_\theta}{\nu_\gamma}(1+\Upsilon)\gamma) \\
= \;& \mathrm{Cov}(-\frac{\nu_\theta}{\nu_\gamma}(1-m)\gamma_2, \frac{\nu_\theta}{\nu_\gamma}((1-m)\gamma_2 - \Upsilon\gamma)) \\
= \;& -\frac{\nu_\theta^2}{\nu_\gamma}(1-m-\Upsilon)(1-m)\Delta \\
< \;& 0,
\end{aligned}$$

where the second equality obtains from $E(\theta|\gamma) = \nu_\gamma^{-1}\nu_\theta\gamma$ and $\theta - E(\theta|\gamma)\perp\gamma, \gamma_2$, and the last inequality holds because $1 - m > (1 - m)\Delta > \Upsilon$. It follows that

$$\begin{aligned}
\overline{MOM} &\propto (1+\Upsilon)((1-m)\Delta - \Upsilon) - (1-m-\Upsilon)(1-m)\Delta \\
&= (1-m)m\Delta - \Upsilon(1+\Upsilon-2(1-m)\Delta) \\
&> (1-m)m\Delta - \Upsilon(1-(1-m)\Delta) \\
&\propto \Delta\left[\frac{1}{\Delta} + \frac{m}{1-\Delta} + m(1-m)\frac{\nu_\theta}{\nu_\gamma - \nu_\theta}\right] - \frac{1-(1-m)\Delta}{1-\Delta} \\
&> 0,
\end{aligned}$$

where the first inequality obtains because $(1 - m)\Delta > \Upsilon$. $\square$

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Figure 1: Long-Term Reversals

This graph plots the long-run reversal parameter $\text{Cov}(P_1 - P_0, \theta - P_2)$ as functions of the parameters representing late-informed investors' skepticism about the quality of the early Date-1 information (κ_ϵ), and overestimation of the quality of their own information (ω_μ). We assume the parameter values $m = 0.1$, $A = 1$, $\nu_\theta = 1$, $\nu_\mu = 1$, and $\nu_\epsilon = 1$. We let $\omega_\epsilon = \nu_\epsilon$ so that early-informed investors have rational beliefs about the quality of their information.

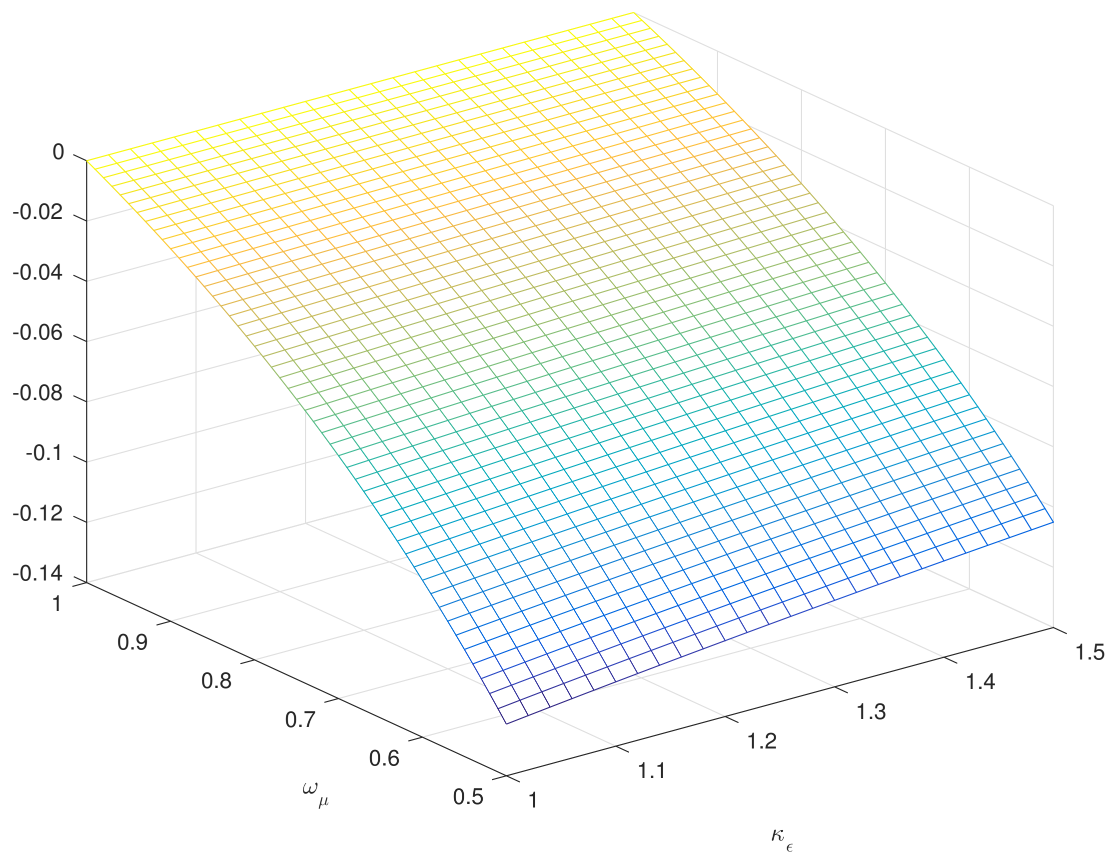


Figure 2: Short-Run Momentum and Reversals

This graph plots the momentum parameter $\overline{MOM}$ as functions of the parameters representing late-informed investors' skepticism about the quality of the early Date-1 information (κ_ϵ), and overestimation of the quality of their own information (ω_μ). We assume the parameter values $m = 0.1$, $A = 1$, $\nu_\theta = 1$, $\nu_\mu = 1$, and $\nu_\epsilon = 1$. We let $\omega_\epsilon = \nu_\epsilon$ so that early-informed investors have rational beliefs about the quality of their information.

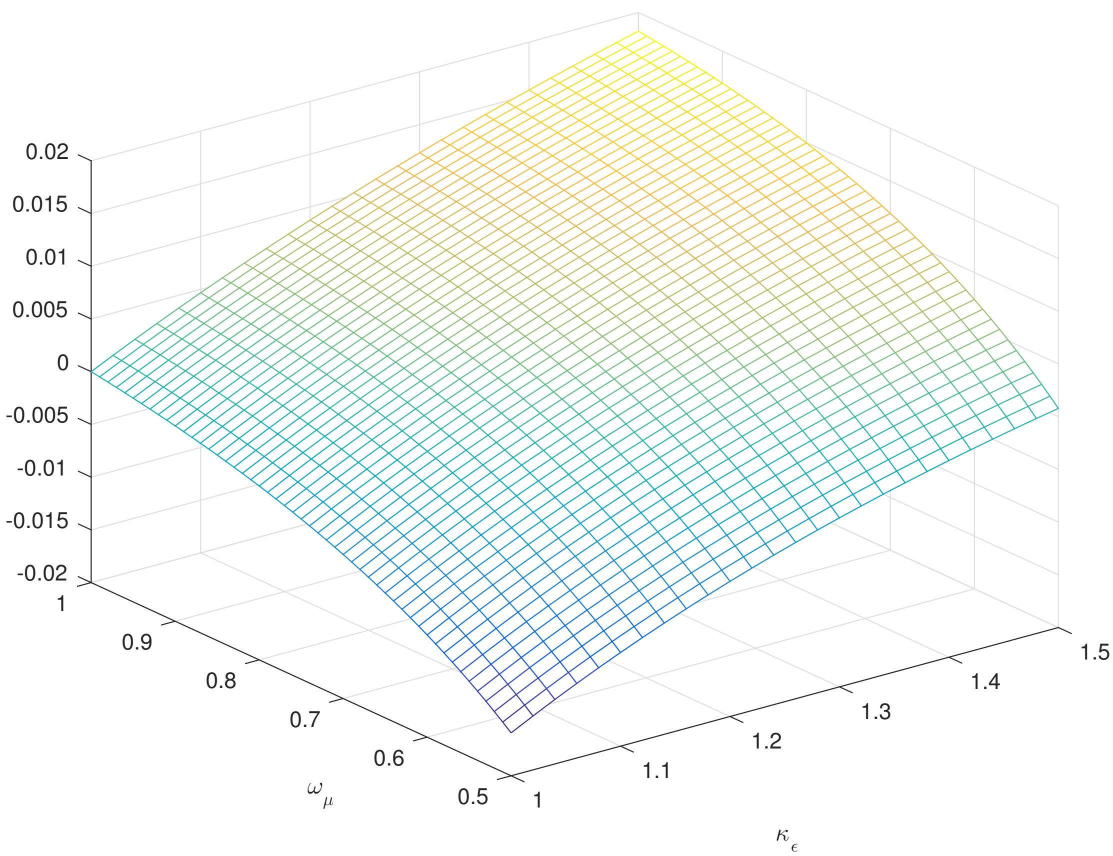


Figure 3: Skepticism and Momentum

This graph plots the momentum parameter $\overline{MOM}$ as a function of late-informed investors' skepticism (κ_ϵ). We assume the other parameter values $m = 0.05$, $A = 1$, $\nu_\theta = 1$, $\nu_\mu = 1$, $\nu_\epsilon = 1$, $\omega_\epsilon = 0.5$, and $\omega_\mu = 1$.

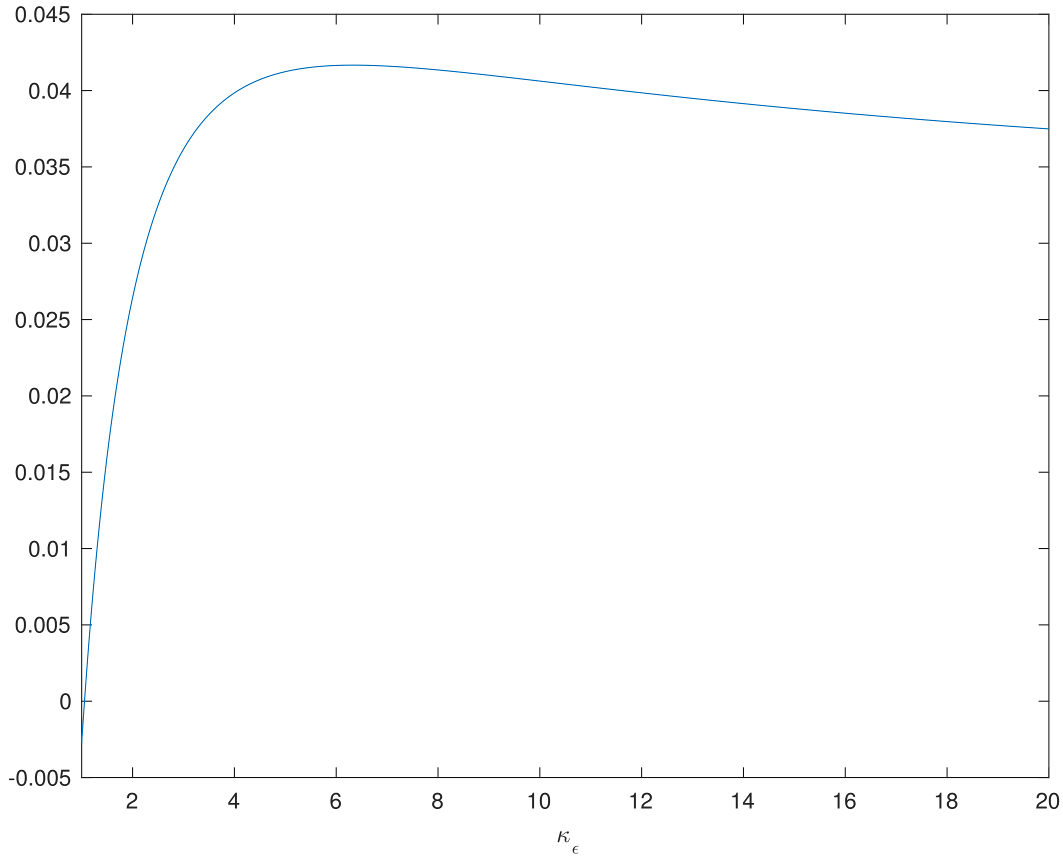


Figure 4: Rational Risk-Averse Market Makers and Momentum

This graph plots the momentum parameter $\overline{MOM}$ as a function of the risk-bearing capacity of rational and risk-averse market makers (λ/A_N). We assume the parameter values $m = 0.05$, $A = 1$, $\nu_\theta = 1$, $\nu_\mu = 1$, $\nu_\epsilon = 1$, $\omega_\epsilon = 0.9$, $\omega_\mu = 1$, and $\kappa_\epsilon = 11$.

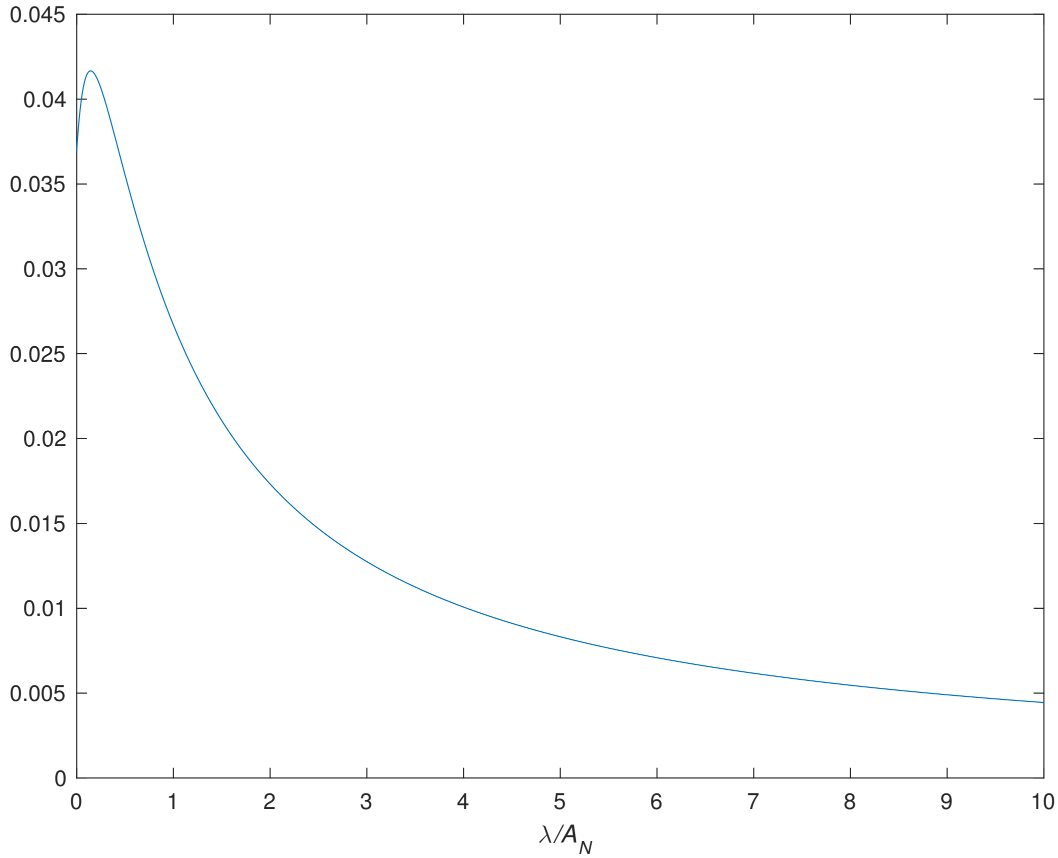


Figure 5: Liquidity and Overconfidence

This graph plots the Date-1 illiquidity measure $\alpha_\tau \delta$ as functions of the parameters representing late-informed investors' skepticism about the quality of the early Date-1 information (κ_ϵ), and overestimation of the quality of their own information (ω_μ). We assume the parameter values $m = 0.1$, $A = 1$, $\nu_\theta = 1$, $\nu_\mu = 1$, $\nu_\epsilon = 1$, $\lambda = 0.1$, $A_N = 1$, and $\nu_z = 0.01$. We let $\omega_\epsilon = \nu_\epsilon$ so that early-informed investors have rational beliefs about the quality of their information.

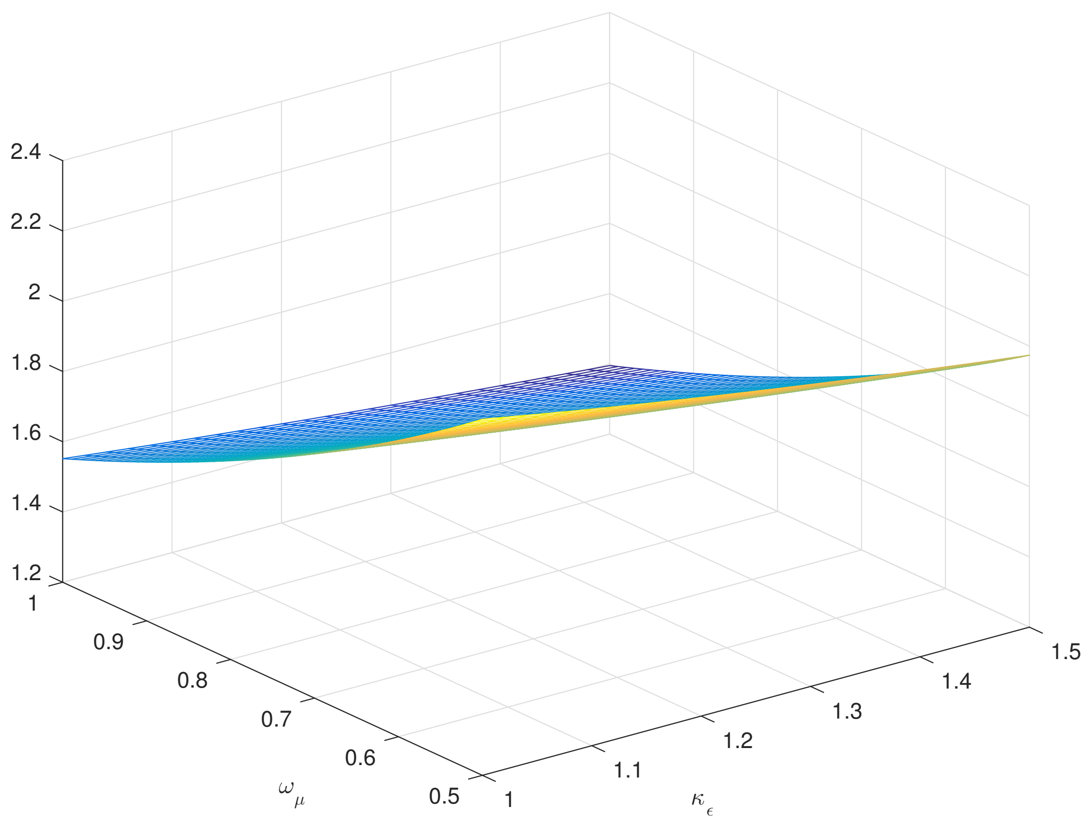


Figure 6: Short-Run Momentum and Overconfidence

This graph plots the momentum parameter $\overline{MOM}$ as functions of the parameters representing late-informed investors' skepticism about the quality of the early Date-1 information (κ_ϵ) and overestimation of the quality of their own information (ω_μ). We assume the parameter values $m = 0.1$, $A = 1$, $\nu_\theta = 1$, $\nu_\mu = 1$, $\nu_\epsilon = 1$, $\lambda = 0.1$, $A_N = 1$, and $\nu_z = 0.01$. We let $\omega_\epsilon = \nu_\epsilon$ so early-informed investors have rational beliefs about the quality of their information.

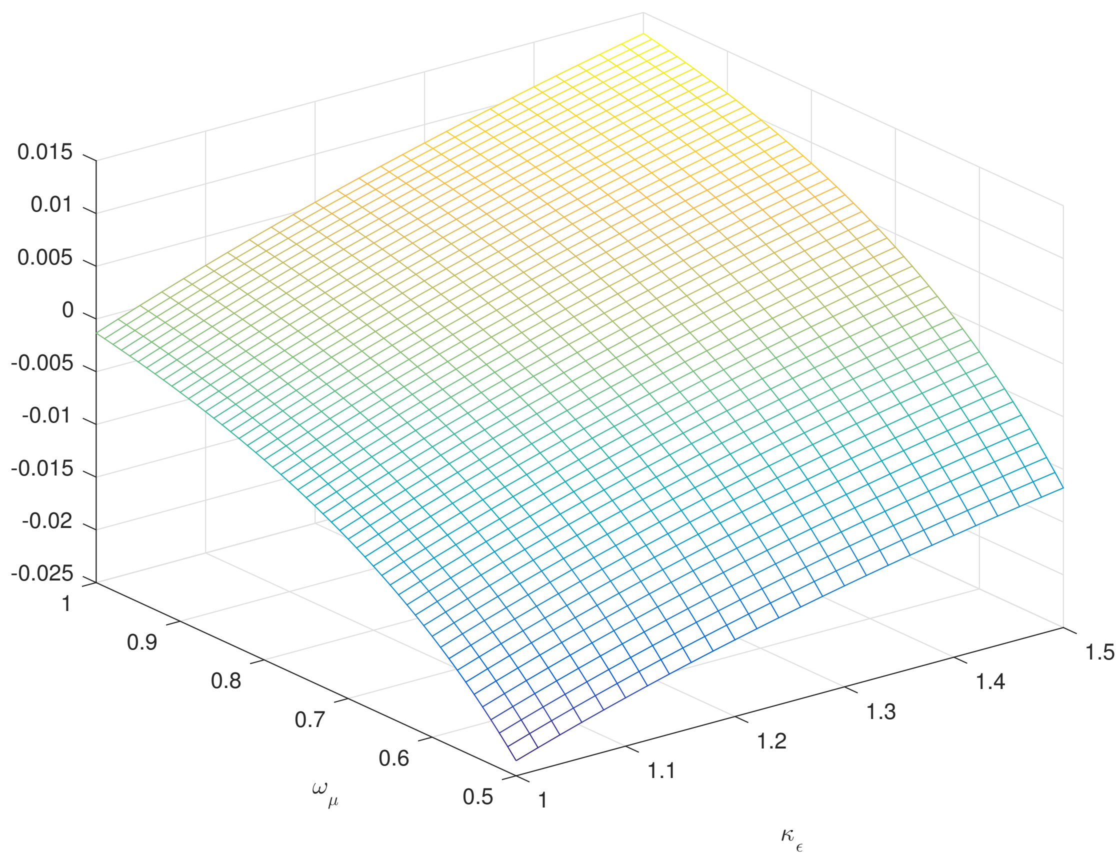


Figure 7: Liquidity, Short-Run Momentum, and Overconfidence

This graph plots the momentum parameter $\overline{MOM}$ and the Date-1 illiquidity measure, $\alpha_\tau\delta$ (denoted as $ILLIQ$), as functions of the summary measure of late-informed investors' overconfidence, $OC \equiv (\kappa_\epsilon - \nu_\epsilon) + (\nu_\mu - \omega_\mu)$. We assume that $\nu_\mu - \omega_\mu = 0.2(\kappa_\epsilon - \nu_\epsilon)$. To generate the values of OC on the horizontal axis, we increase κ_ϵ starting from unity, while simultaneously changing ω_μ. The other parameter values are $m = 0.1$, $A = 1$, $\nu_\theta = 1$, $\nu_\mu = 1$, $\nu_\epsilon = 1$, $\lambda = 0.1$, $A_N = 1$, and $\nu_z = 0.01$. We let $\omega_\epsilon = \nu_\epsilon$ so early-informed investors have rational beliefs about the quality of their information.

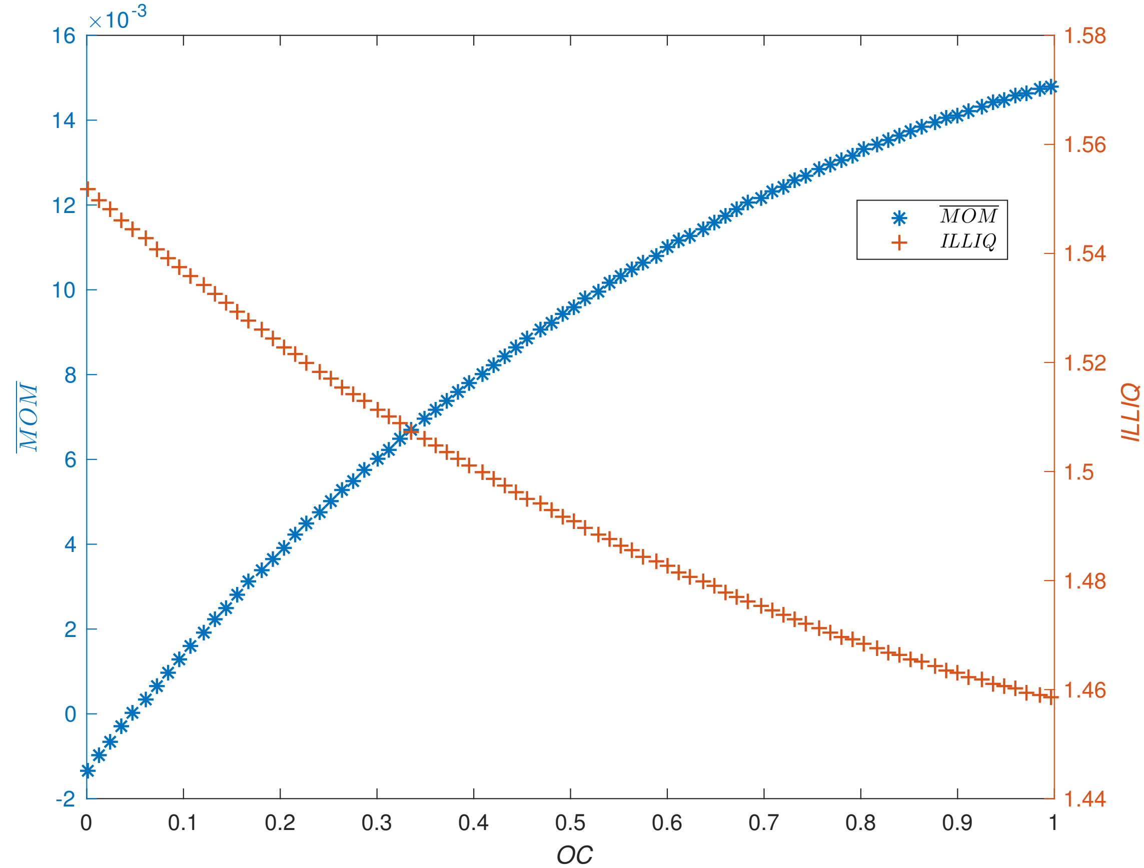


Figure 8: Skepticism About Fundamentals and Long-Term Reversals

This graph plots the long-run reversal parameter $\mathrm{Cov}(P_1 - P_0, \theta - P_2)$ as functions of the noise variances in the early Date-1 signal (ν_ϵ) and the late Date-2 signal (ν_μ). We assume the parameter values $m = 0.1$, $\lambda = 0.1$, $A = 1$, $A_N = 1$, $\nu_\theta = 1$, $\nu_z = 0.01$, $\Delta = 0.5$, $\omega_\epsilon = \nu_\epsilon$, $\omega_\mu = \nu_\mu$, and $\kappa_\epsilon = \nu_\epsilon$. Late-informed investors are skeptical in that they believe the early Date-1 signal contains only limited fundamental information (i.e., $s = \theta_1 + \mu_1 + \epsilon$ and $\gamma = \theta_1 + \mu_1$).

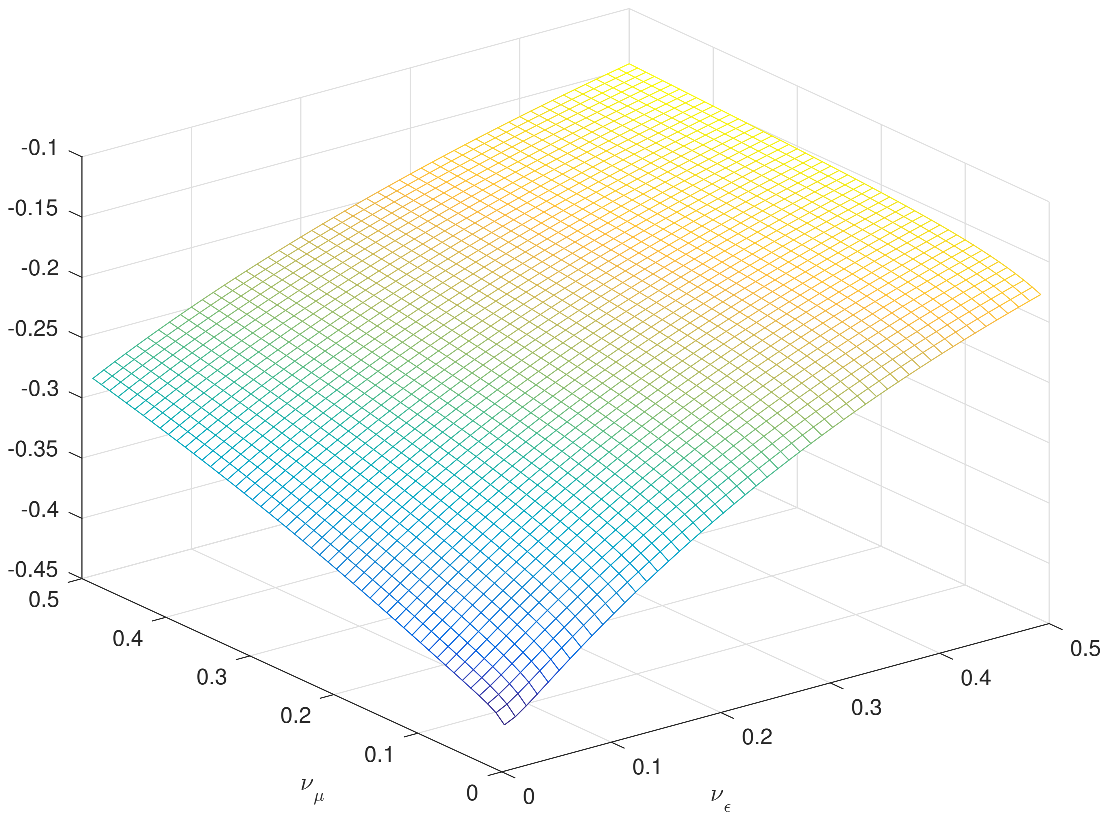


Figure 9: Skepticism About Fundamentals and Short-Term Momentum

This graph plots the short-term momentum parameter $\overline{MOM}$ as functions of the noise variances in the early Date-1 signal (ν_ϵ) and the late Date-2 signal (ν_μ). We assume the parameter values $m = 0.1$, $\lambda = 0.1$, $A = 1$, $A_N = 1$, $\nu_\theta = 1$, $\nu_z = 0.01$, $\Delta = 0.5$, $\omega_\epsilon = \nu_\epsilon$, $\omega_\mu = \nu_\mu$, and $\kappa_\epsilon = \nu_\epsilon$. Late-informed investors are skeptical in that they believe the early Date-1 signal contains only limited fundamental information (i.e., $s = \theta_1 + \mu_1 + \epsilon$ and $\gamma = \theta_1 + \mu_1$).

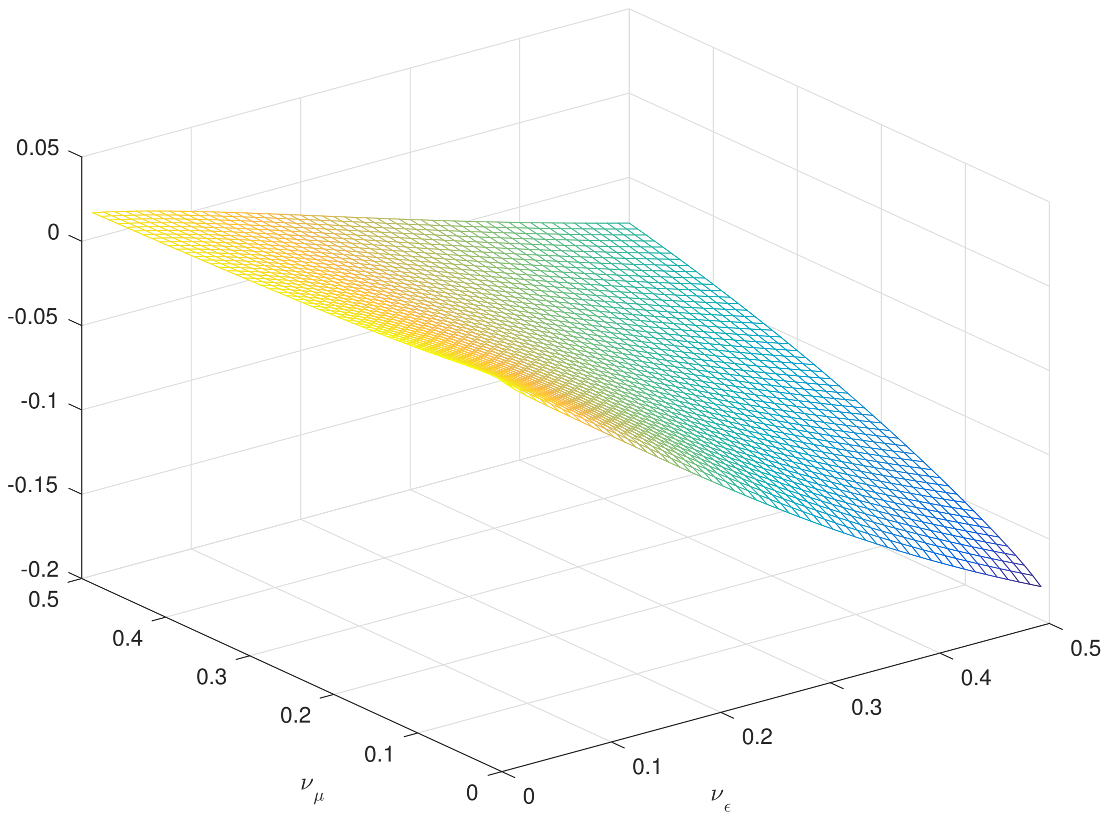
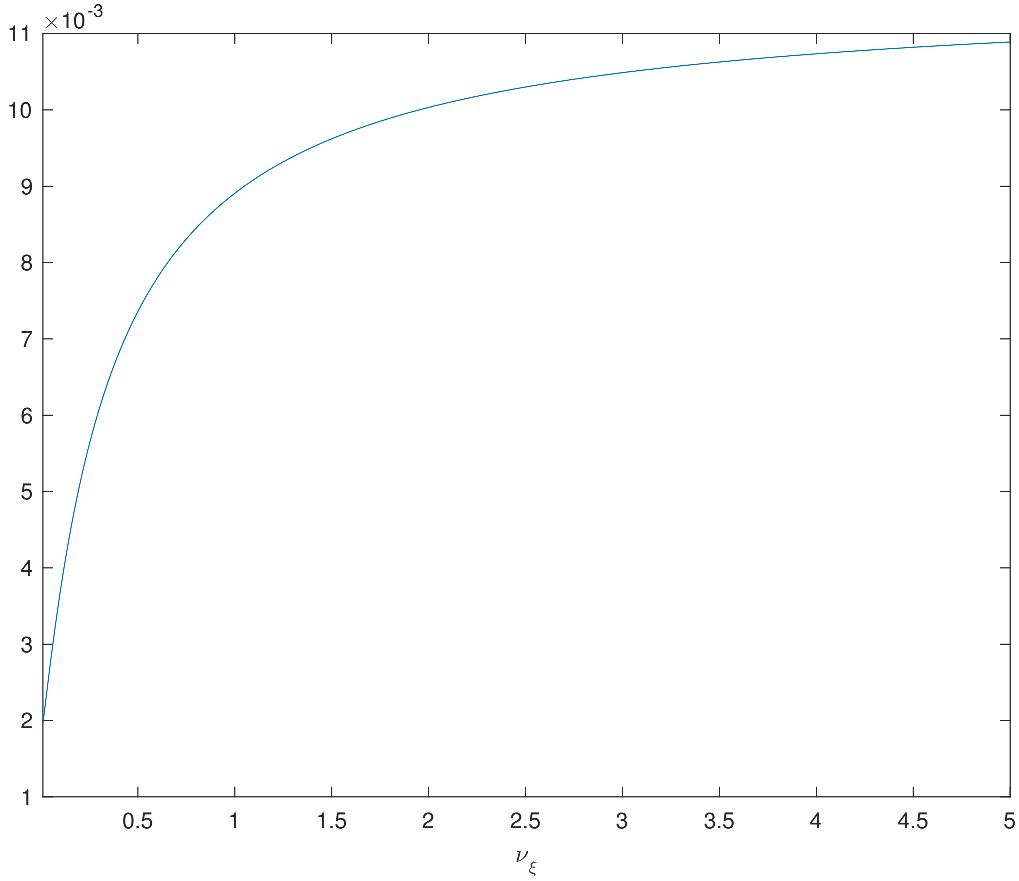


Figure 10: Public Information and Short-Term Momentum In the Presence of Skepticism About Fundamentals

This graph plots the short-term momentum parameter $\overline{MOM}$ as a function of ν_ξ, the variance of the noise in Date-1 public information signal $\gamma_2 + \xi$. We assume the parameter values $m = 0.1$, $\lambda = 0.1$, $A = 1$, $A_N = 1$, $\nu_\theta = 1$, $\nu_\mu = 0.5$, $\nu_\epsilon = 0.01$, $\nu_z = 0.01$, $\Delta = 0.5$, $\omega_\epsilon = \nu_\epsilon$, $\omega_\mu = \nu_\mu$, and $\kappa_\epsilon = \nu_\epsilon$. Late-informed investors are skeptical in that they believe the early Date-1 signal contains only limited fundamental information (i.e., $s = \theta_1 + \mu_1 + \epsilon$ and $\gamma = \theta_1 + \mu_1$).



Internet Appendix for

A Parsmonious Model of Momentum and Reversals in Financial Markets

In this internet appendix, we extend the setting of the main paper as follows. Like in Section 5 of the main paper, both early- and late-informed investors overestimate the quality of their own information (indicated by the parameters ω_ϵ and ω_μ), but are skeptical about the quality of others' information (indicated by κ_ϵ). Late-informed investors are also skeptical about the fundamental information structure; that is, from their stand point, $\gamma = \theta_1 + \mu_1$, and $s = \theta_1 + \mu_1 + \epsilon$. The scale of this skepticism is indicated by the parameter Δ. A public information signal, $t = \theta + \xi$, arrives either at Date 1 or Date 2. Early- and late-informed investors are skeptical about the quality of t about θ in that they believe that the variance of ξ equals $\kappa\nu_\xi$ where $\kappa \geq 1$. There is still a mass λ of rational uninformed investors who serve as market makers. We further assume that both the Date-1 and Date-2 supplies of the risky security are random and are indicated by the normal random variables, z_1 and z_2, which are independent and have the same variance ν_z. We show in Section A.3 of this appendix that if the public information signal arrives at Date 2, the equilibrium prices take the following forms:

$$P_1 = B\tau_1 \tag{A.1}$$

and

$$P_2 = C(\tau_2 + a\gamma_2 + bs) + D\tau_1 + Gt, \tag{A.2}$$

where $\tau_1 \equiv s - \delta_1 z_1$ and $\tau_2 \equiv \gamma - \delta_2 z_2$, and B, δ_1, C, δ_2, a, b, D, and G are constants to be determined, with $P_0 = 0$. If the public information signal arrives at Date 1, we can use a similar analysis to show that equilibrium prices take similar forms except that P_1 also depends linearly on the public signal t.

A.1 Momentum, Reversals, and Public Information

Note that θ, τ_2, γ_2, t, s, and τ_1 follow a multi-variate normal distribution with mean zero and variance-covariance matrix

$$\begin{pmatrix} \nu_\theta & \nu_\theta & \Delta\nu_\theta & \nu_\theta & \nu_\theta & \nu_\theta \\ \nu_\theta & \nu_\gamma + \delta_2^2\nu_z & \Delta\nu_\gamma & \nu_\theta & \nu_\gamma & \nu_\gamma \\ \Delta\nu_\theta & \Delta\nu_\gamma & \Delta\nu_\gamma & \Delta\nu_\theta & \Delta\nu_\gamma & \Delta\nu_\gamma \\ \nu_\theta & \nu_\theta & \Delta\nu_\theta & \nu_\theta + \nu_\xi & \nu_\theta & \nu_\theta \\ \nu_\theta & \nu_\gamma & \Delta\nu_\gamma & \nu_\theta & \nu_s & \nu_s \\ \nu_\theta & \nu_\gamma & \Delta\nu_\gamma & \nu_\theta & \nu_s & \nu_s + \delta_1^2\nu_z \end{pmatrix}.$$

We can use Eqs. (A.1) and (A.2) to compute the long-run reversal parameter,

$$\mathrm{Cov}(P_1 - P_0, \theta - P_2),$$

and the short-run momentum parameter,

$$\overline{MOM} = \left[\mathrm{Cov}(P_1 - P_0, P_2 - P_1) + \mathrm{Cov}(P_2 - P_1, \theta - P_2)\right]/2.$$

Intuitively, if the public signal is not very precise, then stock price patterns we demonstrated in Section 5 still prevail. We verify this intuition using the following example. Suppose that the public signal arrives at Date 2. Figures A.1 and A.2 plot the long-run reversal parameter, $\mathrm{Cov}(P_1 - P_0, \theta - P_2)$, and the short-term momentum parameter, $\overline{MOM}$ as functions of the signal noise variances in the Date-1 signal (ν_ϵ) and Date-2 signal (ν_μ). The other parameter values are in the captions of the figures. Particularly, we assume that the public signal is sufficiently noisy (i.e., $\nu_\xi = 10$), and late- and early-informed investors are sufficiently skeptical about the quality of the public signal (i.e., $\kappa = 2$). This implies that investors put little weight on the public signal when choosing their portfolio. Consequently, the stock price patterns here are almost identical to Figures 8 and 9 when there is no public signal.

If the public signal is very precise, the proposition below replicates Proposition 5 of the main paper.

Proposition A.1 *(i) Suppose that the public signal arrives at Date 2. If the signal is very precise, then long-run reversals go to zero but short-run momentum still prevails.*

(ii) If the public signal arrives at Date 1, then, as the precision of the signal increases, both long-run reversals and short-run momentum go to zero.

We can also use Eqs. (A.1) and (A.2) to compute the post-public-announcement drift parameter

$$\mathrm{Cov}(\theta - P_2, t).$$

Because we no longer have the closed-form solution for P_2 and this drift parameter, we use numerical analysis. Continue to suppose that the public signal arrives at Date 2 (the analysis for the case when the public signal arrives at Date 1 is analogous). Figure A.3 plots $\mathrm{Cov}(\theta - P_2, t)$ as functions of the skepticism about public information quality (κ), and the late informed's skepticism about fundamentals (Δ). Other parameters are presented in the caption of the figure. There are three notable observations. First, $\mathrm{Cov}(\theta - P_2, t)$ is positive if Δ is sufficiently small. This is consistent with Proposition 6 in the main paper in which $\Delta = 0$. Second, $\mathrm{Cov}(\theta - P_2, t)$ increases in κ. Intuitively, as the informed traders are more skeptical about the quality of the public information (high κ), the price P_2 underreacts more to the public information, which is then followed by a greater drift. Third, as Δ increases, $\mathrm{Cov}(\theta - P_2, t)$ decreases and can become negative. The reason is that as the late informed becomes more skeptical about the fundamental of the information s and γ (high Δ), at Date 2 they underreact more to the information γ. This implies that they tend to overweight and overreact to the public information t. Therefore, the drift, $\mathrm{Cov}(\theta - P_2, t)$, is mitigated and eventually becomes negative.

A.2 Liquidity and the Late-Informed

In this internet appendix, the supplies of the risky security at both Dates 1 and 2, z_1 and z_2, are random. From Eqs. (A.1) and (A.2), z_1 affects the prices at both dates, P_1 and P_2, and z_2 affects only P_2. Thus, we compute illiquidity as the total price impact due to z_1 and z_2:

$$ILLIQ \equiv -\left[\frac{d(P_1 + P_2)}{dz_1} + \frac{dP_2}{dz_2}\right] = (B + D)\delta_1 + C\delta_2.$$

Figure A.4 plots *ILLIQ* as functions of the parameters representing late-informed investors' skepticism about the quality of the early Date-1 information (κ_ϵ), and overestimation of the quality of their own information (ω_μ). Other parameters are presented in the caption of the figure. Consistent with Figure 5 of the main paper, as late-informed become more skeptical (i.e., as κ_ϵ increases), liquidity is higher (i.e., $ILLIQ$ decreases). As late-informed investors overestimate the quality of their late information γ to a lesser extent (i.e., as ω_μ increases), liquidity is higher (i.e., $ILLIQ$ decreases).

Figure A.5 verifies the main result of Figure 6 of the main paper: the momentum parameter, $\overline{MOM}$, increases as late informed investors underestimate the quality of early information s to a greater extent (i.e., as κ_ϵ increases), and as they overestimate the quality of their late information γ to a lesser extent (i.e., as ω_μ increases).

Figure A.6 plots the momentum parameter, $\overline{MOM}$, and the illiquidity measure, $ILLIQ$, as functions of a summary measure of late-informed investors' overconfidence, $OC \equiv (\kappa_\epsilon - \nu_\epsilon) + (\nu_\mu - \omega_\mu)$. Like in Figure 7 of the main paper, we assume that $\nu_\mu - \omega_\mu = 0.2(\kappa_\epsilon - \nu_\epsilon)$ and vary κ_ϵ while simultaneously changing ω_μ. Other parameter values are presented in the caption of the figure. Consistent with Figure 7, increasing overconfidence promotes both momentum and liquidity.

Finally, we verify in the proposition below that like in Proposition 7 of the main paper, noise trades can reverse the short-run momentum effect.

Proposition A.2 *If noise trades are sufficiently volatile (i.e., $\nu_z \to \infty$), then we obtain short-run reversals, i.e., $\overline{MOM} \to -\infty$.*

A.3 Proofs

Proof of Eqs. (A.1) and (A.2): Conjecture that the equilibrium prices take the forms in Eqs. (A.1) and (A.2). In what follows, we verify these conjectures and solve for the parameters. We use backward induction.

Date 2: Fix δ_1, we solve for the Date-2 price. At Date 2, τ_1 (from P_1) and t are public knowledge. The i'th early-informed investor learns τ_2 from P_2, and conditions the trade on the information set, $I_{\eta 2} \equiv \{\tau_2, \gamma_2, t, s\}$ (note that s dominates τ_1 as a signal of θ). The investor believes that θ and $I_{\eta 2} \equiv \{\tau_2, \gamma_2, t, s\}$ follow a multi-variate normal distribution with mean zero and variance-covariance matrix

$$\begin{pmatrix} \nu_\theta & \nu_\theta & \Delta\nu_\theta & \nu_\theta & \nu_\theta \\ \nu_\theta & \nu_\gamma + \delta_2^2\nu_z & \Delta\nu_\gamma & \nu_\theta & \nu_\gamma \\ \Delta\nu_\theta & \Delta\nu_\gamma & \Delta\nu_\gamma & \Delta\nu_\theta & \Delta\nu_\gamma \\ \nu_\theta & \nu_\theta & \Delta\nu_\theta & \nu_\theta + \kappa\nu_\xi & \nu_\theta \\ \nu_\theta & \nu_\gamma & \Delta\nu_\gamma & \nu_\theta & \omega_s \end{pmatrix}.$$

Write the wealth at Date 3 as $W_{i3} = W_{i2} + X_{i2}(\theta - P_2)$. The investor needs to choose X_{i2} to maximize

$$\begin{aligned} & E_\eta\left[U_\eta(W_{i3})|I_{\eta 2}\right] \\ = \; & E_\eta\left[-\exp\left[-AW_{i2} - AX_{i2}(\theta - P_2)\right]|I_{\eta 2}\right] \\ = \; & -\exp\left[-AW_{i2} - AX_{i2}(E_\eta(\theta|I_{\eta 2}) - P_2) + 0.5A^2X_{i2}^2\mathrm{Var}_\eta(\theta|I_{\eta 2})\right]. \end{aligned} \tag{A.3}$$

The f.o.c. w.r.t. X_{i2} implies that the investor's demand can be expressed as:

$$X_{\eta 2} = \frac{E_\eta(\theta|I_{\eta 2}) - P_2}{A\mathrm{Var}_\eta(\theta|I_{\eta 2})} = \frac{c_\eta\tau_2 + a_\eta\gamma_2 + g_\eta t + b_\eta s - P_2}{A\nu_{\eta 2}}, \tag{A.4}$$

where c_η, a_η, g_η, b_η, and $\nu_{\eta 2}$ are constants.

At Date 2, a late-informed investor learns γ which dominates s as a signal of θ, and conditions the trade on the information set, $I_{\ell 2} \equiv \{\gamma, \gamma_2, t\}$. The investor believes that θ and $I_{\ell 2} \equiv \{\gamma, \gamma_2, t\}$ follow a multi-variate normal distribution with mean zero and variance-covariance matrix (note that from the investor's standpoint $\gamma = \theta_1 + \mu_1$, and $s = \theta_1 + \mu_1 + \epsilon$)

$$\begin{pmatrix} \nu_\theta & \Delta_1\nu_\theta & \Delta\nu_\theta & \nu_\theta \\ \Delta_1\nu_\theta & \Delta_1\omega_\gamma & 0 & \Delta_1\nu_\theta \\ \Delta\nu_\theta & 0 & \Delta\omega_\gamma & \Delta\nu_\theta \\ \nu_\theta & \Delta_1\nu_\theta & \Delta\nu_\theta & \nu_\theta + \kappa\nu_\xi \end{pmatrix},$$

with $\Delta_1 \equiv 1-\Delta$. From a similar analysis as above, the optimal demand can be expressed as:

$$X_{\ell 2} = \frac{E_\ell(\theta|I_{\ell 2}) - P_2}{A\text{Var}_\ell(\theta|I_{\ell 2})} = \frac{c_\ell \gamma + a_\ell \gamma_2 + g_\ell t - P_2}{A\nu_{\ell 2}},$$

where c_ℓ, a_ℓ, g_ℓ, and $\nu_{\ell 2}$ are constants.

At Date 2, an uninformed rational investor learns τ_1 from P_1 and $\tau_2 + a\gamma_2 + bs$ from P_2, and conditions the trade on the information set, $I_{N2} \equiv \{\tau_2 + a\gamma_2 + bs, t, \tau_1\}$. The investor believes that θ and $I_{N2} \equiv \{\tau_2 + a\gamma_2 + bs, t, \tau_1\}$ follows a multi-variate normal distribution with mean zero and variance-covariance matrix

$$\begin{pmatrix} \nu_\theta & (1+a\Delta+b)\nu_\theta & \nu_\theta & \nu_\theta \\ (1+a\Delta+b)\nu_\theta & \begin{array}{l}(1+b)^2\nu_\gamma + a^2\Delta\nu_\gamma \\ \quad +2(1+b)a\Delta\nu_\gamma + \delta_2^2\nu_z + b^2\nu_\epsilon\end{array} & (1+a\Delta+b)\nu_\theta & \nu_\gamma + a\Delta\nu_\gamma + b\nu_s \\ \nu_\theta & (1+a\Delta+b)\nu_\theta & \nu_\theta + \nu_\xi & \nu_\theta \\ \nu_\theta & \nu_\gamma + a\Delta\nu_\gamma + b\nu_s & \nu_\theta & \nu_s + \delta_1^2\nu_z \end{pmatrix}.$$

The investor's optimal demand can be expressed as:

$$X_{N2} = \frac{E(\theta|I_{N2}) - P_2}{A_N\text{Var}(\theta|I_{N2})} = \frac{c_N(\tau_2 + a\gamma_2 + bs) + g_N t + d_N\tau_1 - P_2}{A_N\nu_{N2}}$$

where c_N, g_N, d_N, and ν_{N2} are constants.

The market clearing condition, $z_2 = mX_{\eta 2} + (1-m)X_{\ell 2} + \lambda X_{N2}$, implies that P_2 takes the form in Eq. (A.2). Particularly, the parameters

$$\begin{aligned}
\delta_2 &= \frac{A\nu_{\ell 2}}{(1-m)c_\ell}, \\
a &= \frac{m\nu_{\eta 2}^{-1}a_\eta + (1-m)\nu_{\ell 2}^{-1}a_\ell}{m\nu_{\eta 2}^{-1}c_\eta + (1-m)\nu_{\ell 2}^{-1}c_\ell}, \\
b &= \frac{m\nu_{\eta 2}^{-1}b_\eta}{m\nu_{\eta 2}^{-1}c_\eta + (1-m)\nu_{\ell 2}^{-1}c_\ell}, \\
C &= \frac{m\nu_{\eta 2}^{-1}c_\eta + (1-m)\nu_{\ell 2}^{-1}c_\ell + (\lambda A_N^{-1}A)\nu_{N2}^{-1}c_N}{m\nu_{\eta 2}^{-1} + (1-m)\nu_{\ell 2}^{-1} + (\lambda A_N^{-1}A)\nu_{N2}^{-1}}, \\
D &= \frac{(\lambda A_N^{-1}A)\nu_{N2}^{-1}d_N}{m\nu_{\eta 2}^{-1} + (1-m)\nu_{\ell 2}^{-1} + (\lambda A_N^{-1}A)\nu_{N2}^{-1}}, \text{ and} \\
G &= \frac{m\nu_{\eta 2}^{-1}g_\eta + (1-m)\nu_{\ell 2}^{-1}g_\ell + (\lambda A_N^{-1}A)\nu_{N2}^{-1}g_N}{m\nu_{\eta 2}^{-1} + (1-m)\nu_{\ell 2}^{-1} + (\lambda A_N^{-1}A)\nu_{N2}^{-1}}.
\end{aligned}$$

Date 1: Write an early-informed investor's wealth at Date 2 as $W_{i2} = W_{i1} + X_{i1}(P_2 - P_1)$. It follows from Eqs. (A.3) and (A.4) that the Date-2 expected utility can be expressed as:

$$
\begin{aligned}
& E_\eta \left[U_\eta(W_{i3}) | I_{\eta 2}\right] \\
= & -\exp\left[-AW_{i1} - AX_{i1}(P_2 - P_1) - 0.5\nu_{\eta 2}^{-1}(c_\eta \tau_2 + a_\eta \gamma_2 + g_\eta t + b_\eta s - P_2)^2\right] \\
= & -\exp\left[-AW_{i1} - AX_{i1}(Cbs + D\tau_1 + F_\eta^T Y_\eta - P_1) - 0.5\nu_{\eta 2}^{-1}(R_\eta^T Y_\eta - (Cbs + D\tau_1 - b_\eta s))^2\right],
\end{aligned}
$$

where

$$
Y_\eta = \begin{pmatrix} \tau_2 \\ \gamma_2 \\ t \end{pmatrix}, \; F_\eta = \begin{pmatrix} C \\ Ca \\ G \end{pmatrix}, \; R_\eta = \begin{pmatrix} c_\eta \\ a_\eta \\ g_\eta \end{pmatrix} - F_\eta.
$$

At Date 1, the investor learns s, and believes that Y_η and s follow a multi-variate normal distribution with mean zero and variance-covariance matrix

$$
\begin{pmatrix}
\nu_\gamma + \delta_2^2 \nu_z & \Delta\nu_\gamma & \nu_\theta & \nu_\gamma \\
\Delta\nu_\gamma & \Delta\nu_\gamma & \Delta\nu_\theta & \Delta\nu_\gamma \\
\nu_\theta & \Delta\nu_\theta & \nu_\theta + \kappa\nu_\xi & \nu_\theta \\
\nu_\gamma & \Delta\nu_\gamma & \nu_\theta & \omega_s
\end{pmatrix},
$$

and thus, $Y_\eta | s \sim N(\iota_\eta s, \nu_{\eta 1})$, where ι_η and $\nu_{\eta 1}$ are constants. It follows that at Date 1, the investor needs to choose X_{i1} to maximize (here and in what follows, we use the fact in Footnote 27 in the main paper to take the expectation)

$$
\begin{aligned}
& E_\eta \left[U_\eta(W_{i3}) | s\right] = E_\eta \left[E_\eta \left[U_\eta(W_{i3}) | I_{\eta 2}\right] | s\right] \\
\propto & -\exp\Big[-AW_{i1} - AX_{i1}(Cbs + D\tau_1 - P_1) \\
& +0.5 \left(AX_{i1}F_\eta - \nu_{\eta 2}^{-1} R_\eta (Cbs + D\tau_1 - b_\eta s) - \nu_{\eta 1}^{-1}\iota_\eta s\right)^T \\
& \left(\nu_{\eta 2}^{-1} R_\eta R_\eta^T + \nu_{\eta 1}^{-1}\right)^{-1} \left(AX_{i1}F_\eta - \nu_{\eta 2}^{-1} R_\eta (Cbs + D\tau_1 - b_\eta s) - \nu_{\eta 1}^{-1}\iota_\eta s\right)\Big].
\end{aligned}
$$

The f.o.c. w.r.t. X_{i1} implies that the investor's optimal demand is given by

$$
\begin{aligned}
X_{\eta 1} = & \left[AF_\eta^T \left(\nu_{\eta 2}^{-1} R_\eta R_\eta^T + \nu_{\eta 1}^{-1}\right)^{-1} F_\eta\right]^{-1} \Big[Cbs + D\tau_1 - P_1 \\
& + F_\eta^T \left(\nu_{\eta 2}^{-1} R_\eta R_\eta^T + \nu_{\eta 1}^{-1}\right)^{-1} \left(\nu_{\eta 2}^{-1} R_\eta (Cbs + D\tau_1 - b_\eta s) + \nu_{\eta 1}^{-1}\iota_\eta s\right)\Big] \\
= & \pi_{\eta s} s + \pi_{\eta \tau}\tau_1 - \pi_{\eta P} P_1,
\end{aligned}
$$

where $\pi_{\eta s}$, $\pi_{\eta \tau}$ and $\pi_{\eta P}$ are constant parameters defined accordingly.

We can use a similar analysis as above to show that a late informed trader's Date-2 expected utility can be expressed as:

$$\begin{aligned} E_\ell\left[U_\ell(W_{i3})|I_{\ell 2}\right] &= -\exp\left[-AW_{i1} - AX_{i1}(P_2 - P_1) - 0.5\nu_{\ell 2}^{-1}(c_\ell\gamma + a_\ell\gamma_2 + g_\ell t - P_2)^2\right] \\ &= -\exp\left[-AW_{i1} - AX_{i1}(F_\ell^T Y_\ell + D\tau_1 - P_1) - 0.5\nu_{\ell 2}^{-1}(R_\ell^T Y_\ell - D\tau_1)^2\right], \end{aligned}$$

where

$$Y_\ell = \begin{pmatrix} \gamma \\ \gamma_2 \\ t \\ \tau_2 \\ s \end{pmatrix},\ F_\ell = \begin{pmatrix} 0 \\ Ca \\ G \\ C \\ Cb \end{pmatrix},\ R_\ell = \begin{pmatrix} c_\ell \\ a_\ell \\ g_\ell \\ 0 \\ 0 \end{pmatrix} - F_\ell.$$

At Date 1, the investor learns τ_1 from P_1, and believes that Y_ℓ and τ_1 follow a multi-variate normal distribution with mean zeros and variance-covariance matrix (note that from the investor's standpoint $\gamma = \theta_1 + \mu_1$, and $s = \theta_1 + \mu_1 + \epsilon$)

$$\begin{pmatrix} \Delta_1\omega_\gamma & 0 & \Delta_1\nu_\theta & \Delta_1\omega_\gamma & \Delta_1\omega_\gamma & \Delta_1\omega_\gamma \\ 0 & \Delta\omega_\gamma & \Delta\nu_\theta & 0 & 0 & 0 \\ \Delta_1\nu_\theta & \Delta\nu_\theta & \nu_\theta + \kappa\nu_\xi & \Delta_1\nu_\theta & \Delta_1\nu_\theta & \Delta_1\nu_\theta \\ \Delta_1\omega_\gamma & 0 & \Delta_1\nu_\theta & \Delta_1\omega_\gamma + \delta_2^2\nu_z & \Delta_1\omega_\gamma & \Delta_1\omega_\gamma \\ \Delta_1\omega_\gamma & 0 & \Delta_1\nu_\theta & \Delta_1\omega_\gamma & \Delta_1\omega_\gamma + \kappa_\epsilon & \Delta_1\omega_\gamma + \kappa_\epsilon \\ \Delta_1\omega_\gamma & 0 & \Delta_1\nu_\theta & \Delta_1\omega_\gamma & \Delta_1\omega_\gamma + \kappa_\epsilon & \Delta_1\omega_\gamma + \kappa_\epsilon + \delta_1^2\nu_z \end{pmatrix},$$

and thus, $Y_\ell|\tau_1 \sim N(\iota_\ell\tau_1, \nu_{\ell 1})$, where ι_ℓ and $\nu_{\ell 1}$ are constants. It follows that at Date 1, the investor needs to choose X_{i1} to maximize

$$\begin{aligned} & E_\ell\left[U_\ell(W_{i3})|\tau_1\right] = E_\ell\left[E_\ell\left[U_\ell(W_{i3})|I_{\ell 2}\right]|\tau_1\right] \\ \propto\ & -\exp\Big[-AW_{i1} - AX_{i1}(D\tau_1 - P_1) + 0.5(AX_{i1}F_\ell - \nu_{\ell 2}^{-1}R_\ell D\tau_1 - \nu_{\ell 1}^{-1}\iota_\ell\tau_1)^T \\ & \left(\nu_{\ell 2}^{-1}R_\ell R_\ell^T + \nu_{\ell 1}^{-1}\right)^{-1}\left(AX_{i1}F_\ell - \nu_{\ell 2}^{-1}R_\ell D\tau_1 - \nu_{\ell 1}^{-1}\iota_\ell\tau_1\right)\Big]. \end{aligned}$$

The f.o.c. w.r.t. X_{i1} implies that the investor's optimal demand is given by

$$\begin{aligned} X_{\ell 1} &= \left[AF_\ell^T\left(\nu_{\ell 2}^{-1}R_\ell R_\ell^T + \nu_{\ell 1}^{-1}\right)^{-1}F_\ell\right]^{-1} \\ &\quad \left[D\tau_1 - P_1 + F_\ell^T\left(\nu_{\ell 2}^{-1}R_\ell R_\ell^T + \nu_{\ell 1}^{-1}\right)^{-1}\left(\nu_{\ell 2}^{-1}R_\ell D\tau_1 + \nu_{\ell 1}^{-1}\iota_\ell\tau_1\right)\right] \\ &= \pi_{\ell\tau}\tau_1 - \pi_{\ell P}P_1, \end{aligned}$$

where $\pi_{\ell\tau}$ and $\pi_{\ell P}$ are constants.

We can use a similar analysis as above to show that a rational uninformed investor's Date-2 expected utility can be expressed as:

$$\begin{aligned}
& E\left[U_N(W_{i3})|I_{N2}\right] \\
= \; & -\exp\left[-A_N W_{i1} - A_N X_{i1}(P_2 - P_1) - 0.5\nu_{N2}^{-1}(c_N(\tau_2 + a\gamma_2 + bs) + g_N t + d_N\tau_1 - P_2)^2\right] \\
= \; & -\exp\left[-A_N W_{i1} - A_N X_{i1}(F_N^T Y_N + D\tau_1 - P_1) - 0.5\nu_{N2}^{-1}(R_N^T Y_N - (D - d_N)\tau_1)^2\right],
\end{aligned}$$

where

$$Y_N = \begin{pmatrix} \tau_2 \\ \gamma_2 \\ t \\ s \end{pmatrix}, \; F_N = \begin{pmatrix} C \\ Ca \\ G \\ Cb \end{pmatrix}, \; R_N = \begin{pmatrix} c_N \\ c_N a \\ g_N \\ c_N b \end{pmatrix} - F_N.$$

At Date 1, the investor learns τ_1 from P_1, and believes that Y_N and τ_1 follow a multi-variate normal distribution with mean zero and variance-covariance matrix

$$\begin{pmatrix}
\nu_\gamma + \delta_2^2\nu_z & \Delta\nu_\gamma & \nu_\theta & \nu_\gamma & \nu_\gamma \\
\Delta\nu_\gamma & \Delta\nu_\gamma & \Delta\nu_\theta & \Delta\nu_\gamma & \Delta\nu_\gamma \\
\nu_\theta & \Delta\nu_\theta & \nu_\theta + \kappa\nu_\xi & \nu_\theta & \nu_\theta \\
\nu_\gamma & \Delta\nu_\gamma & \nu_\theta & \nu_s & \nu_s \\
\nu_\gamma & \Delta\nu_\gamma & \nu_\theta & \nu_s & \nu_s + \delta_1^2\nu_z
\end{pmatrix},$$

and thus, $Y_N|\tau_1 \sim N(\iota_N\tau_1, \nu_{N1})$, where ι_N and ν_{N1} are constants. It follows that at Date 1, the investor needs to choose X_{i1} to maximize

$$\begin{aligned}
& E\left[U_N(W_{i3})|\tau_1\right] = E\left[E\left[U_N(W_{i3})|I_{N2}\right]|\tau_1\right] \\
\propto \; & -\exp\Big[-A_N W_{i1} - A_N X_{i1}(D\tau_1 - P_1) + 0.5\left(A_N X_{i1}F_N - \nu_{N2}^{-1}R_N(D - d_N)\tau_1 - \nu_{N1}^{-1}\iota_N\tau_1\right)^T \\
& \left(\nu_{N2}^{-1}R_N R_N^T + \nu_{N1}^{-1}\right)^{-1}\left(A_N X_{i1}F_N - \nu_{N2}^{-1}R_N(D - d_N)\tau_1 - \nu_{N1}^{-1}\iota_N\tau_1\right)\Big].
\end{aligned}$$

The f.o.c. w.r.t. X_{i1} implies that the investor's optimal demand is given by

$$\begin{aligned}
X_{N1} \; = \; & \left[A_N F_N^T\left(\nu_{N2}^{-1}R_N R_N^T + \nu_{N1}^{-1}\right)^{-1}F_N\right]^{-1} \\
& \left[D\tau_1 - P_1 + F_N^T\left(\nu_{N2}^{-1}R_N R_N^T + \nu_{N1}^{-1}\right)^{-1}\left(\nu_{N2}^{-1}R_N(D - d_N)\tau_1 + \nu_{N1}^{-1}\iota_N\tau_1\right)\right] \\
= \; & \pi_{N\tau}\tau_1 - \pi_{NP}P_1,
\end{aligned}$$

where $\pi_{N\tau}$ and π_{NP} are constants.

The market clearing condition, $z_1 = mX_{\eta 1} + (1-m)X_{\ell 1} + \lambda X_{N1}$, implies that P_1 takes the form in Eq. (A.1). Particularly, the parameters

$$\begin{aligned} \delta_1 &= 1/(m\pi_{\eta s}), \\ B &= \frac{m(\pi_{\eta s} + \pi_{\eta \tau}) + (1-m)\pi_{\ell \tau} + \lambda \pi_{N\tau}}{m\pi_{\eta P} + (1-m)\pi_{\ell P} + \lambda \pi_{NP}}. \end{aligned} \tag{A.5}$$

The right hand side of Eq. (A.5) is also a function of δ_1; therefore, we can use Eq. (A.5) to solve for δ_1 numerically. Based on δ_1 and the above derivation, we can solve the other parameters in Eqs. (A.1) and (A.2).

<u>Date 0:</u> We can use a similar derivation as for the proof of Proposition 1 in the main paper to show that $P_0 = 0$. □

Proof of Proposition A.1: (i) Suppose that the public signal, t, arrives at Date 2. If $\nu_\xi \to 0$ and thus $t \to \theta$, then we can follow the analysis in the proof of Eqs. (A.1) and (A.2) to show that $P_2 \to \theta$. It follows that the long-run reversal parameter, $\text{Cov}(P_1 - P_0, \theta - P_2) \to 0$. Further, $\text{Cov}(P_2 - P_1, \theta - P_2) \to 0$, and thus, $\overline{MOM} \to \text{Cov}(P_1 - P_0, P_2 - P_1)/2$. It can be shown that under certain parameter values (e.g., the parameter values specified in Corollary 3 of the main paper) that the short-run momentum parameter, $\overline{MOM} > 0$.

(ii) Suppose that the public signal, t, reaches investors at Date 1. We can use a similar analysis as in the proof of Eqs. (A.1) and (A.2) to show that if $\nu_\xi \to 0$ and thus $t \to \theta$, then $P_1 = P_2 = \theta$. In this case, all returns equal zero so that momentum and reversals do not obtain. □

Proof of Proposition A.2: Suppose that the public signal arrives at Date 2. We can follow the proof of Proposition A.1 to show that in the equilibrium prices (see Eqs. (A.1) and (A.2)), $D \to 0$, and the other parameters, particularly B and C, are non-zero in general and bounded. It follows immediately that $\text{Cov}(P_1 - P_0, P_2 - P_1), \text{Cov}(P_2 - P_1, \theta - P_2) \to -\infty$; therefore, $\overline{MOM} \to -\infty$.

If the public signal arrives at Date 1, then we can use a similar analysis to prove this proposition. □

Figure A.1: Long-Term Reversals

This graph plots the long-run reversal parameter $\text{Cov}(P_1 - P_0, \theta - P_2)$ as functions of signal noise variances in the early Date-1 information (ν_ϵ) and the late Date-2 information (ν_μ). Late-informed investors are skeptical in that they believe the early Date-1 signal contains only limited fundamental information (i.e., $s = \theta_1 + \mu_1 + \epsilon$ and $\gamma = \theta_1 + \mu_1$). The public signal, $t = \theta + \xi$, arrives at Date 2. Date-1 and Date-2 supplies of the risky security, z_1 and z_2, are normal random variables. We assume the parameter values of $m = 0.1$, $\lambda = 0.1$, $A = 1$, $A_N = 1$, $\nu_\theta = 1$, $\nu_z = 0.01$, $\nu_\xi = 10$, $\kappa = 2$, $\Delta = 0.1$, $\omega_\epsilon = \kappa_\epsilon = \nu_\epsilon$, and $\omega_\mu = \nu_\mu$.

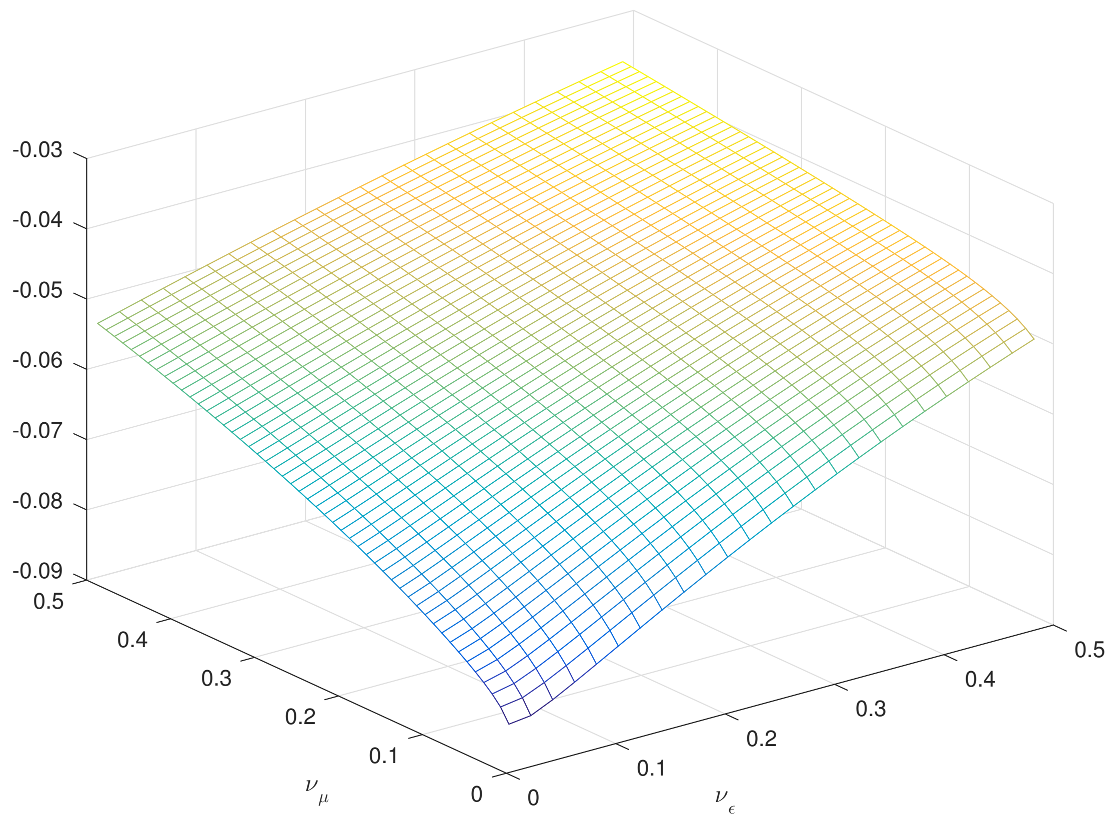


Figure A.2: Short-Term Momentum and Reversals

This graph plots the short-term momentum parameter $\overline{MOM}$ as functions of signal noise variances in the early Date-1 information (ν_ϵ) and the late Date-2 information (ν_μ). Late-informed investors are skeptical in that they believe the early Date-1 signal contains only limited fundamental information (i.e., $s = \theta_1 + \mu_1 + \epsilon$ and $\gamma = \theta_1 + \mu_1$). The public signal, $t = \theta + \xi$, arrives at Date 2. Date-1 and Date-2 supplies of the risky security, z_1 and z_2, are normal random variables. We assume the parameter values of $m = 0.1$, $\lambda = 0.1$, $A = 1$, $A_N = 1$, $\nu_\theta = 1$, $\nu_z = 0.01$, $\nu_\xi = 10$, $\kappa = 2$, $\Delta = 0.1$, $\omega_\epsilon = \kappa_\epsilon = \nu_\epsilon$, and $\omega_\mu = \nu_\mu$.

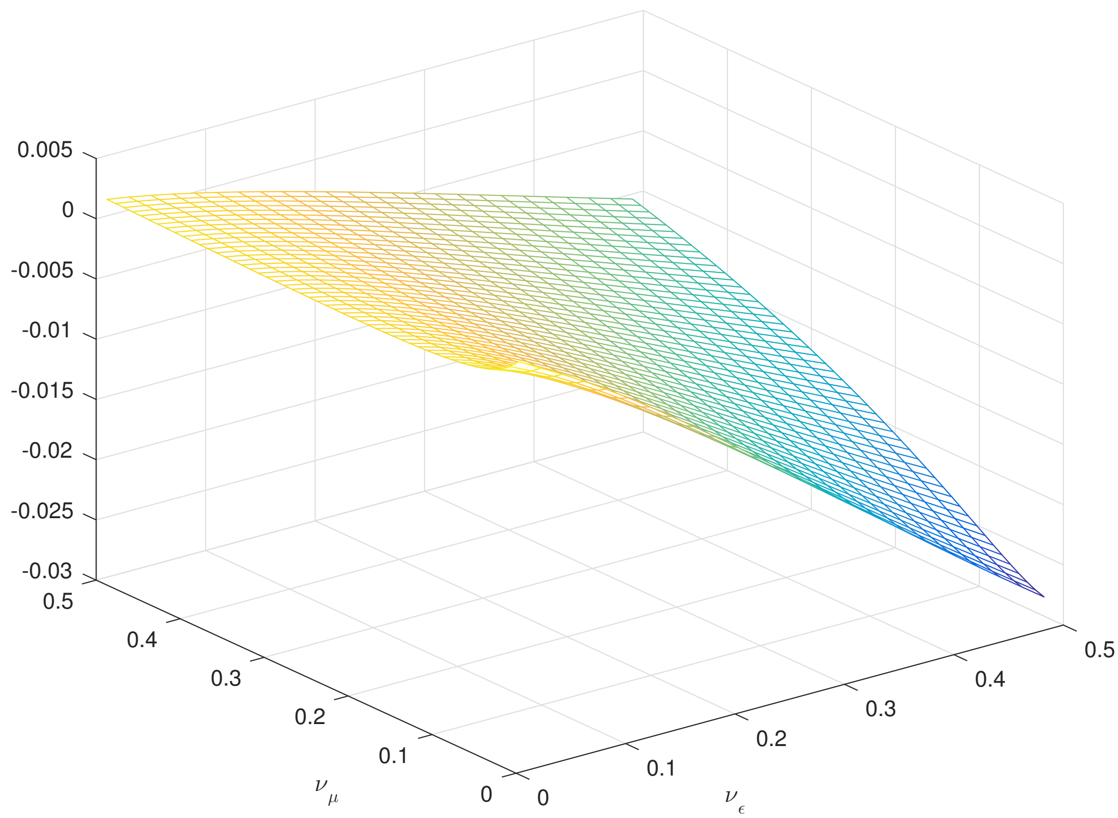


Figure A.3: Post-Public-Announcement Drift

This graph plots the post-public-announcement drift parameter $\text{Cov}(\theta - P_2, t)$ as functions of the skepticism about public information quality (κ), and the late informed's skepticism about fundamentals (Δ). Late-informed investors are skeptical in that they believe the early Date-1 signal contains only limited fundamental information (i.e., $s = \theta_1 + \mu_1 + \epsilon$ and $\gamma = \theta_1 + \mu_1$). The public signal, $t = \theta + \xi$, arrives at Date 2. Date-1 and Date-2 supplies of the risky security, z_1 and z_2, are normal random variables. We assume the parameter values of $m = 0.1$, $\lambda = 0.1$, $A = 1$, $A_N = 1$, $\nu_\theta = 1$, $\nu_z = 0.01$, $\nu_\xi = 10$, $\omega_\epsilon = \kappa_\epsilon = \nu_\epsilon = 0.1$, and $\omega_\mu = \nu_\mu = 0.5$.

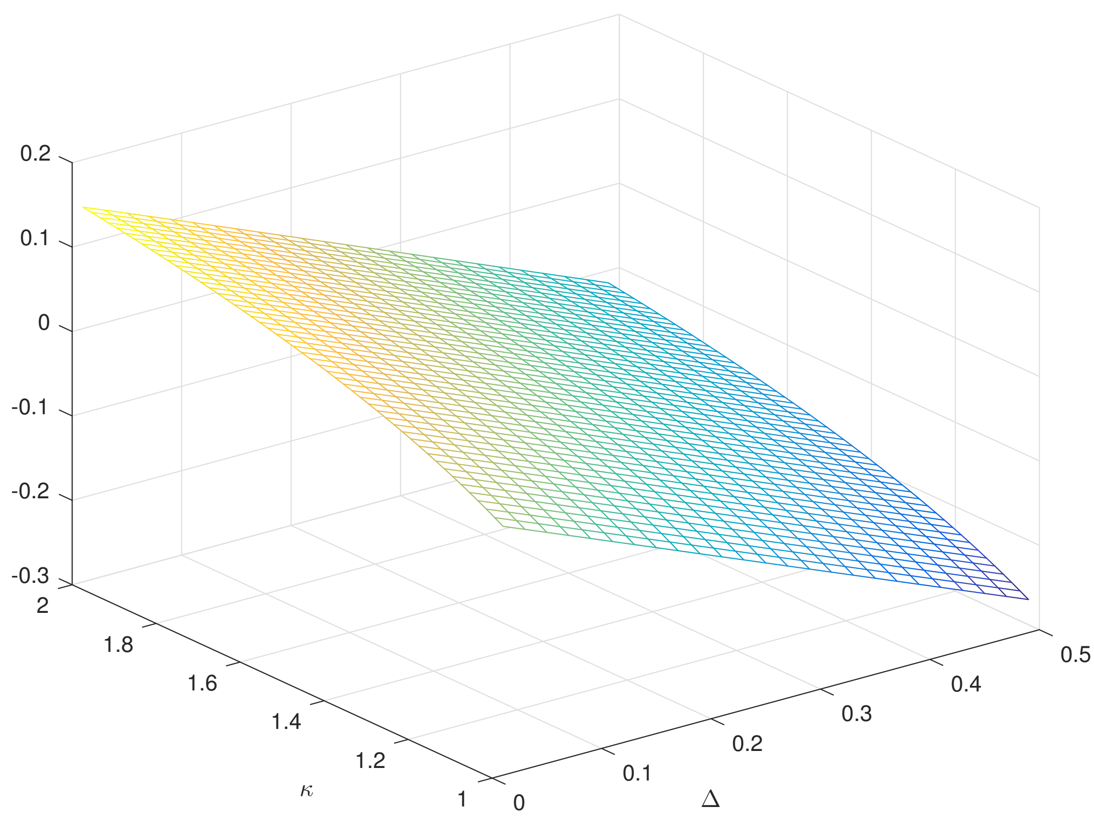


Figure A.4: Liquidity and Overconfidence

This graph plots the illiquidity measure, *ILLIQ*, as functions of the parameters representing late-informed investors' skepticism about the quality of the early Date-1 information (κ_ϵ), and over-estimation of the quality of their own information (ω_μ). Late-informed investors are skeptical in that they believe the early Date-1 signal contains only limited fundamental information (i.e., $s = \theta_1 + \mu_1 + \epsilon$ and $\gamma = \theta_1 + \mu_1$). The public signal, $t = \theta + \xi$, arrives at Date 2. Date-1 and Date-2 supplies of the risky security, z_1 and z_2, are normal random variables. We assume the parameter values of $m = 0.1$, $\lambda = 0.1$, $A = 1$, $A_N = 1$, $\nu_\theta = 1$, $\nu_z = 0.01$, $\nu_\xi = 10$, $\kappa = 2$, $\Delta = 0.1$, $\omega_\epsilon = \nu_\epsilon = 0.1$, and $\nu_\mu = 1$.

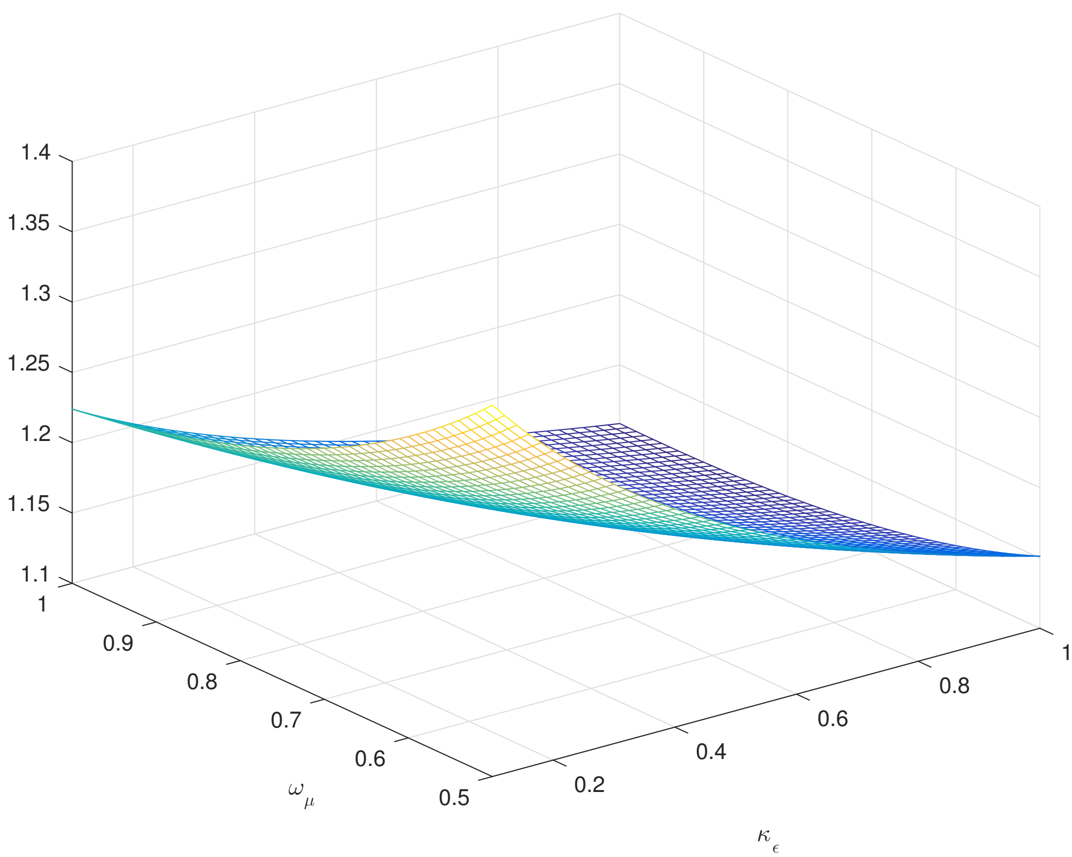


Figure A.5: Short-Run Momentum and Overconfidence

This graph plots the momentum parameter $\overline{MOM}$ as functions of the parameters representing late-informed investors' skepticism about the quality of the early Date-1 information (κ_ϵ), and overestimation of the quality of their own information (ω_μ). Late-informed investors are skeptical in that they believe the early Date-1 signal contains only limited fundamental information (i.e., $s = \theta_1 + \mu_1 + \epsilon$ and $\gamma = \theta_1 + \mu_1$). The public signal, $t = \theta + \xi$, arrives at Date 2. Date-1 and Date-2 supplies of the risky security, z_1 and z_2, are normal random variables. We assume the parameter values of $m = 0.1$, $\lambda = 0.1$, $A = 1$, $A_N = 1$, $\nu_\theta = 1$, $\nu_z = 0.01$, $\nu_\xi = 10$, $\kappa = 2$, $\Delta = 0.1$, $\omega_\epsilon = \nu_\epsilon = 0.1$, and $\nu_\mu = 1$.

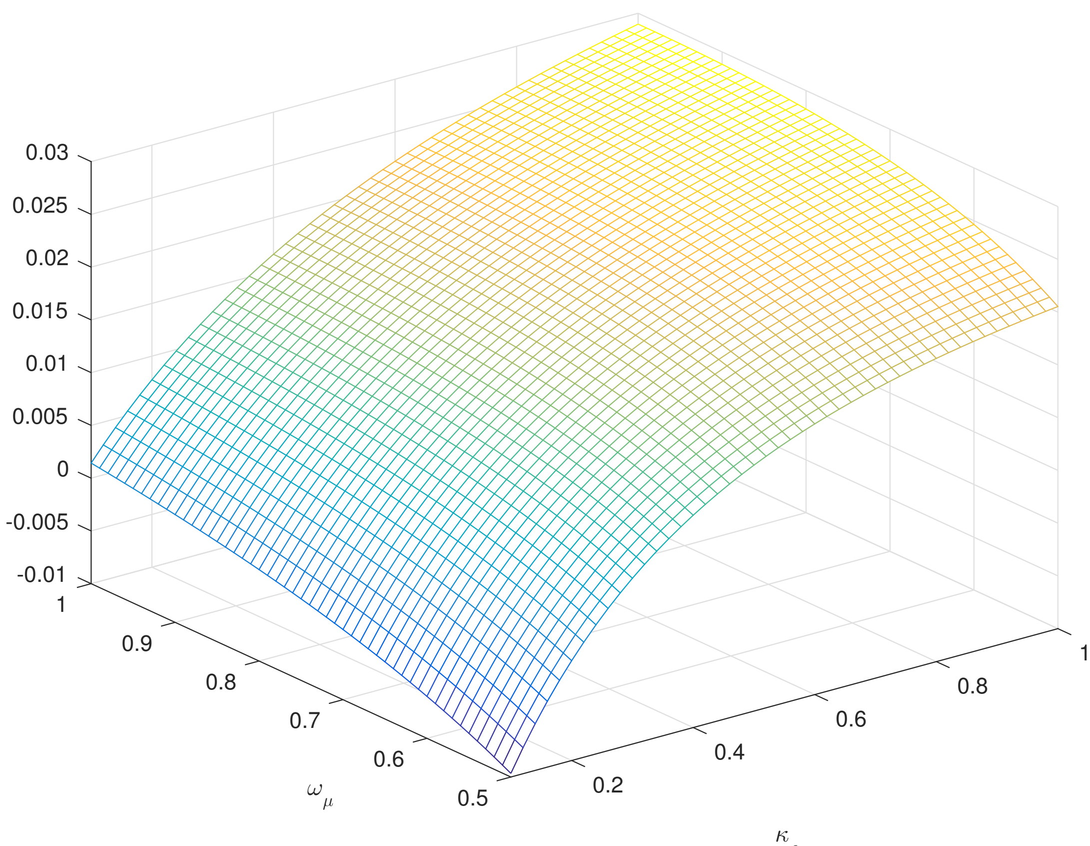


Figure A.6: Liquidity, Short-Run Momentum, and Overconfidence

This graph plots the momentum parameter, $\overline{MOM}$, and the illiquidity measure, $ILLIQ$, as functions of the summary measure of late-informed investors' overconfidence, $OC \equiv (\kappa_\epsilon - \nu_\epsilon) + (\nu_\mu - \omega_\mu)$. We assume that $\nu_\mu - \omega_\mu = 0.2(\kappa_\epsilon - \nu_\epsilon)$. To generate the values of OC on the horizontal axis, we increase κ_ϵ starting from ν_ϵ, while simultaneously changing ω_μ. Late-informed investors are skeptical in that they believe the early Date-1 signal contains only limited fundamental information (i.e., $s = \theta_1 + \mu_1 + \epsilon$ and $\gamma = \theta_1 + \mu_1$). The public signal, $t = \theta + \xi$, arrives at Date 2. Date-1 and Date-2 supplies of the risky security, z_1 and z_2, are normal random variables. We assume the parameter values of $m = 0.1$, $\lambda = 0.1$, $A = 1$, $A_N = 1$, $\nu_\theta = 1$, $\nu_z = 0.01$, $\nu_\xi = 10$, $\kappa = 2$, $\Delta = 0.1$, $\omega_\epsilon = \nu_\epsilon = 0.1$, and $\nu_\mu = 0.5$.