Partnerships in Urban Mobility: Incentive Mechanisms for Improving Public Transit Adoption

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Problem definition: Due to a decline in public transit ridership over the last decade, transit agencies across the United States are facing a financial crisis. To entice commuters to travel by public transit instead of driving personal vehicles, it is essential for municipal governments to address the "last mile" problem by providing convenient and affordable transportation between a commuter's home and a transit station. This challenge raises an important question: Is there a cost-effective mechanism that can improve public transit adoption by solving the last mile problem?
Academic / Practical Relevance: In this paper, we present and analyze two incentive mechanisms for increasing commuter adoption of public transit. In a *direct* mechanism, the government provides a subsidy to commuters who adopt a "mixed mode", which involves taking public transit and hailing rides to/from a transit station. The government funds the subsidy by imposing congestion fees on personal vehicles entering the city center. In an *indirect* mechanism, instead of levying congestion fees, the government secures funding for the subsidy from a private sector partner. We examine the implications of both mechanisms on relevant stakeholders. These two mechanisms are especially relevant because several jurisdictions in the U.S. have begun piloting incentive programs in which commuters receive subsidies for ride-hailing trips that begin or end at a transit station.
Methodology: We present a game-theoretic model to capture the strategic interactions among five self-interested stakeholders (commuters, public transit agency, ride-hailing platform, municipal government, and local private enterprises).
Results: By examining the equilibrium outcomes, we obtain three key findings. First, we characterize how the optimal interventions associated with the direct or the indirect mechanism depend on: (a) the coverage level of the public transit network; (b) the public transit adoption target; and (c) the relative strength of commuter preferences for driving over taking public transit. Second, we show that the direct mechanism cannot be budget neutral without undermining commuter welfare. However, when the public transit adoption target is not too aggressive, we find that the indirect mechanism is budget neutral, and it increases both commuter welfare and sales to the private enterprise. Finally, we show that although the indirect mechanism restricts the scope of government intervention (by eliminating the congestion fee), it can dominate the direct mechanism by leaving all stakeholders better off, especially when the adoption target is modest.
Managerial Implications: Our findings offer cost-effective prescriptions for improving urban mobility and public transit ridership.

Keywords: public transit, public-private partnerships, subsidies, incentives, Mobility as a Service (MaaS).

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1 Introduction

The mission of the United States Federal Transit Administration (FTA) is to "enhance citizens' mobility, accessibility, and economic well-being through the development and management of public transport services". However, over the last decade, the FTA's mission has become increasingly threatened by a decline in public transit ridership across the United States. From 2008 to 2017, per capita ridership on buses, subways, and commuter trains saw a 4% drop in San Francisco, a 5% drop in New York City, and over a 25% drop in both the Los Angeles and Washington DC areas (The Economist, 2019). Moreover, this decline is not isolated to the largest metropolitan areas; nationally, bus ridership has decreased from 5.6 billion trips in 2008 to 4.7 billion trips in 2018 (Kamp, 2020). The widespread decline in public transit ridership is believed to be driven by multiple factors, including lower gasoline prices, changing demographic patterns within cities, and the rise of alternative modes of transportation, such as ride-hailing (Mallett, 2018). As a result of declining fare revenue, over 98% of the 2,200 public transit agencies in the U.S. are in financial crisis (Federal Transit Administration, 2017). Covering these losses is challenging for many municipalities due to limited funding, as well as other competing priorities, such as public safety, education, and affordable housing.

To combat declining ridership and revenue, cities can certainly scale back public transit services. However, doing so would undermine the mission of the FTA, for the following reasons. Reducing access to public transit can hinder mobility, especially for those who cannot afford alternative modes of transportation (e.g., private vehicles, ride-hailing services, and taxis). Moreover, the American Public Transit Association (APTA) has found evidence suggesting that investments in public transit can generate economic returns by lifting business sales and promoting job growth (APTA, 2019c). In addition to the economic benefits, public transit plays a vital role in reducing carbon emissions and alleviating traffic congestion (APTA, 2019c).[1] Some cities have offered free bus rides to increase ridership (e.g., in 2019, Lawrence, Massachusetts and Olympia, Washington began offering free bus rides to improve mobility for the poor and the elderly (Kamp, 2020)); however, the long-term financial viability of fully subsidizing bus fares is questionable. Therefore, there is a clear need for municipalities to develop cost effective – ideally, budget neutral[2] – solutions for increasing transit ridership.

A major barrier to increasing public transit utilization is the "last mile" problem – a challenge caused by the lack of convenient and affordable transit services between an individual's home and a transit station (APTA, 2019b; LA Metro, 2016). Despite increased investments in public transit –

[1]Single-occupancy vehicles emit 1.5 times the CO_2 emissions of buses, and four times the emissions of subways (Hodges et al., 2010). With respect to traffic, public transit has been found to be critical in reducing congestion during peak hours (Anderson, 2014).

[2]The notion of delivering public services in a budget neutral manner has recently been proposed by the Centers for Medicare and Medicaid Services; the reader is referred to CMS (2018) for details.

total inflation-adjusted funding in the U.S. increased from \$60 to \$72 billion between 2007 and 2017 (APTA, 2019a) – commuters continue to eschew public transit in favor of personal vehicles, in part due to the last mile problem. Indeed, in 2017, over 85% of US workers used personal vehicles to commute, with less than 6% relying on public transit (Bureau of Transportation Statistics, 2018). Further, while offering subsidized parking near transit stations may appear to be an attractive solution to the last mile problem, such programs are unlikely to be feasible on a large scale due to limited parking near transit stations, as well as the high cost of building, maintaining, and subsidizing parking spaces. For example, in Los Angeles County, only 24,000 subsidized parking spaces are available, to support approximately 3.6 million daily commuters (Linton, 2016; US Census Bureau, 2018).

To address the last mile gap, various municipal governments are forming partnerships with private transportation companies. For example, in 2019, Los Angeles partnered with the ride-hailing platform Via to launch a pilot project that offers commuters subsidized rides to and from public transit stations, supported by a grant from the Federal Transit Administration (Los Angeles County Metropolitan Transportation Authority, 2019). Several other major cities (e.g., Atlanta, Austin, Detroit, Philadelphia, and Tampa) have also begun testing the use of ride-hailing subsidies as a potential solution to closing the last mile gap (Schwieterman and Livingston, 2018; APTA, 2019d). These subsidy programs have only recently become possible due to advances in information technology, which now enable on-demand ride-hailing, real time tracking of passenger locations, and online fare payments.[3] More generally, the integration between public transit systems and ride-hailing is accelerating – for example, in early 2020, Uber announced a new in-app feature that coordinates trip drop-off times with train schedules (Uber, 2020). This deepening integration between private transportation services and public transit opens the door to new forms of mutually beneficial partnerships.[4]

In this paper, we investigate two incentive mechanisms that aim to increase public transit ridership by addressing the last mile problem. Both mechanisms have a common feature: they both rely on a strategic partnership between a public transit agency and a ride-hailing platform. Specifically, in both mechanisms, commuters receive a subsidy for adopting a *mixed mode* of transportation, in which commuters use the ride-hailing service to travel the last mile distance ("x") between their home and a transit station, and use public transit to travel between the transit station and a final destination (e.g. city center). In this setting, x can be interpreted as a measure of the *cover-*

[3]Information technology has been successfully leveraged to help commuters plan and pay for multi-modal trips within a mobile application; examples include Whim in Helsinki, Citymapper in London, Moovel in Germany, UbiGo in Gothenburg, TAP in Los Angeles, Clipper in San Francisco, and Opal in Sydney (Goodall et al., 2017; Cole, 2017).

[4]There is also evidence to suggest that ride-hailing may be contributing to the decline in public transit ridership, and adding to traffic congestion in city centers (Brown, 2020). Partnerships between public transit agencies and ride-hailing platforms may help move them toward a model of cooperation instead of competition, with benefits to both parties (increased transit ridership in the city center and increased demand for ride-hailing in suburban areas).

age[5] of the public transit system: the last mile x is small (large) if commuters have convenient (inconvenient) access to transit stations from their homes.

Both mechanisms have a common goal: to increase the adoption of the mixed mode by a constant factor ("β"). However, the key differences between the two mechanisms are (a) the role of the local government; and (b) the source of funding for the ride-hailing subsidy. The first incentive program we consider is *direct* in the sense that the the government offers a subsidy ("s") directly to each commuter as an incentive for adopting the mixed mode. To defray the cost associated with the subsidy, the government charges a congestion fee ("e") to commuters who instead choose to drive a personal vehicle into the city.[6] Therefore, under the "direct mechanism" (denoted by [D]), the government uses two levers; namely, the ride-hailing subsidy s and the congestion fee e, to entice commuters to adopt the mixed mode.

In the second mechanism, instead of charging congestion fees, the local government secures funding for the subsidy ("z") from the private sector. This mechanism is *indirect* because the government's role is restricted to facilitating the transfer of the subsidy from a private enterprise to commuters. This scheme is motivated by the fact that many municipalities may not have sufficient funds to further subsidize public transit. To relieve this financial burden, Cole (2017) proposes the following partnership between a municipal government and a private enterprise. The government and public transit agency develop a mobile application to process different transactions, including fares collected from the commuters and subsidies provided by the private enterprise. The private enterprise benefits from higher transit ridership due to in-app advertisements and increased foot traffic at stores near the transit station (e.g., coffee shops, bakeries, health clinics, spas, hair salons). Under this arrangement, the private enterprise can recover the cost associated with the subsidy from the extra revenue derived from increased foot traffic. In contrast to the direct mechanism, this "indirect mechanism" (denoted by [I]) involves just a single lever: the subsidy z, provided by the private enterprise.

Our primary contribution in this paper is to investigate the impact of both subsidy mechanisms on an ecosystem comprising multiple stakeholders: a local municipal government, suburban commuters, city dwellers, a public transit agency, a ride-hailing platform, and a private enterprise. In particular, we address the following questions:

1. How does each mechanism perform with respect to (a) operating cost (borne by the govern-

[5] In the context of public transit, "coverage" broadly refers to the accessibility of transit stations by the general population (i.e., commuters in our context) (Tomer et al., 2011).

[6] Congestion fees have been adopted by many municipal governments, including Singapore, Hong Kong, London, Milan, and Stockholm. Singapore was the first to introduce congestion pricing as a tool to control traffic volume, where citizens pay for fees when they enter city center areas (Development Asia, 2018). London introduced congestion fees in 2003, where there is a charge for entering London's congestion charging zone (13 square miles) between 7 a.m. and 6 p.m. on weekdays (Transport for London, 2019). Since the introduction of congestion fees, traffic congestion and private vehicle usage in London has dropped by 25% and 39%, respectively (Badstuber, 2018). Similarly, in Milan, public ridership increased 12.5% from 2007 to 2013 (Croci, 2016).

ment); (b) commuter welfare; (c) the public transit agency's revenue; (d) the ride-hailing platform's revenue; and (e) the private enterprise's profit?

2. How does the last mile distance (or coverage) x and the mixed mode adoption target β influence prescriptions for (a) the optimal direct subsidy s^* and congestion fee e^* in mechanism [D], and (b) the optimal indirect subsidy z^* in mechanism [I] ?

3. Under what conditions, if any, should the municipal government adopt mechanism [I] over mechanism [D]?

We examine the above questions by developing a game-theoretic framework that captures the interactions among all stakeholders. Specifically, in mechanism [D], we identify the optimal subsidy level s^* and congestion fee e^* that minimizes the government's net spending (subsidy cost less the revenue generated by the congestion fee), subject to improving adoption of the mixed mode by a factor of at least β. In addition, we restrict the set of feasible subsidies and congestion fees to those that satisfy participation constraints associated with commuters, the ride-hailing platform, the public transit agency, and the private enterprise. We also determine the optimal subsidy z^* under mechanism [I], subject to a similar set of stakeholder participation constraints.

Our three key findings can be summarized as follows:

1. Under mechanism [D], it is more effective to rely on congestion fees over ride-hailing subsidies ($e^* \geq s^*$) when public transit coverage is high (i.e., the last mile distance x is small). This prescription is reversed ($s^* > e^*$) when transit coverage is low (i.e., x is large). Moreover, we find that whether the optimal subsidy s^* increases or decreases in x depends on the comparative strength of commuter preferences between driving and taking public transit.

2. Budget neutrality is not attainable under mechanism [D]. In other words, we find that it is not possible for the government to fully recoup the cost of the subsidy through congestion fees, without compromising commuter welfare. In contrast, if the mixed mode adoption target β is not too aggressive, then the government can attain budget neutrality through a partnership with a private enterprise under mechanism [I], while *increasing* commuter welfare. If the adoption target β is aggressive, however, then funding the subsidy through a private sector through mechanism [I] is not viable, due to an erosion of the partner enterprise's profit.

3. Despite the government having access to two levers in mechanism [D] (the subsidy s and the congestion fee e) and only a single lever in mechanism [I] (the subsidy z), mechanism [I] can *dominate* mechanism [D], but only when the adoption target β is modest. Specifically, when β is modest, the commuters, city dwellers, public transit agency, ride-hailing platform, and private enterprise are all better off under mechanism [I].

The remainder of the paper is organized as follows. In §2, we discuss related literature. In §3, we present model preliminaries. In §4, we present mechanism [D] and analyze the optimal incentive scheme (e^*, s^*). In §5, we present mechanism [I], analyze the optimal subsidy z^*, and compare the performance of both mechanisms. In §6, we consider variants of mechanisms [D] and [I] that address the trade-off between operating cost and commuter welfare. In §7, we conclude and discuss potential future research directions.

2 Literature Review

Our paper is related to three streams of literature: government subsidy programs, public-private partnerships, and budget neutral mechanisms.

Government subsidy programs. Within the operations literature, there is a growing focus on subsidy programs that promote the production or adoption of socially beneficial goods or services. This work can be categorized into subsidies for producers (e.g., farmers, manufacturers, healthcare providers) and consumers. Subsidies for producers have been studied in the context of healthcare (Taylor and Xiao, 2014; Levi et al., 2017; Aswani et al., 2019), green technology (Ma et al., 2019; Cohen et al., 2016; Alizamir et al., 2016), and agriculture (Alizamir et al., 2019; Akkaya et al., 2019). Our paper is closer to the literature on consumer-facing subsidies. In the home appliances industry, Yu et al. (2019) determine whether the government should subsidize consumers only, manufacturers only, or both, where the goals of the government are to improve manufacturer profit and consumer welfare in rural areas. Xiao et al. (2019) examine the impact of the same home appliance subsidy program, which they conclude improves both affordability and accessibility for rural customers. In the context of solar energy technology, Chemama et al. (2019) compare the effectiveness of static and dynamic consumer subsidies on supplier behavior. Our paper differs from this existing literature in that we investigate whether the subsidy program can be budget-neutral from the perspective of the subsidy provider.

Previous work in the operations literature on public transit subsidies is sparse. Lodi et al. (2016) consider a setting where operation of the public transit system is outsourced to the private sector, and the government provides a subsidy to offset the operating cost. Yang and Lim (2018) use a field experiment to show that temporarily subsidizing public transit can lead to long-term changes in commuter behavior. The paper that is most similar to ours in this line of research is by Xiao and Zhang (2014), who also consider congestion fees and transit subsidies simultaneously. They focus on the impact that commuter heterogeneity in value-of-time has on the optimal design of congestion fees, and show that transit subsidies can offset the loss in commuter welfare due to the congestion fee. Our work is different in that we focus on the role that congestion fees and transit subsidies can play in improving public transit ridership.

Public-private partnerships. Our work contributes to the modeling of public-private partnerships (PPP), which refers to a private sector partner "financing, constructing, and managing a project in return for a promised stream of payments directly from government or indirectly from users over the projected life of the project or some other specified period" (Weimer and Vining, 2017). Our paper belongs to the stream of PPP literature where the government has all of the bargaining power. In the existing literature, this is often modeled as either a principal-agent problem or a Stackelberg game, where the government is the principal/leader and the firm is the agent/follower; applications include healthcare (So and Tang, 2000; Lee and Zenios, 2012; Gupta and Mehrotra, 2015; Guo et al., 2019; Aswani et al., 2019), disaster management (Guan and Zhuang, 2015; Guan et al., 2018) and risk management (Bakshi and Gans, 2010). With respect to transportation, existing work on public-private partnerships has primarily focused on infrastructure. Lodi et al. (2016) address the government's incentive design problem when management of the public transit service is outsourced to private operators. Gagnepain and Ivaldi (2002) examine the effect on social welfare under different types of contracts between the regulator and the public transit operator. Hansson (2010) considers a multi-principal setting, where the local, regional and county governments interact to regulate public transit procurement. In contrast to these papers, the private partners play a different role in our work, namely, they provide a complementary transportation service (in the case of the ride-hailing platform) and funding for subsidies (in the case of the partner enterprise).

Previous work has also considered settings where the public-private partnership is formed through negotiation. In the transportation context, Kang et al. (2013) investigate royalty bargaining associated with underground railway station construction, and Wang and Zhang (2016) examine road pricing of transportation networks with both public and private roads. Other application areas include natural resource development (Anandalingam, 1987), public procurement (Gur et al., 2017; Saban and Weintraub, 2019) and global supply chain management (Cohen et al., 2018; Cho et al., 2019; Cohen and Lee, 2020).

Budget neutral mechanisms. This paper also contributes to the literature on budget neutral policies. This work has primarily appeared in the public policy literature, and has addressed issues such as social security reform (Burkhauser and Smeeding, 1994), environmental taxation (Goulder, 1995; Murray and Rivers, 2015), and fiscal policy (Correia et al., 2013; D'Acunto et al., 2016). Within the operations management literature, previous work on budget neutral policies is scarce. Guo et al. (2014) optimize a two-tier queuing system with both a free server and fee-based server, in the setting where the system is self-financed by the costly server. Arifoglu and Tang (2019) develop a budget neutral incentive mechanism for coordinating a decentralized influenza vaccine supply chain.

With respect to transportation settings, most existing papers that focus on budget neutrality are based on schemes under which subsidies/rebates are funded by congestion fees, similar to mechanism [D] in our work (see, e.g., Guo and Yang (2010); Nie and Liu (2010); Chen and Yang (2012); Xiao and Zhang (2014)). Our paper differs in that we also consider an indirect mechanism

[I], where the government attains budget neutrality by obtaining funding from a private enterprise, instead of imposing congestion fees. Further, we compare mechanisms [D] and [I] in terms of their impact on the relevant stakeholders.

3 Model Preliminaries

A unit mass of commuters are located a (last mile) distance $x > 0$ from a transit station. All commuters must travel to a city center that is located an additional distance of 1 beyond the transit station. For tractability, we consider a parsimonious model where commuters choose between two modes of travel:[7] *driving* or a *mixed mode*. As depicted in Figure 1, x represents the length of the "last mile" that is not covered by public transit. Hence, commuters who choose the driving mode will drive a personal vehicle for a distance of $1 + x$ to the city center. However, commuters who choose the mixed mode will first travel a distance of x to the transit station via a ride-hailing service, and then travel the remaining distance of 1 by public transit.

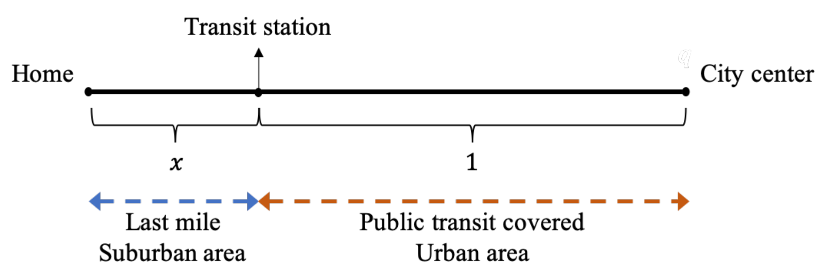

Figure 1: Travel distance of a commuter.

We first describe our base model in the absence of any subsidies or congestion fees. In later sections, we shall extend our base model to incorporate the two incentive schemes. We note here that our focus throughout the paper is on suburban commuters; therefore, all aspects of the model (e.g., transit ridership, operating cost, ride-hailing platform revenue) are defined with respect to this group of commuters only.

Commuter utility. Each commuter obtains a "base" utility B for commuting to the city center. A commuter obtains an "additional" utility V per unit distance traveled via ride-hailing or public transit, or δV if she drives.[8] We assume $\delta > 1$ to reflect a higher intrinsic utility for driving.[9]

[7]We focus on suburban commuters for whom walking to the transit station is prohibitively costly. Accordingly, we exclude the mixed mode that combines walking and public transit. We also exclude the mixed mode that combines driving and public transit, due to limited parking spaces near transit stations in the U.S. Finally, we exclude commuters who exclusively use ride-hailing to commute (only 0.2% of suburban residents commute via taxi or ride-hailing services (Federal Highway Administration, 2017)).

[8]Our main results persist in a model where commuters prefer ride-hailing to public transit, so that the additional value associated with ride-hailing is $l \cdot V$, where $\delta > \ell \geq 1$. For ease of exposition, we assume $\ell = 1$ throughout.

[9]Recent survey data has shown that commuters generally prefer driving to public transit and ride-hailing services

To capture heterogeneity among commuters, we assume $V \sim U[0,1]$. We assume a commuter's cost (or fare) for driving, ride-hailing, and taking public transit are denoted by d, r, and p, respectively. We assume throughout that $r > d > p > 0$, which is supported empirically.[10]

A commuter will travel a distance of $1 + x$ if she chooses the driving mode of transportation. Similarly, under the mixed mode, the commuter will first travel a distance of x via ride-hailing and the remaining distance of 1 via public transit. Therefore, the utilities associated with driving and the mixed mode, denoted by U_d and U_m, respectively, are given by

$$U_d = B + (x+1)(\delta V - d), \tag{1a}$$

$$U_m = B + x(V - r) + (V - p). \tag{1b}$$

Travel mode demand. A commuter will adopt the mixed mode if and only if $U_m \geq U_d$. Note that this inequality holds if and only if:[11]

$$V \leq v_1 \equiv \frac{(d-p) - (r-d)x}{(\delta - 1)(x+1)}. \tag{2}$$

Let D_d and D_m denote the demand for the driving and mixed modes, respectively. Let $[a]^+ = \max\{a, 0\}$. Because $V \sim U[0,1]$, we can apply (2) to show that:

$$D_d = \left[\int_{v_1}^{1} 1 \cdot dV\right]^+ = \left[1 - \frac{(d-p) - (r-d)x}{(\delta-1)(x+1)}\right]^+, \tag{3a}$$

$$D_m = \left[\int_{0}^{v_1} 1 \cdot dV\right]^+ = \left[\frac{(d-p) - (r-d)x}{(\delta-1)(x+1)}\right]^+. \tag{3b}$$

Note that D_m represents the additional public transit ridership generated by the mixed mode commuters. Because $r > d > p > 0$, it is straightforward to verify that D_m is decreasing in x, which implies that commuters that live far from the transit station are more likely to drive and less likely to adopt the mixed mode. Further, D_m also measures the "reduction" in traffic congestion, because $D_m = 1 - D_d$.

To exclude the degenerate cases where *all* commuters adopt the same travel mode, we assume that $x \in (\underline{x}, \bar{x})$, so that $D_m > 0$ and $D_d > 0$, where $\underline{x} = \max\{0, \frac{(d-p)-(\delta-1)}{(\delta-1)+(r-d)}\}$ and $\bar{x} = \frac{d-p}{r-d}$. We also assume that the public transit and the ride-hailing platform have sufficient capacity to accommodate demand for the mixed mode.[12]

Commuter welfare. Because $V \sim U[0,1]$, the commuter welfare in the base model is given by

$$W = \int_{0}^{v_1} U_m dV + \int_{v_1}^{1} U_d dV. \tag{4}$$

(Zhu and Fan, 2018).

[10] The cost of driving is estimated to be \$0.76/mile, assuming a mileage of 10,000 miles per year (American Automobile Association, 2018). Ride-hailing is estimated to cost more than \$1.07/mile (Fare Estimater, 2019). The cost of commuting by public transit is typically much lower; for example, the cost of public transit in Los Angeles is estimated to be \$0.2/mile, assuming an average commute distance of 16 miles (Leonard, 2019).

[11] We assume the base value B is large enough such that $U_d \geq 0$ and $U_m \geq 0$.

[12] The average vehicle occupancy rate of public transit in 2017 was less than 30% (Economist, 2018). Ride-hailing services also have ample capacity: the average idle rate of Uber drivers is 30-40% (Currie, 2018; Brown, 2020).

Transit agency revenue. Note that the unit fare of public transit is p. The revenue generated by the public transit agency from these commuters under the mixed mode demand D_m is:

$$\Pi_p = p \cdot D_m. \tag{5}$$

Ride-hailing platform revenue. Recall that each mixed mode commuter will travel a distance of x via ride-hailing to the transit station, at a unit fare of r. The revenue generated by the ride-hailing platform from these commuters under the mixed mode demand D_m is:

$$\Pi_r = r \cdot x \cdot D_m. \tag{6}$$

Private enterprise profit. Consider a private enterprise who sells to mixed mode commuters who travel through the transit station.[13] We model the enterprise profit as $(k-c)[1-k\alpha]^+D_m$, where k and c represent the unit price and cost, respectively, and $[1-k\alpha]^+$ represents the proportion of mixed mode commuters that purchase the product. Note that $\alpha > 0$ is the commuters' price sensitivity. To avoid the trivial case where the enterprise's optimal profit is non-positive, we assume $\alpha c < 1$.[14] Then, for any given mixed mode demand D_m, the optimal retail price in the base model is $k_b = \frac{\alpha c+1}{2\alpha}$, with a corresponding profit of

$$\Pi_s = \frac{(1-\alpha c)^2}{4\alpha} \cdot D_m. \tag{7}$$

In the next section, we present the first incentive mechanism (i.e., the direct mechanism [D]) that aims to increase commuter adoption of public transit via the mixed mode.

4 Direct Mechanism: Commuter Subsidies and Congestion Fees

In the direct mechanism [D], the government provides a lump sum subsidy s per trip to each commuter who takes the mixed mode, and charges a lump sum congestion fee e to each commuter who travels by a personal vehicle. The sequence of events is as follows. First, the government sets the subsidy s and the congestion fee e. Second, after observing the subsidy and congestion fee, each commuter chooses to commute by either driving or the adopting the mixed mode. Third, the private enterprise (who is passive under the direct mechanism) sets a profit maximizing price, based on the mixed mode demand. Hence, commuter utilities under the incentive (e, s) are given by:

$$U_d(e,s) = U_d - e, \tag{8a}$$

$$U_m(e,s) = U_m + s, \tag{8b}$$

[13] Although we model the private enterprise as a single entity, it can equivalently be interpreted it as a collection of cooperating firms that sell goods and services to commuters who pass through the transit station.

[14] Note that if $\alpha c \geq 1$, then $(k-c)[1-k\alpha]^+D_m \leq 0$ for any $D_m \geq 0$ and $k \geq 0$.

where U_d and U_m are given in (1). A commuter will adopt the mixed mode m under incentive (e, s) if and only if $U_m(e, s) \geq U_d(e, s)$, or equivalently, if her valuation V satisfies:

$$V \leq v_1(e, s) \equiv \min \left\{ 1, \frac{(d-p) - (r-d)x + e + s}{(\delta - 1)(x+1)} \right\}. \tag{9}$$

Similar to the base case, the demand for each mode under the incentive (e, s) is then given by

$$D_d(e, s) = \left[1 - \frac{(d-p) - (r-d)x + e + s}{(\delta - 1)(x+1)} \right]^+, \tag{10a}$$

$$D_m(e, s) = 1 - \left[1 - \frac{(d-p) - (r-d)x + e + s}{(\delta - 1)(x+1)} \right]^+. \tag{10b}$$

Note that if $(e, s) = (0, 0)$, then $v_1(e, s)$, $D_d(e, s)$, and $D_m(e, s)$ simplify to the base values v_1, D_d, and D_m as stated in §3, respectively.

Performance metrics. We next define the metrics by which we evaluate the performance of the incentive (e, s). Because the government offers a subsidy s to each of the $D_m(e, s)$ mixed mode commuters and collects a fee e from each of the $D_d(e, s)$ commuters that drive, the total cost for the government to operationalize the incentive (e, s) is:

$$C(e, s) = sD_m(e, s) - eD_d(e, s). \tag{11}$$

Similar to the base case, the total welfare accrued to all commuters is:

$$W(e, s) = \int_0^{v_1(e,s)} U_m(e, s) dV + \int_{v_1(e,s)}^1 U_d(e, s) dV, \tag{12}$$

the public transit agency's revenue generated from these commuters is:

$$\Pi_p(e, s) = p \cdot D_m(e, s), \tag{13}$$

and the ride-hailing platform's revenue generated from these commuters is:

$$\Pi_r(e, s) = r \cdot x \cdot D_m(e, s). \tag{14}$$

Lastly, the private enterprise's profit generated from these commuters is $\pi_{sD}(k) = (k - c)(1 - k\alpha)D_m(e, s)$, which yields an optimal price of $k_b = \frac{\alpha c + 1}{2\alpha}$, and an accompanying profit of

$$\Pi_{sD}(e, s) = \frac{(1 - \alpha c)^2}{4\alpha} \cdot D_m(e, s). \tag{15}$$

Note that when $(e, s) = (0, 0)$, there is no operating cost $C(e, s) = 0$, and the metrics $W(e, s)$, $\Pi_p(e, s)$, $\Pi_r(e, s)$, and $\Pi_{sD}(e, s)$ reduce to the base values W, Π_p, Π_r, and Π_s (defined in §3), respectively. Figure 2 depicts the relationship among the various stakeholders under the direct mechanism [D].

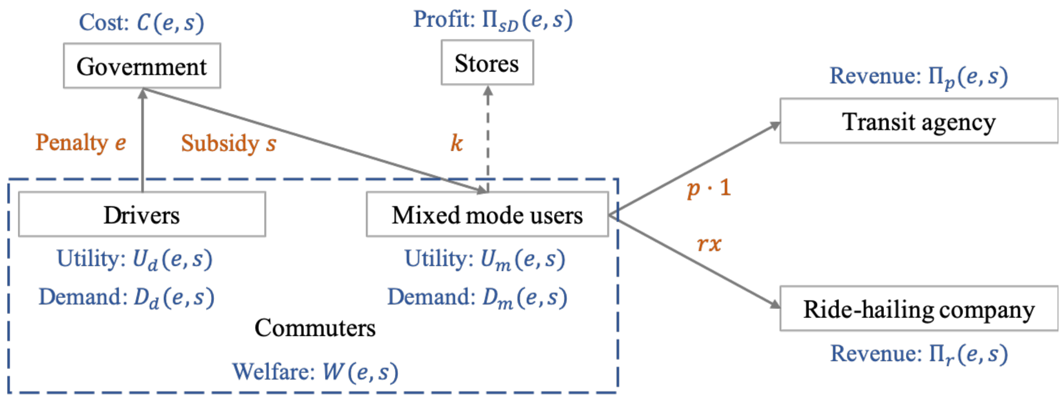

Figure 2: Strategic interactions among stakeholders under the direct mechanism [D].

4.1 Direct Mechanism: Problem Formulation

We now formulate the government's incentive design problem, which accounts for the six aforementioned metrics that affect the stakeholders depicted in Figure 2. In particular, the government selects the subsidy s and congestion fee e according to the following criteria:

I **Minimum operating cost.** Because most municipalities in the U.S. are budget constrained with respect to public transit, we assume the government wishes to minimize the total operating cost, $C(e,s)$.

II **Increase transit ridership.** The intent of the incentive (e,s) is to increase public transit ridership among commuters by a factor by $\beta > 0$. We assume throughout that β is an exogenous adoption target, and that the incentive (e,s) must satisfy:

$$D_m(e,s) - D_m \geq \beta D_m,$$

where D_m is the baseline mixed mode adoption without government intervention, as stated in §3. Note that the increase β is equivalent to a decrease in demand for driving; therefore, β may be equivalently interpreted as the target reduction in traffic congestion. For this reason, the constraint above can also be interpreted as improving the well-being of city dwellers. Analogous to the assumption that $x \in (\underline{x}, \bar{x})$ in the base model, we assume throughout that the target β is restricted to the non-degenerate case where $D_d(e,s) > 0$ and $D_m(e,s) > 0$. To enforce the preceding inequalities, it suffices to assume that $\beta < \tau(x) \equiv \frac{(\delta-1)(x+1)}{(d-p)-(r-d)x} - 1$. Note that if $\beta \geq \tau(x)$, then upon implementation of incentive (e,s), no commuters will drive $(D_d(e,s) = 0)$, which is unlikely to occur in practice.

III **Commuter participation.** To generate buy-in from the public, commuters should collectively be no worse off than prior to the intervention: $W(e,s) \geq W$.

IV **Ride-hailing platform participation.** To ensure that the partnership with the ride-hailing platform is viable, the platform's revenue should not decrease: $\Pi_r(e,s) \geq \Pi_r$.

V **Public-transit participation.** To ensure that the public transit agency is not negatively impacted, public transit revenue from the mixed mode should not decrease: $\Pi_p(e,s) \geq \Pi_p$.

VI **Enterprise participation.** Lastly, to prevent adverse effects on the local economy, it is desirable to ensure that the incentive (e,s) does not reduce the private enterprise's profit: $\Pi_{sD}(e,s) \geq \Pi_s$.

Based on the six criteria above, the optimal incentive (e^*, s^*) is thus given by the solution to the following optimization problem:

$$
\text{Mechanism [D]:} \quad
\begin{aligned}
\min_{e,s\geq 0} \;& C(e,s) && (16a)\\
\text{s.t.} \;& D_m(e,s) - D_m \geq \beta D_m && (16b)\\
& W(e,s) \geq W && (16c)\\
& \Pi_r(e,s) \geq \Pi_r && (16d)\\
& \Pi_p(e,s) \geq \Pi_p && (16e)\\
& \Pi_{sD}(e,s) \geq \Pi_s. && (16f)
\end{aligned}
$$

4.2 Direct Mechanism: Optimal Incentive (e^*, s^*)

By solving problem (16), we obtain the optimal incentive (e^*, s^*) under mechanism [D], as follows.

Proposition 1. *The optimal congestion fee e^* and the optimal subsidy s^* under mechanism [D] are given by:*

$$
\begin{aligned}
e^* &= \frac{\beta(\beta+2)((d-p)-(r-d)x)^2}{2(\delta-1)(x+1)},\\
s^* &= \frac{\beta((d-p)-(r-d)x)(2(\delta-1)(x+1)-(\beta+2)((d-p)-(r-d)x))}{2(\delta-1)(x+1)}.
\end{aligned}
\qquad (17)
$$

Further,

(i) The optimal congestion fee e^* strictly increases in β and strictly decreases in x.

(ii) The optimal subsidy s^* strictly increases in β. If δ is large, s^* strictly decreases in x. However, if δ is small, there exists $\tilde{x}$ such that s^* increases on $x < \tilde{x}$ and decreases on $x \geq \tilde{x}$.

Proposition 1 implies that as the mixed mode adoption target β becomes more aggressive, the commuter subsidy s^* and congestion fee e^* also increase to meet the adoption target, as expected. Additionally, as the last mile distance x increases, the demand for driving also increases, and hence the congestion fee e^* decreases to maintain commuter welfare.

In contrast to the congestion fee e^*, the behavior of the optimal subsidy s^* is not monotonic in the last mile distance x. To see why, let us consider the impact on mechanism [D] when x increases. As x increases, the mixed mode adoption (before any government intervention) D_m (as stated in (3)) decreases, which generates two competing effects. First, note that the transit ridership constraint can be rewritten as $D_m(e, s) \geq (\beta + 1)D_m$. Therefore, as D_m decreases, the requirement for the mixed mode commuters $D_m(e, s)$ becomes less stringent. Consequently, the government can afford to reduce s, which we refer to as the "adoption effect." Second, as D_m decreases, commuter welfare decreases, due to fewer commuters receiving the subsidy. To ensure commuters are not worse off, the government needs to increase s, which we refer to as the "welfare effect." Hence, whether the optimal subsidy s^* increases or decreases in x depends on which of these two effects dominate.

To examine when one effect dominates the other, first consider the case when commuters strongly prefer driving over transit and ride-hailing (i.e., when δ is large). In this case, the welfare effect is weak (because the mixed mode adoption D_m is already low before any intervention), and the adoption effect dominates – leading s^* to decrease in x. However, when commuters are relatively indifferent between the two modes (i.e., when δ is small), the mixed mode adoption D_m is highly sensitive to small changes in x. As a result, as x increases, demand for the mixed mode drops sharply, which makes the welfare effect dominates – leading s^* to increase in x.[15]

Corollary 1. *There exists a threshold $\hat{x}$ such that $e^* \geq s^*$ in mechanism [D] if and only if $x \leq \hat{x}$. Further, there exists a threshold $\bar{\bar{\delta}} > 1$ such that $s^* > e^*$ in mechanism [D] for all $x \in (\underline{x}, \bar{x})$ if and only if $\delta \geq \bar{\bar{\delta}}$.*

Corollary 1 states that for any adoption target β, the congestion fee is a more efficient intervention than the subsidy when the last mile distance x is small. To see why, recall from statement (i) and (ii) of Proposition 1 that, when x is small, e^* is high and s^* is low. Also, observe that when the last mile distance x is small, the mixed mode is already attractive to commuters, which makes providing a subsidy (per commuter) relatively costly; in this setting, the congestion fee e is the preferred lever for promoting the adoption of the mixed mode, due to its cost efficiency. (The explanation for the case where x is large is similar, and so we omit it for conciseness.) Corollary 1 also implies that if δ is large, then $\hat{x} < \underline{x}$, which yields $s^* > e^*$ for all $x \in (\underline{x}, \bar{x})$. This occurs because when δ is large, most commuters prefer to drive, and so it is sub-optimal for the government to set a high congestion fee, due to its degradation of commuter welfare.

Next, we examine whether the government can attain budget neutrality (or positive revenue) by using congestion fees to offset commuter subsidies under mechanism [D].

Corollary 2. *It is impossible for mechanism [D] to be budget neutral, i.e., $C(e^*, s^*) > 0$. Further, the minimal operating cost $C(e^*, s^*)$ is higher when β is large or when x is small.*

[15]Note that even when δ is small, the welfare effect ceases to dominate the adoption target effect when the last mile distance is large ($x \geq \tilde{x}$), due to low demand for the mixed mode.

The result in Corollary 2 is a consequence of the commuter welfare constraint $W(e, s) \geq W$, which is always binding under the optimal incentive (e^*, s^*). Therefore, in order to achieve a budget neutral implementation of mechanism [D], the government must either set a congestion fee that is above e^*, or a subsidy that is below s^*. In either case, the corresponding solution is infeasible because the commuter welfare constraint will be violated. Next, to see why $C(e^*, s^*)$ is higher when β is large or when x is small, observe that, in this case, the mixed mode adoption target βD_m on the right hand side of constraint (16b) is large, which requires the mixed mode adoption $D_m(e, s)$ to also be large. Therefore, as more mixed mode commuters receive the subsidy, mechanism [D] incurs a greater cost to the government. We can now summarize our main finding from mechanism [D] as follows.

Remark 1. *The direct mechanism [D] can enable the government to meet the public transit adoption target β. However, this mechanism is costly: attaining budget neutrality is impossible without undermining commuter welfare.*

The above remark naturally raises the following question: Does there exist an alternate mechanism that can enable the government to attain budget neutrality, without negatively impacting commuter welfare? In the next section, we address this question by examining an alternative incentive program under which the subsidy is funded by a private enterprise, instead of congestion fees.

5 Indirect Mechanism: Funding Subsidies Through a Private Enterprise

Because most municipalities are financially constrained, self-funded or budget neutral projects are preferred. In this section, we present the *indirect* mechanism [I], where the local government secures funding for the subsidy from a private enterprise, and does not charge congestion fees. The indirect mechanism can be operationalized through a mobile app that allows the government to collect fares from riders and subsidies from the private enterprise (Cole, 2017).[16] Also, by providing a subsidy z to each commuter who chooses the mixed mode, the enterprise can generate extra revenue from increased foot traffic at stores near the transit station, which can then be used to recoup the cost of the subsidy.

From the government's perspective, the subsidy z is a pass-through, because it is paid by the participating enterprise "indirectly" through the government. Hence, mechanism [I] is budget neutral by design. By using the same approach as in §4, we now extend the base model (without government intervention) presented in §3 to the case when the government adopts the indirect mechanism [I]. Similar to the direct mechanism, a commuter receives a lump sum subsidy z only

[16]For example, in New York and Chicago, the governments charge ride-hailing companies per-trip fees to support public transit infrastructure and accessibility (Joshi et al., 2019).

if she chooses the mixed mode. The interactions among all stakeholders are depicted in Figure 3. By comparing Figures 2 and 3, one can observe that, aside from the subsidy payment that

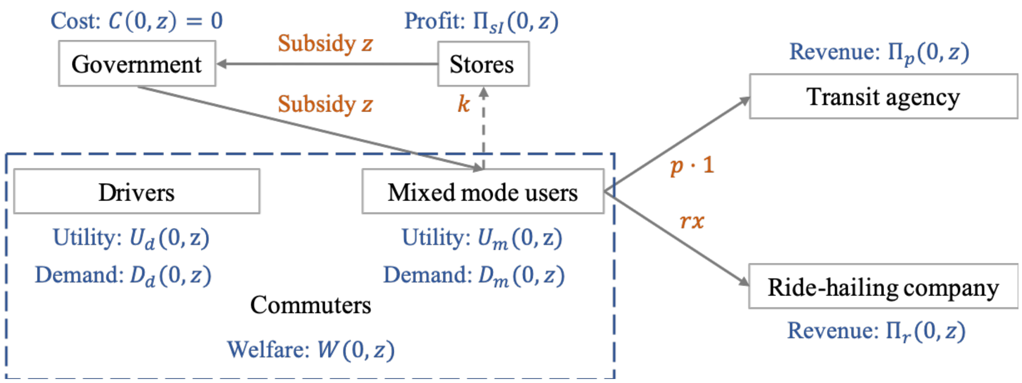

Figure 3: Strategic interactions among stakeholders under the indirect mechanism [I].

the private enterprise has to reimburse the government, mechanism [I] has the same impact on the rest of the stakeholders as mechanism [D]. Hence, by replacing (e, s) in mechanism [D] with $(0, z)$, we can determine the commuter utility associated with the driving mode and the mixed mode under mechanism [I]. Then, by using the same approach as presented in §3, we can determine the demand for driving $D_d(0, z)$ and the mixed mode $D_m(0, z)$, as stated in (10). Similarly, we can evaluate the six performance metrics from §4 as follows. First, the government's operating cost for implementing mechanism [I] is zero, because the cost of the subsidy $zD_m(0, z)$ is reimbursed in full by the private enterprise. Second, the commuter welfare $W(0, z)$, the revenue of public transit $\Pi_p(0, z)$, and the revenue of ride-hailing platform $\Pi_r(0, z)$ can be retrieved from (12), (13) and (14), respectively, by replacing (e, s) with $(0, z)$. Third, for any subsidy level z selected by the government, the subsidy cost $zD_m(0, z)$ is borne by the private enterprise under mechanism [I]. Hence, the private enterprise's profit is given by $\pi_{sI}(k) = (k - c)(1 - k\alpha)D_m(0, z) - zD_m(0, z)$. Then, the corresponding optimal price is $k_I = \frac{\alpha c+1}{2\alpha}$, and the optimal profit is

$$\Pi_{sI}(0, z) = \left(\frac{(1 - \alpha c)^2}{4\alpha} - z\right) D_m(0, z). \tag{18}$$

Note that when $z = 0$, the metric $\Pi_{sI}(0, z)$ reduces to the base value Π_s defined in §3.

5.1 Indirect Mechanism: Problem Formulation

The government's incentive design problem is to determine the optimal subsidy z^*, subject to the criteria I-VI (as presented in §4.1). Recall that the indirect mechanism [I] is budget neutral by design (i.e., $C(0, z) = 0$), so the cost-minimization criterion I is no longer relevant. For this reason, we require a surrogate objective function. Because we impose the participation constraints II-VI in mechanism [I] as well, we shall consider the case where the government chooses the subsidy level z that maximizes the enterprise's profit. We select this objective function to recognize that the viability of such partnerships in practice depends on the benefits accrued to the private enterprise. (In the next section, we consider an alternative formulation that maximizes commuter welfare, and obtain similar results.) The government's problem under the indirect mechanism [I] is then given by

$$\max_{z \geq 0} \Pi_{sI}(0, z) \tag{19a}$$

$$\text{s.t. } D_m(0, z) - D_m \geq \beta D_m \tag{19b}$$

$$W(0, z) \geq W \tag{19c}$$

Mechanism [I]:

$$\Pi_r(0, z) \geq \Pi_r \tag{19d}$$

$$\Pi_p(0, z) \geq \Pi_p \tag{19e}$$

$$\Pi_{sI}(0, z) \geq \Pi_s. \tag{19f}$$

Note that formulation (19) may be infeasible if β is large, for the following reason. If β is large, then the private enterprise must issue a large subsidy to reach the adoption target. However, if the cost of providing a large subsidy exceeds the extra revenue earned by the enterprise (due to increased demand), then the participation constraint (19f) may be violated. Formally, let $\bar{\tau}(x) = \min\{\frac{(1-\alpha c)^2}{4\alpha((d-p)-x(r-d))} - 1, \tau(x)\}$, where $\tau(x) = \frac{(\delta-1)(x+1)}{(d-p)-(r-d)x} - 1$, as defined in §4.1. Then it can be shown that formulation (19) is feasible if and only if $\beta \leq \bar{\tau}(x)$.

5.2 Indirect Mechanism: Optimal Subsidy z^*

In the case where mechanism [I] is feasible (i.e., $\beta \in (0, \bar{\tau}(x)]$), we can solve problem (19) to obtain the optimal subsidy, z^*.

Proposition 2. *Suppose $\beta \leq \bar{\tau}(x)$. Then Mechanism [I] is feasible, and the optimal subsidy is given by:*

$$z^* \quad = \quad \min\{\max\{\underline{z}, \hat{z}\}, \bar{z}\}, \tag{20}$$

where $\underline{z} = \beta((d-p) - x(r-d))$, $\hat{z} = \frac{1}{2}(\frac{(1-\alpha c)^2}{4\alpha} - ((d-p) - x(r-d)))$ and $\bar{z} = \min\{\frac{(1-\alpha c)^2}{4\alpha}, (\delta - 1)(x+1)\} - ((d-p) - x(r-d))$. Further,

(i) When the unit cost c is low, the optimal subsidy z^* strictly increases in x.

(ii) When the unit cost c is high, there exists $\tilde{x}$ such that the optimal subsidy z^* strictly decreases on $x < \tilde{x}$ and strictly increases on $x \geq \tilde{x}$.

The intuition behind Proposition 2 can be understood in a similar fashion as Proposition 1. To elaborate, note that as the last mile distance x increases, the mixed mode adoption (before any government intervention) D_m decreases, which generates two competing effects. First, the "adoption effect" (as described in §4.2) continues to play a role here, meaning the requirement for the corresponding mixed mode commuters $D_m(0,z)$ becomes less stringent as D_m decreases. Consequently, the private enterprise can afford to reduce the subsidy z. Second, as D_m decreases, the private enterprise has incentive to increase z, so that it can recoup the subsidy cost by increasing foot traffic (i.e, a higher $D_m(0,z)$). We shall refer to the latter effect as the "customer effect." Note that the customer effect plays a role in mechanism [I] – because the subsidy z is paid by the private enterprise – but is absent in mechanism [D]. Hence, whether the optimal subsidy z^* increases or decreases in x depends on which effect dominates.

To examine which effect dominates, let us consider the case where the enterprise's unit cost c is small. In this case, the profit margin is high, and the enterprise's profit is highly sensitive to changes in D_m. Therefore, the private enterprise has a stronger incentive to offer a higher subsidy to boost the customer base $D_m(0,z)$, which makes the customer effect dominate the adoption effect. As a result, the subsidy z^* increases in x when c is small. The intuition behind the decrease in z^* when c is large is similar, which we omit for conciseness.

5.3 Comparison: Direct Mechanism [D] versus Indirect Mechanism [I]

By considering Proposition 1 and Proposition 2 presented in §4.2 and §5.2, we can compare the performance of the direct and indirect mechanisms according to each of the six metrics discussed in §4. For ease of reference, we shall use superscript D and I to denote quantities obtained at optimal solutions in mechanisms [D] and [I], respectively. Also, in preparation, let $\tilde{\tau}(x) = \frac{\hat{z}^2-(z^*-\hat{z})^2}{\frac{(1-\alpha c)^2}{4\alpha}((d-p)-(r-d)x)}$. The following result compares the performance of the two mechanisms.

Proposition 3. *Mechanism [I] outperforms mechanism [D] on operating cost ($C^I = 0 < C^D$), mixed mode adoption ($D^I_m \geq D^D_m$), commuter welfare ($W^I > W^D$), ride-hailing platform profit ($\Pi^I_r \geq \Pi^D_r$), and transit agency revenue ($\Pi^I_p \geq \Pi^D_p$). Further,*

- *i) if $\beta \leq \tilde{\tau}(x)$, Mechanism [I] outperforms mechanism [D] on enterprise profit ($\Pi^I_s \geq \Pi^D_s$),*
- *ii) if $\beta \in (\tilde{\tau}(x), \bar{\tau}(x)]$, Mechanism [D] outperforms Mechanism [I] on enterprise profit ($\Pi^I_s < \Pi^D_s$).*

Proposition 3 states that the indirect mechanism [I] dominates the direct mechanism [D] in all performance metrics when the adoption target is not too large, $\beta \leq \tilde{\tau}(x)$. However, in the intermediate case where $\tilde{\tau}(x) < \beta \leq \bar{\tau}(x)$, the direct mechanism [D] will enable the private enterprise to generate a higher profit. Also, recall from Proposition 2 that when the adoption target is high $\beta > \bar{\tau}(x)$, formulation (19) is infeasible, meaning there is no subsidy level z that can satisfy all of the criteria I-VI. This observation enables us to make the following summarizing remark:

Remark 2. *If the public transit adoption target is aggressive $\beta > \bar{\tau}(x)$, then the government should adopt mechanism [D]. If the adoption target is conservative, $\beta \leq \bar{\tau}(x)$, the government should adopt mechanism [I] (even though the private enterprise can earn higher profit under mechanism [D] when β satisfies $\tilde{\tau}(x) < \beta \leq \bar{\tau}(x)$).*

The above remark has the following implications. When the municipal government has a very tight or no budget, the government should set a conservative adoption target $\beta \leq \bar{\tau}(x)$. In this case, the government should adopt the indirect mechanism [I], which leaves all parties better off, and critically, does not require increased spending by the government. However, if the local government has ample funding, then it is feasible to set an aggressive transit adoption target level $\beta \geq \bar{\tau}(x)$, in which case the government should adopt and fund mechanism [D].

6 Alternative Formulation: Maximizing Commuter Welfare Under Budget Neutrality

In §4 and §5, we obtain the optimal incentive schemes based on two different objectives: minimizing operating cost in mechanism [D], and maximizing the private enterprise's profit in mechanism [I]. In this section, we analyze mechanisms [D] and [I] by considering a common objective: maximizing commuter welfare while maintaining budget neutrality.

6.1 Alternative Direct Mechanism [D-A]

Recall from Remark 1 in §4.2 that it is not possible to implement the direct mechanism [D] in a budget neutral manner without undermining commuter welfare. However, when a financially constrained municipality finds it too costly to implement mechanism [D], some loss in commuter welfare may be tolerated (especially if there is public support for policies that lower carbon emissions and traffic congestion). This motivates us to modify the original incentive design problem (16), by instead maximizing commuter welfare subject to a budget neutrality constraint. The modified formulation based on this alternative objective is given by:

Mechanism [D-A]:

$$\max_{e,s\geq 0} \; (W(e,s) - W) \tag{21a}$$

$$\text{s.t. } D_m(e,s) - D_m \geq \beta D_m \tag{21b}$$

$$C(e,s) = 0 \tag{21c}$$

$$\Pi_r(e,s) \geq \Pi_r \tag{21d}$$

$$\Pi_p(e,s) \geq \Pi_p \tag{21e}$$

$$\Pi_{sD}(e,s) \geq \Pi_s. \tag{21f}$$

By solving problem (21), we obtain the following results:

Proposition 4. *The optimal congestion fee e^* and the optimal subsidy s^* under the alternative* *direct mechanism [D-A] are given by:*

$$e^* = \frac{\beta(\beta+1)((d-p)-(r-d)x)^2}{(\delta-1)(x+1)}$$
$$s^* = \frac{\beta((d-p)-(r-d)x)((\delta-1)(x+1)-(\beta+1)((d-p)-(r-d)x))}{(\delta-1)(x+1)}$$

Further,

i) The optimal congestion fee e^* increases in β and decreases in x.

ii) If δ is large, s^* strictly decreases in x. However, if δ is small, there exists $\tilde{x}$ such that s^* increases on $x < \tilde{x}$ and decreases on $x \geq \tilde{x}$.

iii) There exists $\tilde{\beta}$ such that s^* increases on $\beta < \tilde{\beta}$ and decreases on $\beta \geq \tilde{\beta}$.

The above results resemble Proposition 1, except that the optimal subsidy s^* under the alternate mechanism [D-A] is no longer always increasing in β. Instead, the optimal subsidy s^* decreases β when the adoption target β is large. This difference in behavior is driven by the budget neutrality constraint (21c). To elaborate, consider the case when β is large. In this case, the right hand side of (21b) is large, which requires the mixed mode demand $D_m(e, s)$ to be large. As more mixed mode commuters collect the subsidy, the government must reduce the subsidy s in order to maintain budget neutrality, which leads s^* to decrease in the adoption target β.

Next, similar to Corollary 1, we compare the relative size of the optimal congestion fee and subsidy.

Corollary 3. *There exists $\hat{x}$ such that $e^* \geq s^*$ in mechanism [D-A] if and only if $x \leq \hat{x}$. Further, there exists $\bar{\delta} > 1$ such that $s^* > e^*$ in mechanism [D-A] for all $x \in (\underline{x}, \bar{x})$ if and only if $\delta \geq \bar{\delta}$.*

Corollary 3 is analogous to Corollary 1: for any β, the government should set a higher congestion fee and a lower subsidy when the last mile distance x is small, and vice versa when x is large.

Recall from Corollary 2 in §4.2 that the direct mechanism [D] is costly (i.e., $C(e^*, s^*) > 0$) due to the commuter welfare constraint (16c). Next, we present a counterpart to Corollary 2: to maintain budget neutrality, commuter welfare degradation is unavoidable under the alternative direct mechanism [D-A].

Corollary 4. *The alternative direct mechanism [D-A] will always result in commuter welfare degradation; i.e., $W(e^*, s^*) < W$. Further, the reduction in commuter welfare, $W - W(e^*, s^*)$, under mechanism [D-A] is higher when the adoption target β is large and when the last mile distance x is small.*

Corollary 4 reveals that Mechanism [D-A] degrades commuter welfare more severely when the adoption target β is large. To elaborate, consider the case when the adoption target β is large. In this case, the corresponding mixed mode demand $D_m(e, s)$ has to be large in order to meet the adoption target. As more mixed mode commuters collect the subsidy, the government must reduce the subsidy in order to maintain budget neutrality, which causes the commuter welfare to deteriorate. Next, we consider why commuter welfare reduction is higher when x is small. In this case, the mixed mode adoption (before any government intervention) D_m as stated in (3) is large, which makes the requirement for the mixed mode adoption $D_m(e, s)$ also large. Similar to the large β case, as more mixed mode commuters receive the subsidy, the government must increase the congestion fee e in order to maintain budget neutrality, which reduces commuter welfare.

6.2 Alternative Indirect Mechanism [I-A]

In parallel with §6.1, we now consider a variation of mechanism [I], which we shall refer to as mechanism [I-A], where the objective is to maximize commuter welfare while maintaining budget neutrality (which is guaranteed by design under mechanism [I]). To do so, we modify our original program (19) presented in §5.1 into the following formulation:

$$\text{Mechanism [I-A]:} \quad \begin{aligned} &\max_{z\geq 0}\ (W(0,z) - W) && \text{(22a)}\\ &\text{s.t. } W(0,z) \geq W && \text{(22b)}\\ &\quad D_m(0,z) - D_m \geq \beta D_m && \text{(22c)}\\ &\quad \Pi_r(0,z) \geq \Pi_r && \text{(22d)}\\ &\quad \Pi_p(0,z) \geq \Pi_p && \text{(22e)}\\ &\quad \Pi_{sI}(0,z) \geq \Pi_s. && \text{(22f)} \end{aligned}$$

In parallel to Proposition 2, the following proposition characterizes the optimal subsidy under the alternative indirect mechanism [I-A].

Proposition 5. *The alternative indirect mechanism [I-A] is feasible if and only if $\beta \leq \bar{\tau}(x)$. When $\beta \leq \bar{\tau}(x)$, the optimal subsidy is given by*

$$z^* = \frac{(1-\alpha c)^2}{4\alpha} - \max\left\{1, \frac{(1-\alpha c)^2}{4\alpha(\delta-1)(x+1)}\right\}((d-p) - (r-d)x).$$

Further, z^* is constant in β and strictly increases in x.

In contrast to Proposition 2, which showed that z^* can decrease in x when the unit cost c is low, Proposition 5 reveals that under mechanism [I-A], the optimal subsidy z^* is always increasing in x. The intuition is as follows. Because the objective of the alternative indirect mechanism [I-A] is to maximize commuter welfare (instead of enterprise profit as in mechanism [I]), the government has a stronger incentive to set a higher subsidy z than in mechanism [I], provided the enterprise is not made worse-off (as restricted by the participation constraint (22f)).

6.3 Comparison: Two Alternative Mechanisms [D-A] and [I-A]

We now compare the performance of the two alternative mechanisms [D-A] and [I-A]. Using the same approach as presented in §5.3, we can retrieve the relevant performance metrics from (10)-(15) and (18) by substituting $(0, z^*)$ for (e, s) and $(0, z)$.

Proposition 6. *The alternative indirect mechanism [I-A] outperforms the alternative direct mechanism [D-A] on operating cost ($C^I = C^D = 0$), mixed mode adoption ($D_m^I \geq D_m^D$), commuter welfare ($W^I > W^D$), ride-hailing platform profit ($\Pi_r^I \geq \Pi_r^D$), and transit agency revenue ($\Pi_p^I \geq \Pi_p^D$). However, mechanism [D-A] outperforms mechanism [I-A] on enterprise profit ($\Pi_s^D > \Pi_s^I$).*

When the adoption target satisfies $\beta \leq \bar{\tau}(x)$, the alternative direct mechanism [D-A] will enable the private enterprise to generate a higher profit than Mechanism [I-A]. However, when $\beta > \bar{\tau}(x)$, problem (22) becomes infeasible. We summarize our observations in the following remark.

Remark 3. *Suppose in addition to budget neutrality, the government prioritizes commuter welfare. Then the government should adopt the alternative direct mechanism [D-A] when the adoption target satisfies $\beta > \bar{\tau}(x)$, and adopt the alternative indirect mechanism [I-A] when $\beta \leq \bar{\tau}(x)$.*

Remark 2 and Remark 3 imply that the dominance of the indirect mechanism [I] is robust, regardless of whether the goal is to maximize commuter welfare or the enterprise's profit. Therefore, when the municipal government is financially constrained, it is advisable for the government to set a conservative transit adoption target $\beta \leq \bar{\tau}(x)$, and adopt the indirect mechanism [I]. However, if the government has sufficient funding, or if the commuters are willing to accept lower welfare (e.g., in support of a reduction in carbon emissions and traffic congestion), then the government can afford to set an aggressive adoption target $\beta > \bar{\tau}(x)$ and adopt the direct mechanism [D].

7 Conclusion

Motivated by an increased focus on public transit and urban mobility by municipal governments, we analyze two mechanisms for improving public transit ridership. Both mechanisms aim to address the "last mile gap" by providing subsidies to commuters who adopt a mixed mode of transportation that combines ride-hailing with public transit. The main differences between the two mechanisms are the role of the municipal government and the source of funding for the subsidy. In the direct mechanism [D], the government provides a ride-hailing subsidy to commuters that adopt the mixed mode, and charges a congestion fee to commuters who travel by personal vehicle. The congestion fee is used to offset the cost of the subsidy, and also serves as an additional incentive for transit adoption (by making driving more costly). In the indirect mechanism [I], the government partners with a private enterprise that provides the subsidy funding, and does not charge a congestion fee.

We present analytical results that characterize the optimal incentives under each mechanism, as well as the impact on the stakeholders involved.

Our findings offer several prescriptions for policy makers who are interested in increasing public transit ridership through partnerships with the private sector. First, in the direct mechanism, we find that congestion fees are a more efficient intervention when public transit coverage is high (i.e. when the last mile distance is small), while subsidies are more efficient when coverage is low. This result is driven by the opposing effects that subsidies and congestion fees have on commuter welfare and operating cost. We also find that the dependence of the optimal subsidy on the transit coverage level depends on the relative strength of commuter preferences for traveling by a personal vehicle or public transit. In particular, we find that when commuters strongly prefer driving, jurisdictions with larger last mile distances should set lower subsidies, but this behavior can be reversed if commuters only slightly prefer driving over public transit.

Second, we find that the government cannot fully recover the cost of providing subsidies by collecting congestion fees. Specifically, we show that the direct mechanism cannot be budget neutral unless the government is willing to accept a decrease in commuter welfare. This suggests that attempting to implement commuter subsidies and congestion fees in a budget neutral manner may be ill-advised if the government is sensitive to commuter welfare. However, in the event that the government can obtain subsidy funding from a private enterprise (who benefits from increased foot traffic at the transit station) in lieu of charging congestion fees (as in the indirect mechanism), we show that public transit ridership can be increased without degrading commuter welfare. However, because the implementation of an indirect mechanism requires the participation of the private enterprise, we find that this indirect mechanism is only viable if the mixed mode adoption target is modest. In other words, only the direct mechanism can enable the government to achieve an ambitious adoption target, and the implementation of the direct mechanism is not budget neutral: it will require extra funding.

Third, although the government has one less “lever” in the indirect mechanism [I] (due to the absence of the congestion fee), it can dominate the direct mechanism [D] if the adoption target is modest. Specifically, when the adoption target is modest, the indirect mechanism can benefit all stakeholders: the government, commuters, the public transit agency, the ride-hailing platform, and the private enterprise. This finding suggests that, for jurisdictions that wish to increase transit ridership, but are severely budget constrained, it may be more fruitful to fund ride-hailing subsidies through partnerships with the private sector than by charging congestion fees. Moreover, this dominance result is robust to an alternate specification of mechanism [I] (namely, when the objective is to maximize commuter welfare instead of enterprise profit).

Our model has several limitations that deserve further examination as future research. For tractability, we restrict attention to two modes of travel – driving and the mixed mode. Although other means of commuting are less common (such as commuting by ride-hailing), including them make our model intractable and may affect our results. We also do not consider the effect of competition (with respect to either the ride-hailing platform or the private enterprise) in this paper.

We conclude by offering three potential directions for future work. First, it may be fruitful to examine other forms of partnership between the transit agency, ride-hailing platform, and the private enterprise. For example, because the ride-hailing platform benefits from the commuter subsidy (due to increased demand), it may be worthwhile to examine partnerships where the cost of the subsidy is shared between the municipal government and the ride-hailing platform. Second, our work is relevant to other settings where two firms with "complementary capacity" engage in a mutually beneficial partnership. For example, FedEx offers a "SmartPost" delivery service in which FedEx maintains responsibility for the long haul transportation of goods, while the United States Postal Service (USPS) handles the last mile delivery between a USPS center and a customer's home. Our work can serve as a springboard for analyzing partnerships of this nature. Third, budget neutral incentive mechanisms have received relatively little attention in the operations literature, but may be valuable in other settings where government funds are limited. Our findings suggest that an appropriately designed public-private partnership may be a cost-effective solution for governments that wish to improve adoption of other socially beneficial goods or services (e.g. electric vehicles).

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A Proofs

Lemma 1. *In mechanism [D], the following inequality holds for all* (e, s) *such that* $e + s < (\delta - 1)(x + 1) - ((d - p) - (r - d)x)$*:*

$$C(e, s) \geq \frac{(\beta((d - p) - (r - d)x))^2}{2(\delta - 1)(x + 1)}.$$

Proof. Using the definition of $D_m(e, s)$ given in (10), if $e + s < (\delta - 1)(x + 1) - ((d - p) - (r - d)x)$, $D_m(e, s) = \frac{(d-p)-(r-d)x+e+s}{(\delta-1)(x+1)} < 1$ and constraint (16b) implies

$$D_m(e, s) - D_m = \frac{e + s}{(\delta - 1)(x + 1)} \geq \beta D_m = \beta \frac{(d - p) - (r - d)x}{(\delta - 1)(x + 1)}.$$

Because $\delta > 1$ and $x > 0$, it follows that

$$e + s \geq \beta((d - p) - (r - d)x), \tag{23}$$

which holds with equality only if constraint (16b) is binding. Next, using (11),

$$C(e, s) = sD_m(e, s) - eD_d(e, s) = \frac{e((d - p) - (r - d)x - (\delta - 1)(x + 1)) + s((d - p) - (r - d)x)}{(\delta - 1)(x + 1)} + \frac{(e + s)^2}{(\delta - 1)(x + 1)}.$$

Further, using the definition of $U_d(e, s)$ and $U_m(e, s)$ given in (8) and the definition of $W(e, s)$ given in (12), the welfare *increment* is given by

$$W(e, s) - W = \left[\int_0^{v_1(e,s)} U_m(e, s)dV + \int_{v_1(e,s)}^1 U_d(e, s)dV\right] - \left[\int_0^{v_1(0,0)} U_m(0, 0)dV + \int_{v_1(0,0)}^1 U_d(0, 0)dV\right] \tag{24}$$

$$= \frac{e((d - p) - (r - d)x - (\delta - 1)(x + 1)) + s((d - p) - (r - d)x)}{(\delta - 1)(x + 1)} + \frac{(e + s)^2}{2(\delta - 1)(x + 1)}. \tag{25}$$

It follows that for all (e, s),

$$C(e, s) = (W(e, s) - W) + \frac{(e + s)^2}{2(\delta - 1)(x + 1)}. \tag{26}$$

Because constraint (16c) implies $W(e, s) - W \geq 0$, (26) implies $C(e, s) \geq \frac{(e+s)^2}{2(\delta-1)(x+1)}$, which holds with equality only when constraint (16c) is binding. Further, because $x < \bar{x}$, $(d - p) - (r - d)x > 0$. It follows from (23) that for all (e, s),

$$C(e, s) \geq \frac{(e+s)^2}{2(\delta-1)(x+1)} \geq \frac{(\beta((d-p)-(r-d)x))^2}{2(\delta-1)(x+1)}, \tag{27}$$

which holds with equality if and only if both constraints (16b) and (16c) are binding. □

Proof of Proposition 1. The proof is composed of two parts. In the first part, we find the optimal solution (e^*, s^*). In the second part, we prove the comparative statics results. To solve for (e^*, s^*), we must consider two cases from the definition of $D_m(e, s)$ in (10): (i) $D_m(e, s) < 1$ and (ii) $D_m(e, s) = 1$. Then we show that the optimal cost in case (i) is lower than in case (ii).

(i) Note that $D_m(e, s) < 1$ if and only if

$$e + s < (\delta - 1)(x + 1) - ((d - p) - (r - d)x), . \tag{28}$$

Thus, we assume (28) holds in the remainder of the proof of this case. To find the optimal solution, it suffices to construct a feasible solution (e_1, s_1) that attains the lower bound in Lemma 1. Let (e_1, s_1) be given by the unique solution to the equations

$$D_m(e, s) - D_m = \beta D_m \tag{29}$$

$$W(e, s) = W, \tag{30}$$

which correspond to constraints (16b) and (16c). Solving for (e_1, s_1) yields a unique solution:

$$e_1 = \frac{\beta(\beta+2)((d-p)-(r-d)x)^2}{2(\delta-1)(x+1)}$$
$$s_1 = \frac{\beta((d-p)-(r-d)x)(2(\delta-1)(x+1)-(\beta+2)((d-p)-(r-d)x))}{2(\delta-1)(x+1)}.$$

Next, we show that (e_1, s_1) is feasible. By construction (e_1,s_1) satisfies constraints (16b) and (16c) with equality. Note that $e_1 + s_1 = \beta((d-p) - (r-d)x)$. Because $0 < \beta < \tau(x)$, it satisfies constraint (28). Because $x < \bar{x}$, it follows that $e_1 + s_1 = \beta((d-p) - (r-d)x) > 0$. Next, for any (e, s), the revenue *increment* of the transit agency, ride-hailing platform, nearby merchants are given by

$$\Pi_p(e, s) - \Pi_p = p\frac{e+s}{(\delta-1)(x+1)}$$
$$\Pi_r(e, s) - \Pi_r = rx\frac{e+s}{(\delta-1)(x+1)}$$
$$\Pi_{sD}(e, s) - \Pi_{sD}(0, 0) = \frac{(1-\alpha c)^2}{4\alpha}\frac{e+s}{(\delta-1)(x+1)},$$

which follow from (13), (14) and (15), respectively. Note $\Pi_p(e, s) - \Pi_p \geq 0$, $\Pi_r(e, s) - \Pi_r \geq 0$ and $\Pi_{sD}(e, s) - \Pi_{sD} \geq 0$ if $e + s \geq 0$. Because $(e_1, s_1) > 0$, (e_1, s_1) satisfies constraints (16d), (16e) and (16f). Hence, (e_1, s_1) is feasible to (16). Because formulation (16) minimizes $C(e, s)$, (e_1, s_1) achieves the lower bound of $C(e, s)$ presented in Lemma 1, and (e_1, s_1) is feasible to (16), it follows that (e_1, s_1) is optimal to (16), in the case where $D_m(e, s) < 1$.

(ii) In the case where $D_m(e,s)=1$, we have $D_d(e,s)=0$ and $v_1(e,s)=1$. Constraint (16c) can then be rewritten as

$$W(e,s)-W=\int_0^1 U_m(e,s)dV-\left[\int_0^{v_1(0,0)} U_m(0,0)dV+\int_{v_1(0,0)}^1 U_d(0,0)dV\right] \tag{31}$$

$$=s-\frac{((\delta-1)(x+1)-((d-p)-x(r-d)))^2}{2(\delta-1)(x+1)}. \tag{32}$$

Let $\phi=\frac{((\delta-1)(x+1)-((d-p)-x(r-d)))^2}{2(\delta-1)(x+1)}$. Therefore, the constraint (16c) requires that (e,s) satisfies $s\geq\phi$. Note $C(e,s)=s$, using (11) and the fact that $D_m(e,s)=1$. Therefore, $C(e,s)\geq\phi$. Using the fact that $0<\beta<\tau(x)$, it is straightforward to verify algebraically that $\phi>C(e_1,s_1)$. Therefore, the optimal solution to (16) is $(e^*,s^*)=(e_1,s_1)$.

Next, we show the comparative statics results for (e^*,s^*) with respect to x and β.

(i) It is straightforward to verify that $\frac{\partial e^*}{\partial\beta}=\frac{(\beta+1)((d-p)-(r-d)x)^2}{(\delta-1)(x+1)}>0$. By taking the first derivative of e^* with respect to x, we have $\frac{\partial e^*}{\partial x}=\frac{\beta(\beta+2)\left((x+1)^2(r-d)^2-(r-p)^2\right)}{2(\delta-1)(x+1)^2}$. Because $x\in(\underline{x},\bar{x})$ implies

$$\max\left\{0,\frac{(d-p)-(\delta-1)}{(\delta-1)+(r-d)}\right\}<x<\frac{d-p}{r-d}, \tag{33}$$

and $r>d>p>0$, it follows that $(x+1)^2(r-d)^2<(r-d+d-p)^2=(r-p)^2$. Hence $\frac{\partial e^*}{\partial x}<0$. Therefore, e^* strictly increases in β and strictly decreases in x.

(ii) Using the expression for s^* from Proposition 1, we have $\frac{\partial s^*}{\partial\beta}=\frac{((d-p)-(r-d)x)((\delta-1)(x+1)-(\beta+1)((d-p)-(r-d)x))}{(\delta-1)(x+1)}$. Because $\delta>1$, $\beta<\tau(x)$ implies $\beta\leq\frac{(\delta-1)(x+1)}{(d-p)-(r-d)x}-1$ and $x\in(\underline{x},\bar{x})$ implies (33) holds, it follows from straightforward algebra that $\frac{\partial s^*}{\partial\beta}>0$. For the derivative of s^* with respect to x, first note that $\frac{\partial^2 s^*}{\partial x^2}=-\frac{\beta(\beta+2)(r-p)^2}{(\delta-1)(x+1)^3}<0$, meaning s^* is strictly concave in x and $\frac{\partial s^*}{\partial x}$ decreases in x. We next check the first derivatives at the boundaries, i.e. $\left.\frac{\partial s^*}{\partial x}\right|_{x=\bar{x}}$ and $\left.\frac{\partial s^*}{\partial x}\right|_{x=\underline{x}}$. Note $\left.\frac{\partial s^*}{\partial x}\right|_{x=\bar{x}}=-\beta(r-d)<0$. If $\delta<\delta_1=1+d-p$, then $\underline{x}=\frac{(d-p)-(\delta-1)}{(\delta-1)+(r-d)}$, and thus $\left.\frac{\partial s^*}{\partial x}\right|_{x=\underline{x}}=\frac{1}{2}\beta((\beta+2)(\delta-1)+2(\beta+1)(r-d))>0$; if $\delta\geq\delta_1$, then $\underline{x}=0$, and thus $\left.\frac{\partial s^*}{\partial x}\right|_{x=\underline{x}}=-\frac{\beta(2(\delta-1)(r-d)-(\beta+2)(d-p)(2r-d-p))}{2(\delta-1)}$. Similarly, if $\delta<\delta_2=\frac{(\beta+2)(d-p)(2r-d-p)}{2(r-d)}+1$, $\left.\frac{\partial s^*}{\partial x}\right|_{x=0}>0$; if $\delta\geq\delta_2$, $\left.\frac{\partial s^*}{\partial x}\right|_{x=0}\leq 0$. Because $\delta_2-\delta_1=\frac{\beta(d-p)(2r-d-p)+2(d-p)(r-p)}{2(r-d)}>0$, we have $\delta_2>\delta_1$. Therefore, if $\delta\geq\delta_2$, $\left.\frac{\partial s^*}{\partial x}\right|_{x=\underline{x}}\leq 0$, and if $1<\delta<\delta_2$, $\left.\frac{\partial s^*}{\partial x}\right|_{x=\underline{x}}>0$. Therefore, by continuity of $\frac{\partial s^*}{\partial x}$, if $\delta\geq\delta_2$, then s^* strictly decreases in $x\in(\underline{x},\bar{x})$. If $\delta<\delta_2$, then there exists $\tilde{x}\in(\underline{x},\bar{x})$ such that s^* strictly increases in x on $(\underline{x},\tilde{x})$ and strictly decreases in x on $[\tilde{x},\bar{x})$.

The result in Proposition 1 follows. □

Proof of Corollary 1. Using the expressions for (e^*,s^*) in Proposition 1, we have

$$e^*-s^*=\beta((d-p)-(r-d)x)\left(\frac{(\beta+2)((d-p)-(r-d)x)}{(\delta-1)(x+1)}-1\right).$$

Because $x\in(\underline{x},\bar{x})$, $(d-p)-(r-d)x>0$. Further, because $\delta>1$, we have $e^*\geq s^*$ if and only if

$x \leq \bar{x} - \frac{(\delta-1)(r-p)}{(r-d)((\beta+2)(r-d)+\delta-1)}$. The result follows by setting $\hat{x} = \bar{x} - \frac{(\delta-1)(r-p)}{(r-d)((\beta+2)(r-d)+\delta-1)}$. Next, note that because $\delta > 1$, $r > p$ and $r > d$, $\hat{x} < \bar{x}$. Define $\bar{\delta} = (\beta+2)(d-p)+1$, which satisfies $\bar{\delta} > 1$. Then it remains to show that $\underline{x} \leq \hat{x}$ if and only if $\delta \geq \bar{\delta}$. From the definition of $\underline{x}$, if $\delta < 1+d-p$, then $\underline{x} = \frac{(d-p)-(\delta-1)}{(\delta-1)+(r-d)}$. It is straightforward to check that $\hat{x} \in (\underline{x}, \bar{x})$. If $\delta \geq 1+d-p$, then $\underline{x} = 0$ and it's follows that $\hat{x} \leq \underline{x}$ only if $\delta \geq \bar{\delta}$, where $\bar{\delta} > 1+d-p$. To conclude, $s^* > e^*$ holds for all $x \in (\underline{x}, \bar{x})$ if and only if $\hat{x} \leq \underline{x}$, which holds if and only if $\delta \geq \bar{\delta}$. □

Proof of Corollary 2. Using the expressions for e^* and s^* from Proposition 1 and the cost function given in (11), the total cost under (e^*, s^*) is $C(e^*, s^*) = \frac{\beta^2((d-p)-(r-d)x)^2}{2(\delta-1)(x+1)}$. Because $x < \bar{x}$ and $\delta > 1$, $C(e^*, s^*) > 0$. By noting that $\frac{\partial C(e^*,s^*)}{\partial \beta} = \frac{\beta((d-p)-x(r-d))^2}{(\delta-1)(x+1)} > 0$, we conclude that $C(e^*, s^*)$ increases in β. Next, $\frac{\partial C(e^*,s^*)}{\partial x} = \frac{\beta^2\left((x+1)^2(r-d)^2-(r-p)^2\right)}{2(\delta-1)(x+1)^2}$. Then from the assumption $x \in (\underline{x}, \bar{x})$, we have $0 < (x+1)(r-d) < (r-p)$. It follows that $\frac{\partial C(e^*,s^*)}{\partial x} < 0$ and that $C(e^*, s^*)$ decreases in x. □

Proof of Proposition 2. The proof is composed of three steps: (i) constructing the sufficient and necessary condition for problem (19) to be feasible, (ii) solving for its optimal solution z^* and (iii) conducting the comparative statics analysis of z^*.

(i) First, define $\tilde{z} = (\delta-1)(x+1) - ((d-p)-(r-d)x)$. To find all the feasible solutions to problem (19), we divide the proof into two cases: (i.a) when $D_m(0,z) \leq 1$, or equivalently, $z \leq \tilde{z}$ and (i.b) when $D_m(0,z) = 1$, or equivalently, $z > \tilde{z}$. In preparation, because $\underline{x} < x < \bar{x}$ and $\delta > 1$, note that $(\delta-1)(x+1) > ((d-p)-x(r-d)) > 0$.

(i.a) With an additional constraint of $z \leq \tilde{z}$, from the definition in (10), $D_m(0,z) = \frac{(d-p)-(r-d)x+z}{(\delta-1)(x+1)}$. We first show that any $z \in [0, \tilde{z}]$ satisfies constraints (19c)-(19e). Using the welfare expression given in (12), the commuter welfare increment is $W(0,z) - W = \frac{z(2(d-p)-2(r-d)x+z)}{2(\delta-1)(x+1)}$. Because $\delta > 1$ and $x \in (\underline{x}, \bar{x})$, we have $W(0,z) - W \geq 0$ if $z \geq 0$. Similarly, using the demand expression given in (3), $D_m(0,z) - D_m = \frac{z}{(\delta-1)(x+1)} \geq 0$ if $z \geq 0$. Using (13) and (14), $\Pi_p(0,z) - \Pi_p = p(D_m(0,z) - D_m)$ and $\Pi_r(0,z) - \Pi_r = rx(D_m(0,z) - D_m)$. Therefore, if $z \geq 0$, $\Pi_p(0,z) - \Pi_p \geq 0$ and $\Pi_r(0,z) - \Pi_r \geq 0$. Next, we define $\underline{z} = \beta((d-p)-x(r-d))$ and $\bar{z} = \min\{\frac{(1-\alpha c)^2}{4\alpha}, (\delta-1)(x+1)\} - ((d-p)-x(r-d))$. Then we show that constraint (19b) is satisfied if $z \geq \underline{z}$, and constraint (19f) is satisfied if $z \leq \bar{z}$. First, using the definition of D_m, it is straightforward to verify that constraint (19b) holds if and only if $\frac{z}{(\delta-1)(x+1)} \geq \beta\frac{(d-p)-(r-d)x}{(\delta-1)(x+1)}$. Because $\delta > 1$, it follows that constraint (19b) holds if and only if $z \geq \beta((d-p)-x(r-d))$, or equivalently, if and only if $z \geq \underline{z}$. Given that $x \leq \bar{x}$, we obtain $\underline{z} > 0$. Lastly, using the revenue expression in (18), when restricting $z \leq \tilde{z}$, it is straightforward to verify that constraint (19f) holds if and only if $z \leq \frac{(1-\alpha c)^2}{4\alpha} - ((d-p)-x(r-d))$, which can be rewritten as $z \leq \bar{z} = \min\{\frac{(1-\alpha c)^2}{4\alpha}, (\delta-1)(x+1)\} - ((d-p)-(r-d)x)$. Thus, given $z \leq \tilde{z}$, problem (19) has feasible region $[\underline{z}, \bar{z}]$.

(i.b) With an additional constraint of $z > \tilde{z}$, $D_m(0,z) = 1$. We first prove all $z > \tilde{z}$ satisfies constraints (19b)-(19e). Note

$$D_m(0,z) - D_m = 1 - \frac{(d-p)-(r-d)x}{(\delta-1)(x+1)} \geq \beta\frac{(d-p)-(r-d)x}{(\delta-1)(x+1)}$$

holds if and only if $\beta \leq \tau(x)$. From the definition in (10), if $z > \tilde{z}$, then

$$\begin{aligned} W(0,z) - W &= \frac{(\delta-1)(x+1)((x+1)(2d-\delta) - 2p - 2rx + 2z + x + 1) - (-d(x+1) + p + rx)^2}{2((\delta-1)(x+1))} \\ &> \frac{(\delta-1)^2(x+1)^2 - ((d-p) - x(r-d))^2}{2((\delta-1)(x+1))} \\ &\geq 0. \end{aligned}$$

Similarly, using (13) and (14), $\Pi_p(0,z) - \Pi_p = p\,(1 - D_m) > 0$ and $\Pi_r(0,z) - \Pi_r = rx\,(1 - D_m) > 0$. Next, let $\dot{z} = \frac{(1-\alpha c)^2}{4\alpha}(1 - \frac{(d-p)-(r-d)x}{(\delta-1)(x+1)})$. Using (18) and (7), given $z > \tilde{z}$, constraint (19f) is satisfied if and only if $z \leq \dot{z}$. Thus given $z > \tilde{z}$, problem (16) has feasible region $(\tilde{z}, \dot{z}]$. To summarize, the feasible region is $[\underline{z}, \bar{z}] \cup (\tilde{z}, \dot{z}]$. Next, note that if $\frac{(1-\alpha c)^2}{4\alpha} < (\delta-1)(x+1)$, then $\bar{z} = \frac{(1-\alpha c)^2}{4\alpha} - ((d-p) - (r-d)x)$ and $\dot{z} < (\delta-1)(x+1) - ((d-p) - (r-d)x) = \tilde{z}$, in which case the feasible region is $[\underline{z}, \frac{(1-\alpha c)^2}{4\alpha} - ((d-p) - (r-d)x)]$. However, if $\frac{(1-\alpha c)^2}{4\alpha} \geq (\delta-1)(x+1)$, then $\bar{z} = (\delta-1)(x+1) - ((d-p) - (r-d)x) = \tilde{z} \leq \dot{z}$, in which case the feasible region is $[\underline{z}, \frac{(1-\alpha c)^2}{4\alpha}(1 - \frac{(d-p)-(r-d)x}{(\delta-1)(x+1)})]$. Let

$$\bar{\bar{z}} = \frac{(1-\alpha c)^2}{4\alpha} - \max\left\{1, \frac{(1-\alpha c)^2}{4\alpha(\delta-1)(x+1)}\right\}((d-p) - (r-d)x).$$

Then the feasible region can be expressed as $[\underline{z}, \bar{\bar{z}}]$. Note that formulation (19) is feasible if and only if $\underline{z} \leq \bar{\bar{z}}$ and $\beta \leq \tau(x)$. With some effort, it can be shown that $\underline{z} < \bar{\bar{z}}$ and $\beta \leq \tau(x)$ if and only if $\beta \leq \min\{\frac{(1-\alpha c)^2}{4\alpha((d-p)-(r-d)x)} - 1, \tau(x)\}$. By definition, $\bar{\tau}(x) = \min\{\frac{(1-\alpha c)^2}{4\alpha((d-p)-(r-d)x)} - 1, \tau(x)\}$. Therefore, (19) is feasible if and only if $\beta \leq \bar{\tau}(x)$.

(ii) Next we solve for the optimal solution z^*. To do so, we first find the equivalent formulation to problem (19) by reducing the constraints to a single feasible region constraint. From part (i) above, for any β that satisfies $0 < \beta \leq \bar{\tau}(x)$, if $\frac{(1-\alpha c)^2}{4\alpha} \geq (\delta-1)(x+1)$, $\bar{\bar{z}} = \frac{(1-\alpha c)^2}{4\alpha}\left(1 - \frac{(d-p)-(r-d)x}{(\delta-1)(x+1)}\right) \geq \tilde{z}$ and it follows that the feasible region is $\left[\underline{z}, \frac{(1-\alpha c)^2}{4\alpha}\left(1 - \frac{(d-p)-(r-d)x}{(\delta-1)(x+1)}\right)\right]$. Note that when $z \in [\tilde{z}, \frac{(1-\alpha c)^2}{4\alpha}\left(1 - \frac{(d-p)-(r-d)x}{(\delta-1)(x+1)}\right)]$, from the definition in (18), we have $\Pi_{sI}(0,z) = \frac{(1-\alpha c)^2}{4\alpha} - z$, which decreases in z. It follows that $\Pi_{sI}(0,\tilde{z}) > \Pi_{sI}(0,z)$ for all $z > \tilde{z}$. Therefore, when $\frac{(1-\alpha c)^2}{4\alpha} \geq (\delta-1)(x+1)$, to maximize $\prod_{sI}(0,z)$, the optimal solution z^* should satisfy $z^* \in [\underline{z}, \tilde{z}]$. Also from part (i) above, if $\frac{(1-\alpha c)^2}{4\alpha} < (\delta-1)(x+1)$, then $\bar{\bar{z}} = \frac{(1-\alpha c)^2}{4\alpha} - ((d-p) - (r-d)x) < \tilde{z}$ and the optimal solution z^* lies in the feasible region and satisfies $z^* \in \left[\underline{z}, \frac{(1-\alpha c)^2}{4\alpha} - ((d-p) - (r-d)x)\right]$. To conclude, z^* must satisfy $z^* \in [\underline{z}, \min\left\{\frac{(1-\alpha c)^2}{4\alpha}, (\delta-1)(x+1)\right\} - ((d-p) - x(r-d))]$, or equivalently, $z^* \in [\underline{z}, \bar{z}]$. Thus formulation (19) can be expressed as:

$$\max \Pi_{sI}(0,z) \text{ s.t. } z \in [\underline{z}, \bar{z}]. \tag{34}$$

Using the profit expression given in (18), and the fact that $\delta > 1$, we have $\frac{\partial^2 \Pi_{sI}(0,z)}{\partial z^2} = -\frac{2}{(\delta-1)(x+1)} < 0$. Hence, $\Pi_{sI}(0,z)$ is strictly concave in z. The solution to the first order condition $\frac{\partial \Pi_{sI}(0,z)}{\partial z} = 0$ is

$\hat{z} = \frac{(1-\alpha c)^2}{8\alpha} - \frac{1}{2}((d-p) - (r-d)x)$. Therefore, the optimal subsidy z^* is given by

$$z^* = \begin{cases} \underline{z} & \text{if } \hat{z} < \underline{z}, \\ \hat{z} & \text{if } \underline{z} \le \hat{z} \le \bar{z}, \\ \bar{z} & \text{if } \bar{z} < \hat{z}, \end{cases}$$

or, equivalently, $z^* = \min\{\max\{\underline{z}, \hat{z}\}, \bar{z}\}$.

(iii) Because $\bar{z} = \min\left\{\frac{(1-\alpha c)^2}{4\alpha}, (\delta-1)(x+1)\right\} - ((d-p)-x(r-d))$, we can rewrite z^* as $z^* = \min\{\max\{\underline{z}, \hat{z}\}, \bar{z}_1, \bar{z}_2\}$, where $\bar{z}_1 = \frac{(1-\alpha c)^2}{4\alpha} - ((d-p) - x(r-d))$ and $\bar{z}_2 = (\delta-1)(x+1) - ((d-p) - x(r-d))$. Note $\frac{\partial \underline{z}}{\partial x} = -\beta(r-d) < 0$, $\frac{\partial \hat{z}}{\partial x} = \frac{r-d}{2} > 0$, $\frac{\partial \bar{z}_1}{\partial x} = r-d > 0$ and $\frac{\partial \bar{z}_2}{\partial x} = (\delta-1) + (r-d) > 0$. Because $\underline{z}, \hat{z}, \bar{z}_1$, and $\bar{z}_2$ are all continuous in x, z^* is continuous in x. Therefore, if $z^* = \underline{z}$, z^* decreases in x; otherwise, z^* increases in x. It remains to show the conditions under which $z^* = \underline{z}$. In preparation, define $\pi \equiv \frac{(1-\alpha c)^2}{4\alpha}$. Let $\tilde{x} = \bar{x} - \frac{\pi}{(2\beta+1)(r-d)}$, $\pi_1 = (\bar{x} - \underline{x})(2\beta+1)(r-d)$ and $c_1 = \frac{1-\sqrt{4\alpha(\bar{x}-\underline{x})(2\beta+1)(r-d)}}{\alpha}$. Note that $\tilde{x} < \bar{x}$ by definition of $\tilde{x}$ and because $r > d$. Also, it can be verified that $\pi \le \pi_1$ if and only if $c \ge c_1$ because $\alpha c < 1$. First, note that $z^* = \underline{z}$ if and only if $\hat{z} \le \underline{z}$ (because $\underline{z} \le \bar{z}$ given $\beta \le \bar{\tau}(x)$), which holds if and only if $x \le \tilde{x}$. Because $x \in (\underline{x}, \bar{x})$, $x \le \tilde{x}$ can hold only if $\tilde{x} > \underline{x}$. It can be verified that $\tilde{x} > \underline{x}$ if and only if $\pi < \pi_1$, or equivalently, $c > c_1$. Therefore, if $c > c_1$, $z^* = \underline{z}$ decreases in x on $(\underline{x}, \tilde{x}]$ and increases in x on $(\tilde{x}, \bar{x})$; if $c \le c_1$, z^* increases in x on $(\underline{x}, \bar{x})$.

The result follows. □

Proof of Proposition 3. By Corollary 2, we have $C^D = C(e^*, s^*) > 0$, and by construction of mechanism [I], $C^I = C(0, z^*) = 0$. Hence, $C^I = 0 < C^D$. Next, using the expressions for e^* and s^* given in Proposition 1, and the expression for z^* given in Proposition 2, it can be shown that $D_m(0, z^*) - D_m \ge \beta D_m = D_m(e^*, s^*) - D_m$, or equivalently, $D_m^I \ge D_m^D$. Because $D_m^I \ge D_m^D$, from (5) and (6) we immediately obtain $\Pi_p^I \ge \Pi_p^D$ and $\Pi_r^I \ge \Pi_r^D$, respectively.

To compare the commuter welfare in mechanism [D] and mechanism [I], it's easy to check $W^D = W$ because constraint (16c) is binding in equilibrium. Furthermore, $W^I - W = W(0, z^*) - W = \frac{z^*(2(d-p)-2(r-d)x+z^*)}{2(\delta-1)(x+1)}$, which satisfies $W^I - W > 0$ if and only if $z^* > 0$. To see $z^* > 0$ holds, let us first define $\gamma_1 = \frac{(1-\alpha c)^2}{4\alpha} > 0, \gamma_2 = (d-p) - x(r-d) > 0$, and $\gamma_3 = (\delta-1)(x+1) > 0$ for ease of exposition. Then $\underline{z} = \beta\gamma_2 > 0$, $\hat{z} = \frac{\gamma_1 - \gamma_2}{2}$, $\bar{z} = \min\{\gamma_1 - \gamma_2, \gamma_3 - \gamma_2\}$ and $\bar{\tau}(x) = \min\{\frac{\gamma_1}{\gamma_2} - 1, \frac{\gamma_3}{\gamma_2} - 1\}$. For mechanism [I] to be feasible and correspondingly $0 < \beta \le \bar{\tau}(x)$ to hold, we only focus on the regime where $\bar{\tau}(x) > 0$. Using the definitions of γ_1, γ_2, and γ_3, it can be shown that $\bar{\tau}(x) > 0$ holds if and only if $\gamma_1 > \gamma_2$ and $\gamma_3 > \gamma_2$. It follows $\hat{z} = \frac{\gamma_1 - \gamma_2}{2} > 0$ and $\bar{z} = \min\{\gamma_1 - \gamma_2, \gamma_3 - \gamma_2\} > 0$. Therefore, $z^* = \min\{\max\{\underline{z}, \hat{z}\}, \bar{z}\} > 0$ and it follows that $W^I > W = W^D$.

As for the enterprise's profit, using definitions in (15) and (18), we calculate $\Pi_s^I - \Pi_s^D = \frac{\hat{z}^2 + \gamma_1\gamma_2 - (z^* - \hat{z})^2}{\gamma_3} - \frac{\gamma_1\gamma_2(\beta+1)}{\gamma_3}$. Define $\tilde{\tau}(x) = \frac{\hat{z}^2 - (\hat{z} - z^*)^2}{\gamma_1\gamma_2}$. Then $\Pi_s^I < \Pi_s^D$ holds if and only if $\beta > \tilde{\tau}(x)$. Because $\beta \in (0, \bar{\tau}(x)]$ holds for feasibility of mechanism [I], to see there exists β such that $\beta > \tilde{\tau}(x)$, it remains to show $0 \le \tilde{\tau}(x) \le \bar{\tau}(x)$, for all x. By definition of z^*, we prove $0 \le \tilde{\tau}(x) \le \bar{\tau}(x)$ is true no matter $z^* = \underline{z}$, $z^* = \hat{z}$ or $z^* = \bar{z}$, as follows. Before that, observe because $0 < \beta \le \bar{\tau}(x)$ by assumption, it follows that $\underline{z} \le \bar{z}$. Next, we consider three cases:

(i) If $0 < \hat{z} \leq \underline{z} \leq \bar{z}, z^* = \underline{z}$. Then it follows that $\tilde{\tau}(x) = \frac{\hat{z}^2-(\hat{z}-\underline{z})^2}{\gamma_1\gamma_2} = \frac{2\underline{z}\hat{z}-\underline{z}^2}{\gamma_1\gamma_2} \leq \frac{\underline{z}^2}{\gamma_1\gamma_2} \leq \frac{\bar{z}^2}{\gamma_1\gamma_2} = \frac{(\min\{\gamma_1,\gamma_3\}-\gamma_2)^2}{\gamma_1\gamma_2} < \frac{(\min\{\gamma_1,\gamma_3\}-\gamma_2)\min\{\gamma_1,\gamma_3\}}{\gamma_1\gamma_2} = \bar{\tau}(x)\min\{1,\frac{\gamma_3}{\gamma_1}\} \leq \bar{\tau}(x)$. Further, because $\beta \leq \bar{\tau}(x) = \min\left\{\frac{\gamma_1}{\gamma_2}-1, \frac{\gamma_3}{\gamma_2}-1\right\}$, it follows that $\beta\gamma_2 \leq \gamma_1-\gamma_2$, or equivalently, $z^* = \underline{z} \leq 2\hat{z}$. Thus $\tilde{\tau}(x) = \frac{\hat{z}^2-(\underline{z}-\hat{z})^2}{\gamma_1\gamma_2} \geq 0$.

(ii) If $0 < \underline{z} \leq \hat{z} \leq \bar{z}, z^* = \hat{z}$, $\tilde{\tau}(x) = \frac{\hat{z}^2}{\gamma_1\gamma_2} \leq \frac{\bar{z}^2}{\gamma_1\gamma_2}$. From 1, it follows that $\tilde{\tau}(x) \leq \frac{\bar{z}^2}{\gamma_1\gamma_2} < \bar{\tau}(x)$. It's also easy to check $\tilde{\tau}(x) > 0$.

(iii) If $0 < \underline{z} \leq \bar{z} \leq \hat{z}, z^* = \bar{z}$. Because $\bar{z} = \min\{\gamma_1-\gamma_2, \gamma_3-\gamma_2\}$ and $\hat{z} = \frac{\gamma_1-\gamma_2}{2} < \gamma_1-\gamma_2$, then for $\bar{z} \leq \hat{z}$ to hold, it must be true that $\bar{z} = \gamma_3-\gamma_2 < \gamma_1-\gamma_2$, or equivalently, $\gamma_3 < \gamma_1$. Then $\bar{\tau}(x) = \frac{\gamma_3}{\gamma_2}-1$. It follows $\tilde{\tau}(x) = \frac{2\bar{z}\hat{z}-\bar{z}^2}{\gamma_1\gamma_2} = \frac{2(\gamma_3-\gamma_2)\frac{\gamma_1-\gamma_2}{2}-(\gamma_3-\gamma_2)^2}{\gamma_1\gamma_2} = (\frac{\gamma_3}{\gamma_2}-1)(1-\frac{\gamma_3}{\gamma_1}) < \bar{\tau}(x)$. It's also easy to check $\tilde{\tau}(x) > 0$.

From above, we then conclude that if $\beta \leq \tilde{\tau}(x)$, $\Pi_s^I \geq \Pi_s^D$ and if $\beta \in (\tilde{\tau}(x), \bar{\tau}(x)]$, $\Pi_s^I < \Pi_s^D$. □

Proof of Proposition 4. Similar to Proposition 1, the proof is composed of two parts. First, we find the optimal solutions (e^*, s^*). Second, we prove the comparative statics results.

To solve for (e^*, s^*), we consider two cases: (i) when $D_m(e,s) < 1$, and (ii) when $D_m(e,s) = 1$. We then show that the optimal commuter welfare is higher in case (i) than in case (ii).

(i) First consider the case where $D_m(e,s) < 1$. The proof proceeds in two steps. First, we construct an upper bound for the optimal value of the objective function (21a), and second, we construct a feasible solution that attains this bound. From (26) and constraint (21c), we have

$$C(e,s) = (W(e,s) - W) + \frac{(e+s)^2}{2(\delta-1)(x+1)} = 0.$$

Constraint (21b) here is the same as constraint (16e) in formulation (16), which can be rewritten as $e + s \geq \beta((d-p)-(r-d)x)$. Note $\beta((d-p)-(r-d)x) > 0$ because $x \in (\underline{x}, \bar{x})$. Therefore,

$$W(e,s) - W = -\frac{(e+s)^2}{2(\delta-1)(x+1)} \leq -\frac{(\beta((d-p)-x(r-d)))^2}{2(\delta-1)(x+1)},$$

where the inequality holds with equality if and only if constraint (21b) is binding.

Next, we prove that the upper bound is attainable. We first solve for (e,s) such that constraint (21b) is binding and constraint (21c) is satisfied:

$$\begin{aligned} &D_m(e,s) - D_m = \beta D_m \\ &C(e,s) = 0. \end{aligned} \tag{35}$$

This yields the unique solution (e_2, s_2), where

$$\begin{aligned} e_2 &= \frac{\beta(\beta+1)((d-p)-(r-d)x)^2}{(\delta-1)(x+1)} \\ s_2 &= \frac{\beta((d-p)-(r-d)x)((\delta-1)(x+1)-(\beta+1)((d-p)-(r-d)x))}{(\delta-1)(x+1)}. \end{aligned}$$

It follows that $e_2 + s_2 < (\delta-1)(x+1) - ((d-p)-(r-d)x)$ from $0 < \beta < \tau(x)$. To see that (e_2, s_2) is feasible, we first prove $e_2 > 0$ and $s_2 > 0$. Because $\delta > 1$ and $x < \bar{x}$, we have $e_2 > 0$.

Because $0 < \beta < \tau(x)$, $\beta + 1 \leq \frac{(\delta-1)(x+1)}{(d-p)-(r-d)x}$, and so $s_2 > 0$. Using a parallel argument to the proof of Proposition 1, it can be verified that constraints (21d), (21e) and (21f) are satisfied by (e_2, s_2). Hence, (e_2, s_2) is feasible. Because the objective is to maximize the commuter welfare increment $\Delta W(e,s) = W(e,s) - W$, (e_2, s_2) achieves the upper bound, and (e_2, s_2) is feasible to (21), (e_2, s_2) is the unique optimal solution to formulation (21), in the case where $D_m(e,s) < 1$. The corresponding optimal commuter welfare increment is $\Delta W(e_2, s_2) = -\frac{(\beta((d-p)-x(r-d)))^2}{2(\delta-1)(x+1)}$.

(ii) In the case where $D_m(e,s) = 1$, we have $D_d(e,s) = 0$. Then the budget neutrality constraint (21c) can be rewritten as

$$C(e,s) = sD_m(e,s) - eD_d(e,s) = s = 0. \tag{36}$$

From $D_m(e,s) = 1$, it follows that $e \geq (\delta-1)(x+1) - ((d-p)-(r-d)x)$. From definition in (12), the commuter welfare increment is

$$\Delta W(e,s) = \int_0^1 U_m(e,s)dV - \left[\int_0^{v_1(0,0)} U_m(0,0)dV + \int_{v_1(0,0)}^1 U_d(0,0)dV\right] \tag{37}$$

$$= -\frac{((\delta-1)(x+1) - ((d-p)-x(r-d)))^2}{2(\delta-1)(x+1)}. \tag{38}$$

Note $\Delta(e,s) < \Delta W(e_2, s_2)$, because $0 < \beta < \tau(x)$. Therefore, the unique optimal solution to (21) is $(e^*, s^*) = (e_2, s_2)$.

Next, we show the comparative statics results for (e^*, s^*) with respect to x and β.

(i) We first take the first derivative of e^* with respect to x and β. Because $\delta > 1$, $0 < x < \bar{x}$ and $r > d > p > 0$, it follows that $\frac{\partial e^*}{\partial \beta} = \frac{(2\beta+1)((d-p)-(r-d)x)^2}{(\delta-1)(x+1)} > 0$. Thus, e^* increases in β. Next, $\frac{\partial e^*}{\partial x} = \frac{\beta(\beta+1)\left((x+1)^2(r-d)^2-(r-p)^2\right)}{(\delta-1)(x+1)^2}$. Because $r - p = (d-p) + (r-d) > (r-d)(x+1)$, it follows that $\frac{\partial e^*}{\partial x} < 0$. Therefore, e^* decreases in x.

(ii) Next, for the optimal subsidy s^*, we have $\frac{\partial s^*_{D2}(x,\beta)}{\partial \beta} = \frac{((d-p)-(r-d)x)((\delta-1)(x+or1)-(2\beta+1)((d-p)-(r-d)x))}{(\delta-1)(x+1)}$. Then when $\beta < \frac{(\delta-1)(x+1)}{2((d-p)-(r-d)x)} - \frac{1}{2} = \frac{\beta_D(x)}{2}$, $\frac{\partial s^*}{\partial \beta} > 0$, and when $\beta \geq \frac{\beta_D(x)}{2}$, $\frac{\partial s^*}{\partial \beta} \leq 0$. Therefore, s^* increases in $\beta \in (0, \frac{\tau(x)}{2})$ and decreases in $\beta \in [\frac{\tau(x)}{2}, \tau(x))$. Next, for the change in s^* with respect to x, first note that s^* is concave in x: $\frac{\partial^2 s^*}{\partial x^2} = -\frac{2\beta(\beta+1)(p-r)^2}{(\delta-1)(x+1)^3} < 0$. Then to show the result,it suffices to check the first derivatives at the boundaries, i.e. $\left.\frac{\partial s^*(x,\beta)}{\partial x}\right|_{x=\bar{x}}$ and $\left.\frac{\partial s^*(x,\beta)}{\partial x}\right|_{x=\underline{x}}$. If $\delta < \delta_1 = 1 + d - p$, then $\underline{x} = \frac{(d-p)-(\delta-1)}{(\delta-1)+(r-d)}$, and $\left.\frac{\partial s^*(x,\beta)}{\partial x}\right|_{x=\underline{x}} = \beta((\beta+1)(\delta-1) + (2\beta+1)(r-d)) > 0$; if $\delta \geq \delta_1$, then $\underline{x} = 0$, and $\left.\frac{\partial s^*_{DA}(x,\beta)}{\partial x}\right|_{x=\underline{x}} = \frac{\beta((\beta+1)(d-p)(2r-d-p)-(\delta-1)(r-d))}{\delta-1}$. If $\delta < \delta_3 = \frac{(\beta+1)(d-p)(2r-d-p)}{r-d} + 1$, $\left.\frac{\partial s^*}{\partial x}\right|_{x=\underline{x}} > 0$; if $\delta \geq \delta_3$, $\left.\frac{\partial s^*}{\partial x}\right|_{x=\underline{x}} \leq 0$. Notice $\delta_3 - \delta_1 = \frac{(d-p)(\beta(2r-d-p)+r-p)}{r-d} > 0$, or $\delta_3 > \delta_1$. We then conclude that if $\delta \geq \delta_3$, $\left.\frac{\partial s^*(x,\beta)}{\partial x}\right|_{x=\underline{x}} \leq 0$ and if $1 < \delta < \delta_3$, $\left.\frac{\partial s^*(x,\beta)}{\partial x}\right|_{x=\underline{x}} > 0$. Therefore, by continuity of $\frac{\partial s^*}{\partial x}$, it follows that when $\delta \geq \delta_3$, s^* decreases in $x \in (\underline{x}, \bar{x})$. When $\delta < \delta_3$, there exists $\tilde{x} \in (\underline{x}, \bar{x})$ such that s^* strictly increases in x on $(x, \tilde{x})$ and strictly decreases in x on $[\tilde{x}, \bar{x})$.

The result follows. □

Proof of Corollary 3. Note $e^* - s^* = \beta((d-p)-(r-d)x)\left(\frac{2(\beta+1)((d-p)-(r-d)x)}{(\delta-1)(x+1)} - 1\right)$. Because $x \in (\underline{x}, \bar{x})$, it follows $(d-p)-(r-d)x > 0$. Therefore, $e^* \geq s^*$ if and only if $\frac{2(\beta+1)((d-p)-(r-d)x)}{(\delta-1)(x+1)} - 1 \geq 0$. It follows that $e^* \geq s^*$ if and only if $x \leq \bar{x} - \frac{(\delta-1)(r-p)}{(r-d)(2(\beta+1)(r-d)+\delta-1)}$. Defining $\hat{x} = \bar{x} - \frac{(\delta-1)(r-p)}{(r-d)(2(\beta+1)(r-d)+\delta-1)}$ yields the main result. Following a parallel argument to the proof of Corollary 1, define $\bar{\bar{\delta}} = 2(\beta+1)(d-p)+1$, which satisfies $\bar{\bar{\delta}} > 1$. It can then be shown that $s^* > e^*$ for all $x \in (\underline{x}, \bar{x})$ if and only if $\hat{x} \leq \underline{x}$, which holds if and only if $\delta \geq \bar{\bar{\delta}}$. □

Proof of Corollary 4. By plugging in the optimal solutions (e^*, s^*) in Proposition 4, the change in commuter welfare is given by

$$\Delta W(e^*, s^*) = W(e^*, s^*) - W = -\frac{\beta^2((d-p)-(r-d)x)^2}{2(\delta-1)(x+1)} < 0.$$

Also, by noting that $\frac{\partial \Delta W(e^*,s^*)}{\partial \beta} = -\frac{\beta((d-p)-x(r-d))^2}{(\delta-1)(x+1)} < 0$, we conclude that $\Delta W(e^*, s^*)$ strictly decreases in β. Next, $\frac{\partial \Delta W(e^*,s^*)}{\partial x} = \frac{\beta^2\left((r-p)^2-(x+1)^2(r-d)^2\right)}{2(\delta-1)(x+1)^2}$. Because $x \in (\underline{x}, \bar{x})$, then $0 < (x+1)(r-d) < (r-p)$. It follows that $\frac{\partial \Delta W(e^*,s^*)}{\partial x} > 0$ and that $\Delta W(e^*, s^*)$ strictly increases in x. Therefore, the commuter welfare decrease $W - W(e^*, s^*) = -\Delta W(e^*, s^*)$ is higher when the adoption target β is large, or when the last mile distance x is small. □

Proof of Proposition 5. Because formulation (19) and (22) share the same set of constraints, mechanism [I-A] is feasible if and only if mechanism [I] is feasible, which holds if and only if $\beta \leq \bar{\tau}(x)$, where $\bar{\tau}(x) = \min\{\frac{(1-\alpha c)^2}{4\alpha((d-p)-x(r-d))} - 1, \frac{(\delta-1)(x+1)}{(d-p)-(r-d)x} - 1\}$ (as defined in §5.1). From the proof of Proposition 2, the feasible region of problem (19) is $[\underline{z}, \bar{\bar{z}}]$, which is also the feasible region of (22). From the definition in (10) and (12), the objective function is

$$W(0,z) - W = \begin{cases} \frac{z(2(d-p)-2x(r-d)+z)}{2(\delta-1)(x+1)}, & \text{if } z < (\delta-1)(x+1) - ((d-p)-(r-d)x) \\ z - \frac{((\delta-1)(x+1)-((d-p)-x(r-d)))^2}{2(\delta-1)(x+1)} & \text{if } z \geq (\delta-1)(x+1) - ((d-p)-(r-d)x) \end{cases} \tag{39}$$

which is continuous and increasing in z. Therefore, the optimal subsidy is

$$z^* = \bar{\bar{z}} = \frac{(1-\alpha c)^2}{4\alpha} - \max\left\{1, \frac{(1-\alpha c)^2}{4\alpha(\delta-1)(x+1)}\right\}((d-p)-(r-d)),$$

which, by inspection, is constant in β and increasing in x. □

Proof of Proposition 6. First, we compare the performances of the two mechanisms in metrics I $\sim$ V: By design, we have $C^I = C^D = 0$. Next, using the expressions for e^* and s^* given in Proposition 4, and the expression for z^* given in Proposition 5, it can be shown that $D_m(0,z^*) - D_m \geq \beta D_m = D_m(e^*, s^*) - D_m$, or equivalently, $D_m^I \geq D_m^D$. Because $D_m^I \geq D_m^D$, from (5) and (6) we immediately obtain $\Pi_p^I \geq \Pi_p^D$ and $\Pi_r^I \geq \Pi_r^D$, respectively. Recall from Corollary 4 that, $W(e^*, s^*) - W < 0$, and then $W^D < W$. Using the same argument from the proof of Proposition 3, because $\bar{\tau}(x) > 0$, it follows that $z^* = \bar{\bar{z}} > 0$ and thus

$$W^I - W = W(0, z^*) - W = \frac{z^*(2(d-p) - 2(r-d)x + z^*)}{2(\delta-1)(x+1)} > 0.$$

Therefore $W^I > W > W^D$. Finally, to compare the enterprise's profit under the two mechanisms, we first calculate the profit of mechanism [I-A] using the definition in (18). We consider two cases: when $D_m(0, z^*) < 1$ and $D_m(0, z^*) = 1$. It can be shown that $D_m(0, z^*) < 1$ implies $z^* = \frac{(1-\alpha c)^2}{4\alpha} - ((d-p) - (r-d)x)$, and thus

$$\Pi_{sI}(0, z^*) = \left(\frac{(1-\alpha c)^2}{4\alpha} - z^*\right) D_m(0, z^*) = \frac{(1-\alpha c)^2}{4\alpha} \frac{(d-p) - (r-d)x}{(\delta - 1)(x+1)} = \Pi_s.$$

If instead $D_m(0, z^*) = 1$, then $z^* = (\delta - 1)(x+1) - ((d-p) - (r-d)x)$, and thus

$$\Pi_{sI}(0, z^*) = \left(\frac{(1-\alpha c)^2}{4\alpha} - z^*\right) = \frac{(1-\alpha c)^2}{4\alpha} \frac{(d-p) - (r-d)x}{(\delta - 1)(x+1)} = \Pi_s.$$

Therefore, the enterprise's optimal profit under mechanism [I-A] is $\Pi_s^I = \Pi_s$. Next, the enterprise's profit under mechanism [D-A] is $\Pi_s^D = \Pi_{sD}(e^*, s^*) = (1+\beta)\Pi_s$. Because $\beta > 0$, we conclude $\Pi_s^D > \Pi_s^I$. □