Gender-Based Operational Issues Arising from On-Demand Ride-Hailing Platforms: Safety Concerns, Service Systems, and Pricing and Wage Policy

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Problem definition: Motivated by the recent development of "women only" ride-hailing platforms that aim to address the safety concerns raised by female riders and drivers, we examine whether a platform such as Uber, Lyft or Didi should offer gender-specific services (i.e., a "dedicated" system in which riders are matched with drivers of the same gender) instead of the current gender-neutral services (i.e., a "pooling" system in which riders are matched with drivers randomly without taking their gender into consideration).

Academic/Practical Relevance: We make an initial attempt to examine if and when gender-specific services can enable the platform to address female safety concerns and improve its profitability.

Methodology: We use a game theoretic framework to model two system configurations: (a) a gender-neutral "pooling" system in which riders and drivers are matched randomly without taking their gender into consideration; and (b) a gender-specific "dedicated" system in which riders are matched with drivers of the same gender. For each system configuration, we analyze a two-stage queueing game by first determining the respective equilibrium joining and participating behavior of riders and drivers. Then, we derive the platform's optimal pricing and wage decisions to maximize its profit.

Results: First, we obtain the following two findings regarding the pooling system: one, enhancing the rider-side safety confidence level increases the effective arrival rate of female riders while enhancing that of the driver side may reduce the number of joined female riders. Two, the platform's profit is convex and increasing with the rider-side safety confidence level but concave and increasing with that of the driver side. Second, when female riders and drivers have a higher safety confidence level, the gender-neutral pooling system yields the highest profit for the platform because of the underlying "pooling effect". However, when female riders (or drivers) have a lower safety confidence level, the gender-specific dedicated system is optimal. More importantly, when operating a gender-specific dedicated system under a gender-based pricing and wage policy, we show that, relative to their male counterparts, it is optimal for the platform to charge female riders a lower price and pay female drivers a higher wage especially when female drivers are in short supply.

Managerial Implications: Gender-specific dedicated systems can be a profitable way for the ride-hailing platform to address safety concerns raised by female riders and drivers. We pinpoint that a key obstacle for the platform to move to the dedicated system is the limited female labor size, which reduces the system's economies of scale. The "equal policy" under which the platform has to impose the same price and wage on both the female subsystem and the male subsystem

further burdens the platform.

Keywords: Sharing Economy; Pooling System; Dedicated System; Gender-based Pricing and Wage; Strategic Queueing Behavior.

1 Introduction

On-demand ride-hailing platforms such as Didi, Uber, Lyft, etc. offer convenience for riders and work flexibility for independent drivers; however, Uber, Lyft and Didi received complaints from female riders (drivers, respectively) for being harassed sexually by male drivers (riders, respectively) (Feeney 2015). Over the last 3 years, there were a series of reports about female riders (drivers, respectively) who were sexually assaulted, raped, or murdered by male drivers (riders, respectively) (Samuels 2016, Levin 2017, ChinaDaily 2016, Zhang and Munroe 2018). As female riders and drivers raised safety concerns (Feng et al. 2017), O'Brien (2018) argued that ride-hailing platforms must address female safety problems. In 2018, Uber pledged to re-run background checks on drivers and enhanced its app with new ability to share trip with "trusted contacts" and a button to call the police (Devine 2018). There are gender-specific solutions: Didi announced in 2018 that, for its car-pooling service, drivers can only pick up riders of the same gender in early morning and late evening hours (Al-Heeti 2018). Also, to capitalize on female's safety concerns, new startups SheTaxis, Safr, and Chariot for Women (United States), She Cabs (India), and She'Kab (Pakistan) offer women-only (riders and drivers) ride-hailing services.

The recent development of gender-specific ride-hailing services has motivated us to examine and compare the operational and financial performances between the current gender-neutral "pooling" system (that matches riders and drivers randomly without taking their gender into consideration) and a gender-specific "dedicated system" (that matches riders and drivers of the same gender). Clearly, the pooling system has a shorter waiting time due to the "extra" pooling of both genders, but it may lose some riders and drivers due to their safety concerns. However, while female's safety concerns are absent in the dedicated system, it may lose some female riders due to longer waiting time because the pooling effect is absent and female drivers are often in short supply. Note that females account for only 2% of drivers but 60% of customers in the taxi and delivery industry (SheRides 2016). By taking these tradeoffs and challenges into consideration, we examine the following research questions:

1. When operating the gender-neutral pooling system, what is the impact of female safety concerns on the rider's demand, the driver's supply, and the platform's profitability?

2. Under what conditions should a platform adopt the gender-specific dedicated system?

3. Given female drivers are in short supply, should the gender-specific dedicated system adopt a gender-based pricing and wage policy?[1]

As an initial attempt to explore the aforementioned questions, we consider a situation wherein riders are price- and waiting-time-sensitive and drivers are wage-sensitive. Also, relative to male counterparts, female riders (and drivers) have safety concerns when they are matched with male drivers (and riders). We use the safety confidence level of the riders (drivers, respectively) to measure the degree of safety for riders (drivers, respectively). Given this backdrop, we develop a two-stage queueing game model (Hassin and Haviv 2003, Hassin 2016) to examine the following two system configurations: (a) a gender-neutral "pooling" system wherein riders and drivers are matched randomly without taking their gender into consideration. Riders pay the same price and drivers receive the same wage regardless of their gender. That is, the pooling system adopts the "*gender-neutral pricing and wage policy*"(in short, the "*equal policy*"); and (b) a gender-specific "dedicated" system under which riders are matched with drivers of the same gender. In the dedicated system, the platform may adopt the "gender neutral equal policy" to decide its price and wage (same as that under the pooling system) or makes the " *gender-based pricing and wage* decisions" (in short, the "*unequal policy*"). For each system configuration, we analyze a two-stage game by first deriving the respective equilibrium joining and participating behavior of riders and drivers. We then determine the platform's optimal pricing and wage decisions to maximize its profit.[2]

As one more driver joins the workforce, it brings a negative *same-side externality* to other drivers due to fiercer competition and brings a positive *cross-side externality* to riders because more riders can be attracted to join. The *aggregated externality*, measured by the impact on the demand rate per driver, is shown to be positive in Taylor (2018) where the gender-based safety concern is absent. Here, the riders and drivers in our system exhibit gender-based safety concerns and thus, their relationship is more complicated. Due to females' safety concerns, the cross-side externality is not only affected by the supply- and demand-side size but also by their *gender composition*. We show that a marginal increase of the female driver size still results in a positive aggregated externality on other female drivers. However, a marginal increase of the male driver size may result in a negative aggregated externality on female drivers because it reduces the female riders' expected

[1]Because the pooling system operates as a single legal entity to coordinate riders and drivers of both genders, gender-based pricing and wage policy is deemed discriminatory and it may be illegal. However, in a dedicated system, the platform can always operate two gender-specific services separately under two different legal entities. As such, it is possible for the platform to adopt the gender-based pricing and wage policy.

[2]Under the dedicated system with the gender-based pricing and wage policy, the price and wage are gender-based.

reward due to their safety concerns. These externality relationships, coupled with the platform's profit-maximizing decision, yield the following two interesting results. One, under the platform's profit-maximizing pricing and wage decisions, enhancing the rider-side safety confidence level can indeed attract more female riders to join whereas enhancing the driver-side safety confidence level can actually reduce the number of joined female riders. The main reason is as follows. On the one hand, a higher driver-side safety confidence level entices more female drivers to participate and hence attracts more female riders to join. One the other hand, the platform is less worried about keeping the size of the female driver workforce by maintaining a high fraction of female riders. Thus, the platform can increase its price, which turns away some female riders. Two, we show that the platform's profit is *convex and increasing* with the rider-side safety confidence level but *concave and increasing* with the driver-side safety confidence level. Note that the platform's profit is the product of two terms, its profit margin (per-service price minus per-service wage) and the total effective arrival rate of joining riders. We show that increasing the rider-side safety confidence level enlarges both terms, leading to the convexity result. However, increasing the driver-side safety confidence level enlarges the profit margin but reduces the total effective arrival rate. The net effect of these two counter-forces yields the concavity result. These findings might be helpful for the ride-hailing platform to conduct its cost-benefit analysis when enhancing the driver- and rider-side safety levels.

We find that the platform's preference over pooling and dedicated systems is jointly determined by two effects, namely *mismatch effect* and *pooling effect*. The mismatch effect is mainly determined by the rider- and driver-side safety confidence levels. As the safety confidence levels increase, the gender-mismatch effect diminishes and we show that only when they are higher than certain threshold levels is the pooling system preferred over the dedicated system. As platforms like Didi often operate multiple business modules such as Didi Premier, Didi Express and Didi Carpool (Hitch), and the platform has different degrees of control over drivers such as background checking and training, the platform can adopt dedicated systems for those low-safety-level business module such as Carpool and maintain pooling for those high-safety-level modules such as Didi Premier. The pooling effect in our two-sided pooling system comes from two sources. One, as demonstrated in the traditional queueing literature, it comes from the stochastic characteristics of arrival and service processes: in the dedicated systems, it is possible that at one moment, the male subsystem has idle riders while the female subsystem faces the shortage of drivers, but at the other moment, things are reversed. Two, it comes from the economies of scale as the pooling system has a larger size of drivers and riders. With a larger pool size of drivers, the demand rate per driver is larger and thus the platform only needs to pay a relatively low wage. However, when the pooling system is divided into two gender-specific dedicated systems, the female dedicated system will face a heavy shortage of supply and the demand rate per driver is also significantly reduced. The platform has to offer a much high wage to compensate female drivers, and hence has less incentives to operate such a dedicated system. The gender-neutral "equal policy" will add insult to injury: under this policy, the platform has to pay the same high wage to the male drivers. In summary, the shortage of female workforce is a key factor hindering the adoption of female and male dedicated systems and the "equal policy" makes it worse.

We also find that, when operating gender-specific dedicated systems under a gender-based pricing and wage policy, relative to their male counterparts, it is optimal for the platform to charge female riders a lower price and pay female drivers a higher wage especially when female drivers are in short supply. Our findings provide some insights into the conditions when the gender-specific ride-hailing system and the gender-based pricing and wage policy are beneficial to the riders, drivers, and platforms.

The paper is organized as follows. Section 2 surveys the relevant literature. Section 3 presents our model setup and notations. Two dedicated systems associated with the "equal" and "unequal" pricing and wage policies are studied in Section 4, and the pooling system is analyzed in Section 5. Section 6 compares these three systems. Concluding remarks are provided in Section 7. All the proofs are relegated to the Appendix.

2 Literature Review

Our paper is closely related to two papers that study ride-hailing platforms in a two-sided market. (For the research development about two-sided markets, see Armstrong (2006), Rochet and Tirole (2006), Weyl (2010), Hagiu (2014), Hagiu and Wright (2015), Eisenmann et al. (2006) and the references therein).[3] Taylor (2018) investigates how the rider's delay sensitivity and the driver's self-scheduled independence affect the platform's optimal price and wage decisions, and identifies a positive externality when the number of drivers increases. Our paper differs from Taylor (2018) in that riders and drivers are of two types (male and female) and, due to the safety concern, the externality brought by male drivers on female drivers may be negative. Besides, our focus is to examine the impact on the platform profitability of enhancing safety confidence levels and

[3]By focusing on a one-sided market (in which the driver's decision is not considered), Kostami et al. (2017) study the pricing and capacity allocation decisions of a service provider when customers' utility is affected by the number of customers in the system and the gender composition of the customer group.

re-arranging the service system structure. The second closely-related paper is Benjaafar et al. (2018), which investigate the impact of the labor pool size, delay cost and drivers' opportunity cost variability on labor welfare. Their main conclusion is that expanding the driver pool size is beneficial to the drivers in some cases but hurts them in other cases. Indeed, the labor pool size in our paper is also a critical factor determining the performance of pooling and dedicated structures. In addition, our study also has other driving forces such as mismatch effect between demand and supply and different cross-side externalities between different demand-supply pairs.

Besides Taylor (2018) and Benjaafar et al. (2018), there are many other studies examining various operational issues arising from on-demand service platforms that use different pricing and wage policies to coordinate dynamic rider demand and independent driver supply. Banerjee et al. (2015) and Cachon et al. (2017) investigate the value of dynamic pricing. By using a queueing model, Banerjee et al. (2015) show that static pricing outperforms dynamic pricing when riders have heterogeneous valuation. In contrast, Cachon et al. (2017) find that surge pricing performs well when drivers have heterogeneous opportunity costs. Under market uncertainty, Hu and Zhou (2017) examine the optimality of the fixed commission contract. By considering the case when riders are price- and waiting-time-sensitive, Bai et al. (2018) show that a demand-based commission contract is optimal. Wang et al. (2018) consider the carpooling problem in sharing economy. They investigate a hybrid model under which riders can choose between sharing a ride with others and being served privately. They find a tipping point that characterizes riders' optimal choosing behavior. Jacob and Roet-Green (2017) study the pricing problem in ride-sharing by explicitly considering the cost of sharing, which represents all the disutility induced by sharing a ride with other riders. Gao et al. (2018) consider that the riders and drivers can exchange the roles based on the relative magnitude of supply and demand and study when a user of car sharing shall be a driver/rider. Cohen and Zhang (2018) study a market that contains two two-sided platforms with users being drivers and riders, where the two platforms simultaneously determine their prices and wages to compete for users, and users choose the services based on a multinomial logit choice model. They show that a cooperation between the two platforms through a well-designed profit sharing contract can benefit all the parties in the market.

3 Model Preliminaries

Consider an on-demand service platform that sets the price rate p (measured in terms of price per service) and the wage rate w (measured in terms of wage per service) to coordinate price- and waiting-time-sensitive riders (i.e., demand) and earning-sensitive independent drivers (i.e., supply) of both genders, female (labeled f) and male (labeled m).

Pooling and Dedicated Systems. There are two potential system configurations that the platform may adopt; see Figure 1 for the illustration. In a gender-neutral pooling system (denoted by $\mathcal{P}$), riders are matched with drivers randomly without taking their gender into consideration. Hence, the safety concerns of female riders and drivers are "present" in the pooling system. Moreover, because the pooling system will be operated as a single legal entity, gender-based pricing and wage policy is deemed discriminatory and may be illegal. For this reason, it suffices to consider the "*gender-neutral (equal) pricing and wage policy*" when the platform operates as a pooling system. However, in the gender-specific dedicated systems, female's safety concerns are "absent" because riders are matched with drivers of the same gender. Also, the platform can always operate the dedicated systems as two separate legal entities. Thus, the platform may adopt the "gender-based pricing and wage policy" so that the price p and wage w are gender-specific. For this reason, we shall examine both gender-neutral and gender-based pricing and wage policies when the platform operates as a dedicated system. If the dedicated system adopts the same "equal policy" as that in the pooling system, we denote such system as $\mathcal{DE}$. However, if the dedicated system adopts the "*unequal policy*", i.e., the *gender-based pricing and wage policy*, we denote such system as $\mathcal{DU}$.

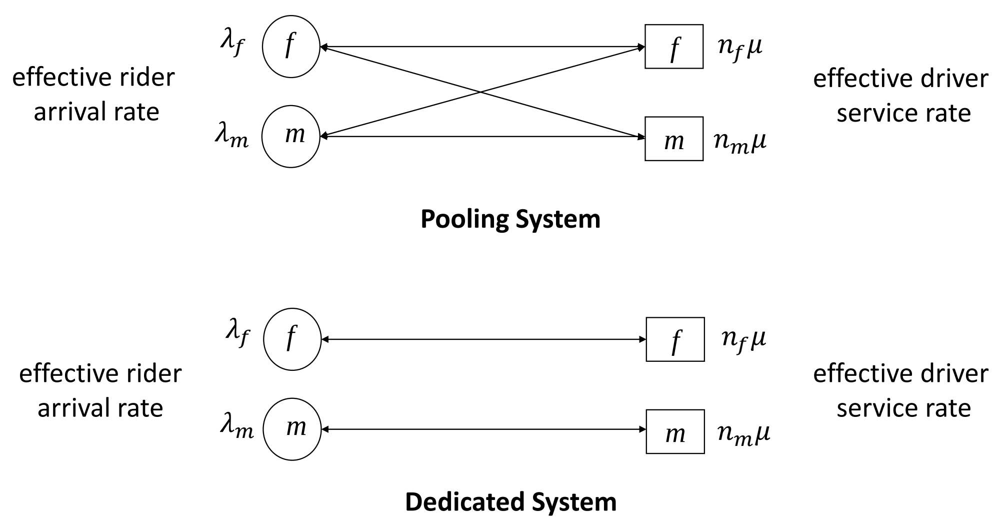

Figure 1: Gender-Neutral Pooling System and Gender-Specific Dedicated Systems

Rider Characteristics. The maximum potential female and male riders may request for the service according to independent Poisson processes with rates Λ_f and Λ_m, respectively. Denote the maximum total potential arrival rate as Λ. Then, $\Lambda = \Lambda_m + \Lambda_f$. Given the price p and anticipating the waiting cost $c \cdot W$, where W is the expected waiting time and c is the waiting cost per unit time, some riders may choose not to request for the service. Denote the "effective" arrival rate of gender-i riders, $i = f, m$ as λ_i, and $\lambda_i \leq \Lambda_i$. To simplify our exposition, we assume that riders of both genders are homogeneous in the sense that they receive the same "base reward" R from the service offered by the dedicated system. However, in the pooling system, due to the presence of females' safety concerns, female riders receive a lower reward αR when they are matched with male drivers. Here, $\alpha \in (0, 1]$ denotes the *female rider's safety confidence level* towards the ride-hailing service offered by the male drivers. A larger α indicates a higher safety confidence level.

Driver Characteristics. There are N_f and N_m female and male registered drivers, each of whom can serve a rider according to an exponential distribution with service rate μ (e.g., the number of riders served per unit time). Throughout this paper, we assume that $N_f < N_m$ as observed in practice (SheRides 2016). We also assume that all registered drivers of both genders are homogeneous in the sense that they have the same "reservation price" r. They will participate and serve if their earning rate is at least as large as r.[4] Due to the safety concern of female drivers, we assume that female drivers will discount the earning rate by a factor $\beta \in (0, 1]$ when they are matched with male riders in the pooling system. Akin to α, parameter β denotes the *female driver's safety confidence level*, which is higher when β is larger. Let n_f and n_m denote the "effective" number of female and male drivers who participate in the platform.

Waiting Time. For any system configuration that has an effective rider arrival rate λ and an effective driver service rate $n\mu$ (that is, n effective drivers), we assume that the system behaves according to an M/M/1 queueing model.[5] Then, the expected waiting time in system $W(\lambda, n)$ satisfies

$$W(\lambda, n) = \begin{cases} \frac{1}{n\mu - \lambda}, & \text{if } \lambda < n\mu \\ +\infty, & \text{otherwise.} \end{cases} \tag{1}$$

Rider and Driver Utilities in the Dedicated System. Recall from Figure 1 that, in a dedicated system, riders are matched with drivers of the same gender so that females' safety concerns are absent. Hence, for a gender-i dedicated system that has an effective rider arrival rate λ_i and an effective driver service rate $n_i\mu$ (that is, n_i effective drivers), $i = f, m$, each rider of gender i

[4]Our model can be extended to the case where drivers have heterogeneous reservation prices, but the analysis becomes intractable.

[5]This assumption has also been adopted by Benjaafar et al. (2018) for obtaining tractable results.

receives a utility $U_i(\lambda_i, n_i)$ as follows:

$$U_i(\lambda_i, n_i) = R - p - cW(\lambda_i, n_i), \quad i = f, m, \tag{2}$$

where R is the base reward, p is the price (which can be gender-specific if the platform adopts the gender-based pricing and wage policy), and $cW(\lambda_i, n_i)$ is the waiting cost associated with the waiting time $W(.,.)$ as stated in (1). Similarly, in the dedicated system, each driver of gender i is busy with utilization rate $\lambda_i/(n_i\mu)$ and has a service rate μ. Along with the wage per service w, each driver of gender i receives a net utility $S_i(\lambda_i, n_i)$ as follows:

$$S_i(\lambda_i, n_i) = \frac{\lambda_i}{n_i\mu} \cdot \mu \cdot w - r = \frac{\lambda_i}{n_i} w - r, \quad i = f, m, \tag{3}$$

where r is the reservation price and w can be gender-specific when the platform adopts the gender-based pricing and wage policy.

Rider and Driver Utilities in the Pooling System. In the pooling system, riders are matched with drivers without taking their gender into consideration. Therefore, females' safety concerns are present. As such, for a pooling system that has an effective rider arrival rate $(\lambda_f + \lambda_m)$ and an effective driver service rate $(n_f + n_m)\mu$, each male rider (with no safety concern) receives a utility $U_m(\lambda_f + \lambda_m, n_f + n_m)$ as follows:

$$U_m(\lambda_f + \lambda_m, n_f + n_m) = R - p - cW(\lambda_f + \lambda_m, n_f + n_m). \tag{4}$$

However, given n_f and n_m effective female and male drivers in the pooling system, a female rider will discount the base reward R by α when she is matched with a male driver with probability $n_m/(n_f + n_m)$. Thus, each female rider receives an expected utility $U_f(\lambda_f + \lambda_m, n_f + n_m)$ that satisfies:

$$U_f(\lambda_f + \lambda_m, n_f + n_m) = \frac{n_m}{n_m + n_f}\alpha R + \frac{n_f}{n_m + n_f} R - p - cW(\lambda_f + \lambda_m, n_f + n_m). \tag{5}$$

By noting that male drivers have no safety concern and yet female drivers have safety concerns if they are matched with male riders, we can use the same approach as above to show that the male driver's net utility $S_m(\lambda_f + \lambda_m, n_f + n_m)$ and the female driver's net utility $S_f(\lambda_f + \lambda_m, n_f + n_m)$ satisfy

$$S_m(\lambda_f + \lambda_m, n_f + n_m) = \frac{\lambda_m + \lambda_f}{n_f + n_m} w - r, \quad \text{and} \tag{6}$$

$$S_f(\lambda_f + \lambda_m, n_f + n_m) = \frac{\lambda_m}{n_f + n_m}\beta w + \frac{\lambda_f}{n_f + n_m} w - r = \frac{\beta\lambda_m + \lambda_f}{n_f + n_m} w - r. \tag{7}$$

For notational convenience, we let

$$d_m(n_f, n_m) := \frac{\lambda_m + \lambda_f}{n_f + n_m} \text{ and } d_f(n_f, n_m) := \frac{\beta\lambda_m + \lambda_f}{n_f + n_m}. \tag{8}$$

Then, $d_m(n_f, n_m)$ can be regarded as the *demand rate* for a male driver while $d_f(n_f, n_m)$ is the *safety-adjusted demand rate* for a female driver.

Sequence of Events and Platform's Problem. For both pooling and dedicated systems, the game sequence is as follows. First, the platform sets the price p and wage w (which is gender-based when appropriate). Then, upon observing the price and wage, riders and drivers of each gender decide whether to join and participate, respectively.

We use backward induction to derive the operational and financial performances associated with three systems, $\mathcal{P}$, $\mathcal{DE}$ and $\mathcal{DU}$. First, for any give price and wage, we use the utilities stated above to derive the effective arrival rate of riders of each gender and the effective participation rate of drivers of each gender in equilibrium. Specifically, in equilibrium, riders will join and drivers will participate only if their net utilities are non-negative. To be consistent with the real situation that the ride-hailing system is *supply-constrained*, we shall assume that $\frac{\Lambda_i}{N_i\mu} > 1$ for $i = f, m$. Similar conditions are also assumed in Banerjee et al. (2015) and Taylor (2018). Besides, we also assume that $\frac{\Lambda_m}{N\mu} < 1$, where $N = N_f + N_m$, so that there is sufficient capacity for female riders to join. Anticipating the equilibrium joining and participating behavior of riders and drivers, the platform then decides the price p_i charged to gender-i riders and wage w_i paid to gender-i drivers, $i \in \{f, m\}$, to maximize its total expected profit in each system K, $K = \mathcal{P}, \mathcal{DE}, \mathcal{DU}$ as follows:

$$\Pi_K = \sum_{i=f,m} \lambda_i(p_i - w_i).$$

Note that under the pooling system $\mathcal{P}$ and the dedicated system $\mathcal{DE}$, $p_f = p_m = p$ and $w_f = w_m = w$. Throughout the paper, we restrict attention to the parameter range in which the platform's expected profit under the optimal price and wage is strictly positive. For ease of reference, we summarize the key notations in Table 1.

4 Analysis of the Dedicated System

In a dedicated system, because riders are matched with drivers of the same gender, females' safety concerns are absent. The corresponding analysis is less intricate than that of the pooling system. To explicate our analysis, we first focus on the dedicated system in this section. We shall move to the pooling system in the next section.

Table 1: Key Notations

i	Gender, $i \in \{f, m\}$
Λ_i	Potential arrival rate of gender i customers
$\Lambda = \Lambda_m + \Lambda_f$	Total potential customer arrival rate
λ_i	Effective arrival rate of gender i riders
N_i	Number of gender i registered drivers
$N = N_m + N_f$	Total number of registered drivers
n_i	Effective number of gender i drivers
μ	Service rate of each independent driver
r	Reservation price of each independent driver
R	Base reward of each rider for using the service
c	Unit-time waiting cost of each rider
α	Female rider's safety confidence level in the ride-hailing service, $0 < \alpha \leq 1$
β	Female driver's safety confidence level in the ride-hailing service, $0 < \beta \leq 1$
p_i	Price per service paid by gender i rider
w_i	Wage per service received by gender i driver
$d_i(n_f, n_m)$	Demand rate of gender i driver in the pooling system
Π_K	Platform's expected profit in system K, $K = \mathcal{P}$, $\mathcal{DE}$, $\mathcal{DU}$.

4.1 The Dedicated System $\mathcal{DE}$: Gender-neutral Pricing and Wage Policy

We now examine the dedicated system $\mathcal{DE}$ wherein the platform adopts the gender-neutral pricing and wage policy. That is, the price p and the wage w are the same for both genders. As observed from Figure 1, in a dedicated system, the platform has the option to operate only the male-subsystem, only the female-subsystem, or both male- and female-subsystems.

On the demand side, in each subsystem of gender i, $i = f, m$, given the price p and the number of participating drivers n_i, riders will join until the utility (2) hits zero so that, in equilibrium, the last rider who joins is indifferent between joining and balking (Hassin and Haviv 2003, Hassin 2016). By applying (1) and (2), in equilibrium, the effective arrival rate of gender i riders, $\lambda_i^e(p, n_i)$, satisfies

$$R - p - c \cdot \frac{1}{n_i\mu - \lambda_i^e(p, n_i)} = 0.$$

We then have

$$\lambda_i^e(p, n_i) = n_i\mu - \frac{c}{R - p}, \quad i = f, m.$$

On the supply side, note that gender $i(= f, m)$ drivers participate if and only if their utility expressed in (3) is non-negative. In each subsystem of gender i, the demand rate per driver, $\lambda_i^e(p, n_i)/n_i$ can be shown to be increasing with the number of participating drivers, n_i; that is, "the equilibrium demand allocated to an driver strictly increases with the number of participating drivers" as stated in Taylor (2018). Note that in each subsystem, a marginal increase of the driver labor size, on the one hand, attracts more riders to join and thus leads to the positive cross-side externality; while on the other hand, it induces the negative same-side externality on the other drivers as the competition among drivers becomes fiercer. However, due to the economies of scale embedded in a queueing system, the *aggregated externality* of a marginal increase of the driver labor size is positive, which leads to the following lemma.

Lemma 1. *In a dedicated system $\mathcal{DE}$, the number of participating drivers of gender i, n_i^e, $i = f, m$, possesses the following properties:*

1. *If the wage $w < \frac{r}{\lambda_m^e(p,1)}$, no drivers participate, i.e, $n_f^e = n_m^e = 0$.*
2. *If the wage $w \in \left[\frac{rN_m}{\lambda_m^e(p,N_m)}, \frac{rN_f}{\lambda_f^e(p,N_f)}\right)$, all male drivers participate but no female drivers participate, i.e., $n_m^e = N_m$ and $n_f^e = 0$.*
3. *If the wage $w \geq \frac{rN_f}{\lambda_f^e(p,N_f)}$, all drivers participate, i.e., $n_f^e = N_f$ and $n_m^e = N_m$.*

Lemma 1 reveals that no drivers (and hence no riders) are in the system if the wage w is too low. When the wage w is moderate, only male subsystem exists. When the wage w is high enough, both male subsystem and female subsystem exist. Due to the positive aggregated externality brought by a marginal increase of the labor size, in each gender-specific subsystem, drivers either all participate or nobody participates. Moreover, as the demand rate per driver increases in the size of participating drivers, the system with a larger labor size has a stronger positive externality and hence, its platform can pay a lower wage. As $N_f < N_m$, the platform needs to offer a high wage to entice female drivers to participate and hence female drivers are more 'expensive' than male drivers even though the gender-mismatch issue does not exist.

Recall that the platform aims to select its price p and wage w to maximize its total profit $\Pi_{\mathcal{DE}} = \sum_{i=f,m} \lambda_i(p_i - w_i)$. A close look at Lemma 1 implies that the platform earns nothing if the wage is too low so that no driver participates. Moreover, from the platform's perspective, it has no incentive to offer a wage above $\frac{rN_m}{\lambda_m^e(p,N_m)}$ if it wants to just entice all male drivers to participate. It also has no incentive to offer a wage above $\frac{rN_f}{\lambda_f^e(p,N_f)}$ so as to entice all drivers to participate. Thus, the platform just needs to compare its profit associated with the case when $w = \widehat{w}^l(p) \equiv \frac{rN_m}{\lambda_m^e(p,N_m)}$ with that when $w = \widehat{w}^h(p) \equiv \frac{rN_f}{\lambda_f^e(p,N_f)}$ to determine its optimal price and wage. By noting these values of w depend on p and that only male-subsystem exists in the former case and both male- and female-subsystems coexist in the latter case, the optimal profit under the dedicated system $\mathcal{DE}$ can be written as

$$\Pi_{\mathcal{DE}}^* = \text{MAX}\{\max_{\widehat{w}^l(p)<p<R-\frac{c}{N_m\mu}} \Pi_{\mathcal{DE}}^l(p), \max_{\widehat{w}^h(p)<p<R-\frac{c}{N_f\mu}} \Pi_{\mathcal{DE}}^h(p)\},$$

where $\Pi^l_{\mathcal{DE}}(p) = (p - \widehat{w}^l(p))\lambda^e_m(p, N_m)$ and $\Pi^h_{\mathcal{DE}}(p) = (p - \widehat{w}^h(p))(\lambda^e_m(p, N_m) + \lambda^e_f(p, N_f))$. Here, we impose an upper bound on price, $p < R - c/(N_i\mu)$, $i \in \{f, m\}$ to ensure that a rider who joins an empty queue can still get a positive utility; that is, the utility given in (2) is positive when $\lambda_i = 0$. The constraints $p > \widehat{w}^j(p)$, $j = h, l$ assure that the optimal profit is strictly positive. Denote $p^{j*}_{\mathcal{DE}}$ as the optimal price for the corresponding subproblem $\Pi^j_{\mathcal{DE}}(p)$, $j \in \{l, h\}$, and let $w^{j*}_{\mathcal{DE}} = \widehat{w}^j(p^{j*}_{\mathcal{DE}})$ be the corresponding optimal wage. We then have the following results.

Proposition 1. *In the dedicated system $\mathcal{DE}$, there exists a threshold value $\widehat{N}_f$ such that the optimal price and wage $(p^*_{\mathcal{DE}}, w^*_{\mathcal{DE}})$ can be characterized as follows.*

1. *If the number of registered female driver $N_f \le \widehat{N}_f$, the platform prefers operating the male-subsystem only and sets*

$$(p^*_{\mathcal{DE}}, w^*_{\mathcal{DE}}) = (p^{l*}_{\mathcal{DE}}, w^{l*}_{\mathcal{DE}}) = \left(R - \sqrt{\frac{cR}{N_m\mu}}, \frac{r}{\mu - \sqrt{\frac{c\mu}{N_m R}}} \right).$$

2. *If the number of registered female driver $N_f > \widehat{N}_f$, the platform prefers operating both female- and male-subsystems and sets $(p^*_{\mathcal{DE}}, w^*_{\mathcal{DE}}) = (p^{h*}_{\mathcal{DE}}, w^{h*}_{\mathcal{DE}})$, where $p^{h*}_{\mathcal{DE}}$ solves the following first-order condition,*[6]

$$\frac{d\Pi^h_{\mathcal{DE}}}{dp} = N\mu - \frac{2cR}{(R-p)^2} - \frac{cr\mu N_f(N_m - N_f)}{(N_f\mu(R-p) - c)^2} = 0,$$

and $w^{h*}_{\mathcal{DE}} = rN_f \Big/ \left(N_f\mu - \frac{c}{R - p^{h*}_{\mathcal{DE}}} \right)$.

Proposition 1 reveals that the limited labor size of female drivers is a key obstacle of operating two dedicated gender-specific subsystems. If the female labor size N_f is below a threshold, then due to the diseconomies of scale, the platform has to pay high wage to female drivers. The equal pay policy further dampens the situation because the same high wage has to be paid to male drivers as well. Also, due to the gender-neutral pricing and wage policy, the platform cannot afford to increase the price without defeating the original intent by turning away female riders. By weighing these tradeoffs, it turns out that it is not beneficial for the platform to increase the wage w. Thus, the platform is better off by abandoning the female-subsystem entirely and operating the male-subsystem only. When N_f is above the threshold, the platform can afford to increase the wage w to entice more female drivers to participate, which in turn attracts more female rides to join. Consequently, the platform is better off by operating both male- and female-subsystems.

[6]We can show that $p^{h*}_{\mathcal{DE}} \in \left(R - \sqrt{\frac{cR}{N_f\mu}}, \min\left\{ R - \sqrt{\frac{cR}{N_m\mu}}, R - \frac{c}{N_f\mu} \right\} \right)$.

Proposition 1 shows that the gender-neutral pricing and wage policy restricts the platform from offering a higher wage in the female-subsystem so as to attract female drivers to participate. This observation motivates us to examine the implications of the gender-based pricing and wage policy next.

4.2 The Dedicated System $\mathcal{DU}$: Gender-based Pricing and Wage Policy

In this section, we examine the dedicated system $\mathcal{DU}$ in which the platform adopts the gender-based pricing and wage policy. By using the same approach as that in §4.1 and considering the fact that the price p_i and wage w_i are now gender-i-specific, $i = f, m$, we can easily show that in equilibrium, the effective arrival rate of gender i riders is

$$\lambda_i^e(p_i, n_i) = n_i\mu - \frac{c}{R - p_i}, \quad i = f, m.$$

The number of participating drivers depends on the wage set by the platform, as shown in the following lemma.

Lemma 2. *In the dedicated system $\mathcal{DU}$ with gender-based pricing and wages, the number of participating drivers of gender i, n_i^e, $i = f, m$, possesses the following properties:*

1. *If $w_i < \frac{r}{\lambda_i^e(p_i,1)}$, no gender-i driver participates, i.e., $n_i^e = 0$.*
2. *If $w_i \geq \frac{rN_i}{\lambda_i^e(p_i,N_i)}$, all gender-$i$ drivers participate, i.e., $n_i^e = N_i$.*

Again, by using the same argument as that in §4.1, to optimize its own profit, the platform has no incentive to offer a wage w_i above $\frac{rN_i}{\lambda_i^e(p_i,N_i)}$. Hence, we can nail down the optimal wage $w_i^*(p_i) = \frac{rN_i}{\lambda_i^e(p_i,N_i)}$ for any given price p_i, $i = f, m$. Then, it remains to find the optimal prices, p_m and p_f associated with the respective male- and female-subsystems. For each subsystem of gender i, $i = f, m$, we obtain the optimal price p_i^* by solving

$$\max_{w_i(p_i) < p_i < R - \frac{c}{\mu N_i}} \Pi_i(p_i) = (p_i - w_i(p))\lambda_i^e(p_i, N_i).$$

The platform's optimal profit under the dedicated system $\mathcal{DU}$ is thus

$$\Pi_{\mathcal{DU}}^* = \Pi_m(p_m^*) + \Pi_f(p_f^*).$$

Proposition 2. *Under the dedicated system $\mathcal{DU}$ with gender-based pricing and wage policy, the platform will operate the subsystem of gender i ($i = f, m$) profitably only if the number of registered* *drivers of gender i, $N_i > \widetilde{N}_i = \frac{c}{(\sqrt{R\mu}-\sqrt{r})^2}$. Moreover, when the platform operates the dedicated subsystem of gender i, the optimal gender-based price and wage satisfy*

$$(p_i^*, w_i^*) = \left(R - \sqrt{\frac{cR}{N_i \mu}}, \frac{r}{\mu - \sqrt{\frac{c\mu}{N_i R}}} \right), i = f, m.$$

By noting that $N_m > N_f$ (as female drivers are in short supply) , Proposition 2 yields an interesting result: when the platform adopts the gender-based pricing and wage policy, compared to their male counterparts, female riders pay a lower price (i.e., $p_f^* < p_m^*$) but female drivers receive a higher wage (i.e., $w_f^* > w_m^*$). Contrary to that stated in Proposition 1, Proposition 2 reveals that when the platform has the flexibility to set gender-based prices and wages, it can afford to operate the female subsystem profitably by offering female drivers a higher wage to entice them to participate while charging female riders a lower price to incentivize them to join (as a way to compensate female riders for their longer waiting time due to the shortage of female drivers).

4.3 Gender-neutral vs. Gender-based Pricing and Wage Policies

We now compare the operational performance between the two dedicated systems, $\mathcal{DE}$ and $\mathcal{DU}$, which adopt the gender-neutral (equal) and gender-based (unequal) pricing and wage policies, respectively. We use subscripts "$\mathcal{DU}$" and "$\mathcal{DE}$" to represent the equilibrium prices and wages in these two dedicated systems, respectively. By using the results as stated in Propositions 1 and 2, we can shown that the thresholds satisfy $\widehat{N}_f > \widetilde{N}_f$[7], which demonstrates that the platform has higher incentives to operate a female subsystem under the gender-based policy. We further obtain the following results.

Proposition 3. *When the platform operates the gender-specific dedicated systems, if the number of registered female drivers $N_f > \widehat{N}_f$, compared to that under the gender neutral equal policy,*

1. *female drivers earn a lower wage under the gender-based pricing and wage policy, i.e., $w_{f,\mathcal{DU}}^* < w_{\mathcal{DE}}^{h*}$.*
2. *female riders pay a lower price under the gender-based pricing and wage policy, i.e., $p_{f,\mathcal{DU}}^* < p_{\mathcal{DE}}^{h*}$.*
3. *the effective arrival rate of female riders is higher under the gender-based pricing and wage policy, i.e., $\lambda_{f,\mathcal{DU}}^* > \lambda_{f,\mathcal{DE}}^*$.*

[7] We show this relationship in the proof of Proposition 3.

Despite Proposition 2 asserts that, relative to male drivers, female drivers earn a higher wage w_f under the gender-based policy, Proposition 3 reveals that, relative to the gender-neutral policy, female drivers earn a lower wage under the gender-based policy. However, even though the wage is lower, female drivers are compensated with a higher arrival rate of female riders (due to lower price).

5 Analysis of the Pooling System $\mathcal{P}$

In this section, we analyze the pooling system $\mathcal{P}$ that engage riders and drivers of both genders as depicted in Figure 1. Unlike the dedicated system in which riders are matched with drivers of the same gender, the pooling system matches riders and drivers without taking their gender into consideration. As such, females' safety concerns are present so that the safety confidence levels of female riders and drivers, α and β will play an important role in our analysis. The analysis of the pooling system is also more complex because it involves arrivals (and participation) of riders (and drivers) of both genders with different utilities. Moreover, because the pooling system is operated under a single legal entity, gender-based pricing and wage policy is deemed discriminatory and may be illegal. For this reason, we focus only on the gender-neutral policy; that is, all riders pay the same price and all drivers receive the same wage regardless of their gender.

5.1 Equilibrium Behavior of Riders and Drivers in the Pooling System

In the pooling system $\mathcal{P}$, each rider of gender i, $i = f, m$, will continue to join the system until the utility U_i given in (4)/(5) hits zero. Relative to female riders, male riders have no safety concern. A close look at (4) and (5) implies that $U_m \geq U_f$. Similarly, because male drivers have no safety concern, a close look at (6) and (7) implies that $S_m \geq S_f$. These observations yield the following implications.

Implication 1. *Consider a pooling system that serves riders and drivers of both genders in a single system. If some female riders (drivers, respectively) join (participate in, respectively) the system, then all male riders (drivers, respectively) will join (participate in, respectively) the system.*

Note that male riders and drivers are more eager to join and participate than their female counterparts. It is likely to have all male riders and drivers joining and participating in the system. However, our assumption $\frac{\Lambda_m}{N_m \mu} > 1$ stipulates that, when this happens, the pooling system must engage some female drivers to participate so as to ensure that the queueing system is stable. This observation and Implication 1 enable us to conclude the following:

Implication 2. *If some female riders join the system, then all male riders join the system (due to Implication 1). Also, some female drivers must participate to ensure the stability of the system (due to the above observation). Hence, based on Implication 1, we can conclude that when some female riders join the system, all male drivers must participate in the system.*

Based on these two implications, we can conclude that, in order for the platform to retain both female and male riders in the pooling system, it must be the case that all male riders (and drivers) and some female riders (and drivers) join (and participate in) the system.

Let $\phi_\alpha = \frac{\alpha N_m + N_f}{N}$. Then, $\phi_\alpha \in (0, 1]$ represents the *safety-adjusted reward factor* of a female rider when all the registered drivers participate, and $\phi_\alpha R$ is the *safety-adjusted reward*. Let λ_i^e denote the effective arrival rate of gender-i riders. Denote n_j^e as the number of participating drivers of gender j in equilibrium, $i = f, m$. By focusing on the equilibrium outcome that has all male riders and some female riders joining the system, we now develop the conditions under which this equilibrium will exist in the following proposition.

Proposition 4. *Consider a pooling system in which the platform sets the price $p < \bar{p} \equiv \phi_\alpha R - \frac{c}{N\mu - \Lambda_m}$ and the wage $w \geq \underline{w} \equiv \frac{rN}{(\beta-1)\Lambda_m + N\mu - c/(\phi_\alpha R - p)}$. Then, in equilibrium, all drivers of both genders will participate so that $n_m^e = N_m$ and $n_f^e = N_f$. Also, all male riders and some female riders will join so that $\lambda_m^e = \Lambda_m$ and $\lambda_f^e(p) = N\mu - \Lambda_m - \frac{c}{\phi_\alpha R - p}$.*

Note that in equilibrium, all drivers of both genders participate. By Implication 2, we know that all male drivers participate as long as there are some female drivers participate. The "all join" behavior of female drivers can be explained by the positive aggregated externality among female drivers. It can be shown that the *safety-adjusted demand rate* for a female driver, $d_f(n_f, N_m)$[8] as stated in (8) is increasing in the participating number of female drivers, i.e., $\partial d_f(n_f, N_m)/\partial n_f > 0$. That is, the aggregated externality brought by a female driver on other female drivers is positive. Moreover, it can be shown that the positive externality, measured by $\partial d_f(n_f, N_m)/\partial n_f$, is decreasing in both α and β, implying that a high safety confidence level reduces the degree of the positive externality. In contrast, a marginal increase of the labor size of male drivers does not necessarily increase the safety-adjusted demand rate for a female driver, as it can reduce female riders' reward. It can be easily shown that $\partial d_f(n_f, N_m)/\partial N_m < 0$ when the female rider's safety confidence level α is small and the female driver's safety confidence level $\beta \to 1$. Therefore, the aggregated externality over female drivers, brought by male drivers, could be negative, a finding different from the positive

[8]For brevity, we provide the detailed analysis regarding the properties of $d_f(n_f, N_m)$ discussed hereafter in the proof of Proposition 4; see the Appendix.

aggregated externality result obtained in Taylor (2018) where drivers have no gender-based safety concerns.

5.2 Optimal Pricing and Wage Decisions for the Pooling System

When the price and wage are within a certain range, Proposition 4 asserts that the effective arrival rate of gender i riders, λ_i^e and the number of participating gender i drivers, n_i^e satisfy Implication 2; namely, drivers of both genders all participate, male riders all join and some female riders join. We now examine the platform's pricing and wage problem that aims to maximize its profit, subject to the constraints $0 < p < \bar{p}$ and $w \geq \underline{w}$. Parameter $\bar{p}$ is the price to attract a single female rider to join, given that all male riders have joined and all the drivers of both genders have participated; and $\underline{w}$ is the minimum wage to attract all the drivers of both genders to participate, given price p and that male riders all join. Clearly, for any given $p \in (0, \bar{p})$, there is no incentive for the platform to offer a wage that is above $\underline{w}$. Hence, it is optimal for the platform to set

$$w(p) = \underline{w} = \frac{rN}{(\beta - 1)\Lambda_m + N\mu - \frac{c}{\phi_\alpha R - p}}. \tag{9}$$

Combining this formulation along with the result as stated in Proposition 4, we can formulate the platform's problem as:

$$\Pi_{\mathcal{P}}^* = \max_{w(p)<p<\bar{p}} \Pi_{\mathcal{P}}(p) = \max_{w(p)<p<\bar{p}} (p - w(p))(\Lambda_m + \lambda_f^e(p)),$$

where $w(p)$ is given in (9) and $\lambda_f^e(p) = N\mu - \Lambda_m - \frac{c}{\phi_\alpha R - p}$.

Proposition 5. *In a pooling system, the platform's profit function $\Pi_{\mathcal{P}}(p)$ is concave in the price p over the range $(0, \bar{p})$. Let $p_{\mathcal{P}}^*$ be the solution of the first-order condition*

$$\frac{d\Pi_{\mathcal{P}}(p)}{dp} = N\mu - \frac{c\phi_\alpha R}{(\phi_\alpha R - p)^2} + \frac{crN(\beta - 1)\Lambda_m}{((N\mu + (\beta - 1)\Lambda_m)(\phi_\alpha R - p) - c)^2} = 0.$$

Then, $p_{\mathcal{P}}^*$ is an interior optimal solution if and only if

$$\frac{d\Pi_{\mathcal{P}}}{dp} \mid_{p \to 0} = \kappa_1(\alpha, \beta) > 0 \text{ and } \frac{d\Pi_{\mathcal{P}}}{dp} \mid_{p \to \bar{p}} = \kappa_2(\alpha, \beta) < 0,$$

where the detailed expressions of $\kappa_1(\alpha, \beta)$ and $\kappa_2(\alpha, \beta)$ are presented in equations (17) and (18) in the Appendix.

Proposition 5 indicates that the sufficient condition for the existence of the interior optimum is that the safety confidence level parameters α and β must fall into certain ranges. Note that when $R > \frac{c\mu N}{(\mu N - \Lambda_m)^2}$, $\kappa_1(1, 1) > 0$ and $\kappa_2(1, 1) < 0$. Besides, we can show that $\kappa_1(\alpha, \beta)$ increases in both α and β while $\kappa_2(\alpha, \beta)$ decreases in α but increases in β (see the proof of Proposition 5 in the appendix for the detail). Therefore, we can always find a certain region constructed by (α, β), satisfying conditions stated in Proposition 5. In other words, when both rider-side and driver-side safety confidence levels are not so low, the interior optimal price exists.

By using the optimal price $p^*_{\mathcal{P}}$, we can retrieve the effective arrival rate of female riders $\lambda^*_{f,\mathcal{P}} = \lambda^e_f(p^*_{\mathcal{P}})$ from Proposition 4, the optimal wage $w^*_{\mathcal{P}}$ from (9), and the corresponding optimal profit $\Pi^*_{\mathcal{P}}$. Also, by applying the implicit function theorem, we can establish the following result.

Proposition 6. *The pooling system's performance exhibits the following characteristics.*

1. *The optimal price $p^*_{\mathcal{P}}$ is increasing in both α and β while the optimal wage $w^*_{\mathcal{P}}$ is decreasing in both α and β.*

2. *The effective arrival rate of female riders $\lambda^*_{f,\mathcal{P}}$ is increasing in α but decreasing in β.*

3. *The platform's optimal profit $\Pi^*_{\mathcal{P}}$ is convex and increasing in α but concave and increasing in β.*

Proposition 6 indicates that enhancing the rider-side and driver-side safety confidence levels have dramatically different impact on the number of joined female riders and the platform profitability. One might believe that enhancing safety confidence levels always encourages female users to join. Proposition 6 implies that enhancing the driver-side safety confidence level actually reduces the number of joined female riders. This is mainly caused by the positive cross-side externality between female riders and female drivers and the platform's selfish profit-maximizing decision. With a low driver-side safety confidence level, the platform has to keep its price sufficiently low to attract enough female riders to join so as to sustain the female driver pool. Once the driver-side safety confidence level is enhanced, the platform has incentives to increase its price without worrying about losing female drivers, which drives away some female riders.

Interestingly, Proposition 6 also shows that the rider-side safety confidence level α affects the platform's profit in a convex way while the driver-side safety confidence level β does that in a concave way. Recall that the platform's profit is the product of two terms, profit margin $p^*_{\mathcal{P}} - w^*_{\mathcal{P}}$ and the demand size $\Lambda_m + \lambda^*_{f,\mathcal{P}}$. Enhancing the rider-side safety confidence level α increases both the profit margin and the demand size, resulting in a convexity relationship between the profit and α; whereas enhancing the driver-side safety confidence level β increases the profit margin but decreases the demand size, resulting in a concavity relationship. These imply that the platform's profit is more elastic toward a larger α and a smaller β, a result helpful for the ride-hailing platform to conduct the cost-benefit analysis when enhancing its safety levels.

6 Pooling vs. Dedicated Systems

By using our results presented in the last two sections, we now compare the operational and financial performances of three systems, namely, the pooling system $\mathcal{P}$, the dedicated system under the gender-neutral policy $\mathcal{DE}$, and the dedicated system under the gender-based pricing and wage policy $\mathcal{DU}$. Through the direct comparison, we obtain the following results.

Proposition 7. *When female riders and drivers have complete confidence in safety so that $\alpha = \beta = 1$, the pooling system $\mathcal{P}$ dominates the dedicated system, i.e., $\Pi^*_{\mathcal{P}} > \Pi^*_{\mathcal{DU}} > \Pi^*_{\mathcal{DE}}$. Moreover, the pooling system can entice more riders of both genders to join; that is, $\lambda^*_{f,\mathcal{P}} + \lambda^*_{m,\mathcal{P}} > \lambda^*_{f,\mathcal{DU}} + \lambda^*_{m,\mathcal{DU}}$.*

Proposition 7 reveals that, when female riders and drivers have complete confidence in safety, it is optimal for the platform to operate according to a pooling system so that the platform can pool the riders (of both genders) and drivers (of both genders). In other words, if a platform can take corrective measures to address females' safety concerns completely, there is no need for the platform to migrate towards a gender-specific dedicated system. Instead, the platform should keep its gender-neutral pooling system by leveraging the pooling of riders and drivers of both genders to achieve a higher profitability.

Next, let us consider the scenario in which the females' safety concerns cannot be fully addressed so that safety confidence levels of female riders and drivers satisfy $\alpha < 1$ and $\beta < 1$. In this case, the safety confidence levels α and β will play an important role as Proposition 6 has shown that, in a pooling system, the platform's profit decreases as females' safety confidence levels deteriorate (i.e., as α and β become smaller). Regarding the dedicated system where females' safety concerns are absent (so that the safety confidence levels can be modeled as $\alpha = 1$ and $\beta = 1$), the platform can earn a higher profit by adopting the gender-based pricing and wage policy; i.e., $\Pi^*_{\mathcal{DU}} > \Pi^*_{\mathcal{DE}}$ because the gender-based pricing and wage policy relaxes the equal policy constraints on price and wage. Furthermore, both of them are independent of α and β. The following conclusion then follows straightforwardly.

Proposition 8. *Given the female driver's safety confidence level β, there exist two thresholds $\widetilde{\alpha}_E(\beta)$ and $\widetilde{\alpha}_U(\beta)$ with $\widetilde{\alpha}_E(\beta) \le \widetilde{\alpha}_U(\beta)$ so that*[9]

[9] While Proposition 8 focuses on the impact of the female riders' safety confidence level α, we can draw the similar conclusion if we vary the female drivers' safety confidence level β. To avoid repetition, we omit details here.

1. $\Pi^*_{\mathcal{P}} < \Pi^*_{\mathcal{DE}} < \Pi^*_{\mathcal{DU}}$ *if* $\alpha < \widetilde{\alpha}_E(\beta)$.

2. $\Pi^*_{\mathcal{DE}} \leq \Pi^*_{\mathcal{P}} \leq \Pi^*_{\mathcal{DU}}$ *if* $\widetilde{\alpha}_E(\beta) \leq \alpha \leq \widetilde{\alpha}_U(\beta)$.

3. $\Pi^*_{\mathcal{DE}} < \Pi^*_{\mathcal{DU}} < \Pi^*_{\mathcal{P}}$ *if* $\widetilde{\alpha}_U(\beta) < \alpha \leq 1$.

When female riders' safety confidence level is high enough (i.e., when α is above a certain threshold $\widetilde{\alpha}_U(\beta)$), Proposition 8 shows that the pooling system is the dominant system. This result is consistent with that of Proposition 7. Proposition 8 also indicates that when female riders' safety confidence level α is below the threshold $\widetilde{\alpha}_U(\beta)$, then it is optimal for the platform to operate a dedicated system by adopting the gender-based pricing and wage policy. This result provides a plausible explanation for the recent development of gender-specific ride-hailing services in the US, India, and Pakistan where females' safety concerns are severe. Also, our result is consistent with the Didi's same-sex policy that was launched immediately after two Didi Hitch female riders were murdered (Al-Heeti 2018).

7 Discussion and Conclusion

Operations of on-demand ride-hailing platforms have some potential safety hazards such as lack of enough background checking on drivers, loose organization of the workforce, lack of enough training for drivers, etc. After two female riders were murdered by Didi drivers in 2018, some mechanisms are under urgent development by Didi to enhance the safety level, such as introducing in-trip audio recording (Dai 2018) and installing one-button emergency call in the apps. To help these platforms in the turmoil, we take a first step to study safety-concerned operations of on-demand ride-hailing platforms. Specifically, we consider two system configurations: a pooling system where females' safety concerns are present and a dedicated system where such safety concerns are absent. We then derive the equilibrium outcomes including the platform's pricing and wage decisions and the joining and participating behaviors of riders and drivers.

We show that as mismatch (due to safety concerns) exists between riders and drivers of different gender, a marginal increase of the male driver labor size may not bring positive externality to female drivers but a marginal increase of the female driver labor size does. Also, female riders and female drivers always have positive cross-side externality, which, coupled with the platform's profit-maximizing behavior, yields two interesting results. One, as the driver safety confidence level is enhanced, more female drivers have incentives to participate, which, however, can reduce the number of joined female riders. The reason is that the platform then has less incentives to attract female riders to join by charging a low price to sustain the female labor size. Two, the platform's optimal profit is convex and increasing in the rider-side safety confidence level but concave and increasing in the driver-side safety confidence level. This elasticity difference can be explained by the aforementioned impact on the female riders' effective arrival rate of enhancing the driver-side safety confidence level: enhancing the rider-side safety confidence level increases both the profit margin and the total effective arrival rate while enhancing the driver-side safety confidence level increases the profit margin but decreases the female-rider effective arrival rate. These results could be helpful for the platform to conduct the cost-benefit analysis when enhancing its safety levels.

Although the platform can improve the safety confidence levels of female riders and drivers by migrating from a pooling system to a gender-specific dedicated system, such a move demands a sufficiently large female driver pool. The limited pool size of female drivers restricts the scale of the dedicated female system, which reduces the economies of scale and the positive externality among drivers. As a result, not only most of female riders cannot be served but also the platform has to pay much higher wage to female drivers. Worse yet, if "equal pay policy" applies, the same high wage shall be paid to male drivers as well, which further hinders the adoption of dedicated systems. In contrast, the pooling system not only benefits from the pooling effect arising from the stochastic characteristics of arrival and service processes, but also benefits from the cross-side positive externality between the large-scale riders and drivers.

The dedicated system is worthy to pursue if the pooling system's safety confidence level is below a threshold. Consider that many ride-hailing platforms run multiple business modules with different levels of safety control over drivers. Platforms can keep the pooling mode for those modules with high safety levels and consider dedicated systems for those modules with low safety levels. To remedy the diseconomies of scale of dedicated systems, the ride-hailing platform can provide higher incentives for female drivers to join the workforce. In our study, the labor sizes of male and female drivers are exogenously given. One can imagine that as the pooling system is changed to the dedicated system, the female labor size can be enlarged due to the enhanced safety confidence level for the female driver and hence, more credits shall be given to the dedicated systems than what we demonstrate in our models.

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Appendix: Proofs

Proof of Lemma 1: Recall that $\lambda_i^e(p, n_i) = n_i\mu - \frac{c}{R-p}, i = f, m$. It can be easily shown that $\lambda_i^e(p, n_i)/n_i$ is increasing in n_i, $i = f, m$. Thus, $n_i/\lambda_i^e(p, n_i)$ is decreasing in n_i. As $N_m > N_f$ (due to the limited supply of the female drivers), we then have

$$\frac{rN_m}{\lambda_m^e(p, N_m)} < \frac{rN_f}{\lambda_f^e(p, N_f)}.$$

Here, similar to that in Taylor (2018), multiple equilibria may exist, and when the platform offers a higher wage, a equilibrium with fewer participating drivers may arise. We then follow Taylor (2018) to assume that all the parties, the platform, the drivers and the riders, would coordinate on the equilibrium that has the most drivers when multiple equilibria exist. That is, we restrict our equilibrium structure to that sated in Lemma 1 of Taylor (2018).

Based on the male driver's utility stated in (3), when in equilibrium only one male driver participates, the wage paid to that male driver must satisfy

$$\frac{\lambda_m^e(p, 1)}{1}w - r = 0.$$

Therefore, when $w < \frac{r}{\lambda_m^e(p,1)}$, no drivers participate. Next, when $w \in [\frac{rN_m}{\lambda_m^e(p,N_m)}, \frac{rN_f}{\lambda_f^e(p,N_f)})$, we can show that a male driver's utility is always positive even when all the male drivers participate as

$$S_m(\lambda_m^e(p, N_m), N_m) = \frac{\lambda_m^e(p, N_m)}{N_m}w - r \geq \frac{\lambda_m^e(p, N_m)}{N_m} \cdot \frac{rN_m}{\lambda_m^e(p, N_m)} - r \geq 0.$$

However, for given any number of participating female drivers $n_f \leq N_f$, we can show that a female driver's utility is always negative as

$$S_f(\lambda_f^e(p, n_f), n_f) = \frac{\lambda_f^e(p, n_f)}{n_f}w - r < \frac{\lambda_f^e(p, n_f)}{n_f} \cdot \frac{rN_f}{\lambda_f^e(p, N_f)} - r \leq 0.$$

Thus, in equilibrium all male drivers participate but no female drivers participate when $w \in [\frac{rN_m}{\lambda_m^e(p,N_m)}, \frac{rN_f}{\lambda_f^e(p,N_f)})$. Similarly, when $w \geq \frac{rN_f}{\lambda_f^e(p,N_f)}$, we can show that $S_m(\lambda_m^e(p, N_m), N_m) > 0$ and $S_f(\lambda_f^e(p, N_f), N_f) \geq 0$. Hence, all drivers participate when $w \geq \frac{rN_f}{\lambda_f^e(p,N_f)}$.

Proof of Proposition 1: First, we solve the subproblem $\max_{\widehat{w}^l(p)<p<R-\frac{c}{\mu N_m}} \Pi_{\mathcal{DE}}^l(p) = (p - \widehat{w}^l(p))\lambda_m^e(p, N_m)$. Substituting $\widehat{w}^l(p) = \frac{rN_m}{\lambda_m^e(p,N_m)}$ and $\lambda_m^e(p, N_m) = N_m\mu - \frac{c}{R-p}$ into $\Pi_{\mathcal{DE}}^l(p)$, we then have

$$\Pi_{\mathcal{DE}}^l(p) = p\left(\mu N_m - \frac{c}{R-p}\right) - rN_m, \tag{10}$$

which is concave in p as $\frac{d^2\Pi^l_{\mathcal{DE}}}{dp^2} = \frac{-2cR}{(R-p)^3} < 0$. Then, based on the first-order condition

$$\frac{d\Pi^l_{\mathcal{DE}}}{dp} = \mu N_m - \frac{cR}{(R-p)^2} = 0,$$

we get the optimal price $p^{l*}_{\mathcal{DE}} = R - \sqrt{\frac{cR}{N_m\mu}}$. It can be easily verified that $p^{l*}_{\mathcal{DE}} < R - \frac{c}{N_m\mu}$. Consequently, the optimal profit of the platform

$$\Pi^{l*}_{\mathcal{DE}} = \Pi^l_{\mathcal{DE}}(p^{l*}_{\mathcal{DE}}) = (\sqrt{R\mu N_m} - \sqrt{c})^2 - rN_m.$$

Next, we solve the subproblem $\max_{\widehat{w}^h(p)<p<R-\frac{c}{\mu N_f}} \Pi^h_{\mathcal{DE}}(p) = (p - \widehat{w}^h(p))(\lambda^e_m(p, N_m) + \lambda^e_f(p, N_f))$. Substituting $\widehat{w}^h(p) = \frac{rN_f}{\lambda^e_f(p,N_f)}$ and $\lambda^e_f(p, N_f) = \mu N_f - \frac{c}{R-p}$ into $\Pi^h_{\mathcal{DE}}(p)$, we obtain

$$\Pi^h_{\mathcal{DE}}(p) = \left(p - \frac{rN_f}{\mu N_f - \frac{c}{R-p}}\right)\left(\mu(N_m + N_f) - \frac{2c}{R-p}\right), \tag{11}$$

based on which we have

$$\frac{d\Pi^h_{\mathcal{DE}}(p)}{dp} = \mu N - \frac{2cR}{(R-p)^2} - \frac{cr\mu N_f(N_m - N_f)}{(\mu N_f(R-p) - c)^2}, \text{ and } \frac{d^2\Pi^h_{\mathcal{DE}}(p)}{dp^2} = -\frac{4cR}{(R-p)^3} - \frac{2cr\mu^2 N_f^2(N_m - N_f)}{(\mu N_f(R-p) - c)^3} < 0.$$

That is, $\Pi^h_{\mathcal{DE}}(p)$ is concave in p. Thus, there exists a unique optimal price $p^{h*}_{\mathcal{DE}}$ that solves the first-order condition $\frac{d\Pi^h_{\mathcal{DE}}(p)}{dp} = 0$. Furthermore, we can show that

$$\frac{d\Pi^h_{\mathcal{DE}}(p)}{dp}\Big|_{p=R-\sqrt{\frac{cR}{\mu N_f}}} = \mu N - 2\mu N_f - \frac{cr\mu N_f(N_m - N_f)}{\left(\mu N_f\sqrt{\frac{cR}{\mu N_f}} - c\right)^2} = \mu(N_m - N_f)\left(1 - \frac{rN_f}{(\sqrt{R\mu N_f} - \sqrt{c})^2}\right) > 0^{10},$$

$$\frac{d\Pi^h_{\mathcal{DE}}(p)}{dp}\Big|_{p=p^{l*}_{\mathcal{DE}}=R-\sqrt{\frac{cR}{\mu N_m}}} = \mu N - 2\mu N_m - \frac{cr\mu N_f(N_m - N_f)}{\left(\mu N_f\sqrt{\frac{cR}{\mu N_m}} - c\right)^2} = \mu(N_f - N_m) - \frac{cr\mu N_f(N_m - N_f)}{\left(\mu N_f\sqrt{\frac{cR}{\mu N_m}} - c\right)^2} < 0,$$

and

$$\lim_{p\to R-\frac{c}{N_f\mu}} \frac{d\Pi^h_{\mathcal{DE}}(p)}{dp} = \lim_{p\to R-\frac{c}{N_f\mu}} \left(\mu N - \frac{2cR}{(R-p)^2} - \frac{cr\mu N_f(N_m - N_f)}{(\mu N_f(R-p) - c)^2}\right) = -\infty < 0.$$

Hence, we have $p^{h*}_{\mathcal{DE}} \in \left(R - \sqrt{\frac{cR}{\mu N_f}}, \min\left\{R - \frac{c}{\mu N_f}, p^{l*}_{\mathcal{DE}}\right\}\right)$.

We now compare $\Pi^h_{\mathcal{DE}}(p^{h*}_{\mathcal{DE}})$ and $\Pi^l_{\mathcal{DE}}(p^{l*}_{\mathcal{DE}})$. First, a close look at (10) and (11) shows that when $N_f \to N_m$, $\Pi^h_{\mathcal{DE}}(p^{h*}_{\mathcal{DE}}) \to 2\Pi^l_{\mathcal{DE}}(p^{l*}_{\mathcal{DE}})$. Next, when N_f is very small such that the female-subsystem has a tiny capacity, that is $N_f \to 0$. Under this situation, due to the long waiting time, the platform needs to set a very low price to attract female riders to join and the revenue is negligible. (Recall

[10] We note that this inequality holds when female-subsystem in $\mathcal{DU}$ generates a strictly positive profit. That is, $(\sqrt{R\mu N_f} - \sqrt{c})^2 - rN_f > 0$.

that we have shown above that $p^{h*}_{\mathcal{DE}} < p^{l*}_{\mathcal{DE}}$.) Meanwhile, due to the positive driver participation externality, the platform needs to set a very high wage to attract female drivers to participate. Under the gender neutral policy, the same high wage has to be paid to male drivers while the same low price has to be charged to male riders. Thus, we must have $\Pi^h_{\mathcal{DE}}(p^{h*}_{\mathcal{DE}}) < \Pi^l_{\mathcal{DE}}(p^{l*}_{\mathcal{DE}})$ when $N_f \to 0$.

Last, we prove that $\Pi^h_{\mathcal{DE}}(p^{h*}_{\mathcal{DE}}(N_f), N_f)$ increases in N_f. Based on (11), we have

$$\frac{\partial \Pi^h_{\mathcal{DE}}(p_0, N_f)}{\partial N_f} = \mu p_0 - r - \frac{\mu N_m - \frac{c}{R-p_0}}{\mu N_f - \frac{c}{R-p_0}} r - \frac{r N_f(-\mu)(\mu N_m - \frac{c}{R-p_0})}{(\mu N_f - \frac{c}{R-p_0})^2} = \mu p_0 - r + \frac{\frac{rc}{R-p_0}(\mu N_m - \frac{c}{R-p_0})}{(\mu N_f - \frac{c}{R-p_0})^2},$$

for any given p_0 that satisfies $\widehat{w}^h(p_0) < p_0 < R - \frac{r}{\mu N_f}$. Since $\widehat{w}^h(p_0) = \frac{r}{\mu - \frac{c}{(R-p_0)N_f}} < p_0 < R - \frac{c}{\mu N_f} < R - \frac{c}{\mu N_m}$, $\mu p_0 > r$ and $\frac{c}{R-p_0} < \mu N_m$. Thus, $\frac{\partial \Pi^h_{\mathcal{DE}}(p_0, N_f)}{\partial N_f} > 0$. That is, if $N_{f_1} < N_{f_2}$, then $\Pi^h_{\mathcal{DE}}(p_0, N_{f_1}) < \Pi^h_{\mathcal{DE}}(p_0, N_{f_2})$. Consequently, we have $\Pi^h_{\mathcal{DE}}(p^{h*}_{\mathcal{DE}}(N_{f_1}), N_{f_1}) < \Pi^h_{\mathcal{DE}}(p^{h*}_{\mathcal{DE}}(N_{f_1}), N_{f_2}) < \Pi^h_{\mathcal{DE}}(p^{h*}_{\mathcal{DE}}(N_{f_2}), N_{f_2})$. That is, $\Pi^h_{\mathcal{DE}}(p^{h*}_{\mathcal{DE}}(N_f), N_f)$ increases in N_f. Combining the above analyses, we conclude that for any given N_m, there exists a threshold $\widehat{N}_f \in (0, N_m)$ such that when $N_f > \widehat{N}_f$, $\Pi^h_{\mathcal{DE}}(p^{h*}_{\mathcal{DE}}) > \Pi^l_{\mathcal{DE}}(p^{l*}_{\mathcal{DE}})$ and $\Pi^h_{\mathcal{DE}}(p^{h*}_{\mathcal{DE}}) \le \Pi^l_{\mathcal{DE}}(p^{l*}_{\mathcal{DE}})$ otherwise.

Proof of Lemma 2: The logic and proof are very similar to that of Lemma 1. We thus omit the details here.

Proof of Proposition 2: By adopting the same method that solves $\max_{\widehat{w}^l(p) < p < R - \frac{c}{\mu N_m}} \Pi^l_{DE}(p)$ in the proof of Proposition 1, we can easily obtain that in the dedicated system of gender i, the optimal price, the effective arrival rate and the wage are, respectively,

$$p_i^* = R - \sqrt{\frac{cR}{\mu N_i}},\ \lambda_i^* = \mu N_i - \sqrt{\frac{c\mu N_i}{R}} \text{ and } w_i^* = \frac{rN_i}{\lambda_i^*} = \frac{r}{\mu - \sqrt{\frac{c\mu}{RN_i}}},\ i = f, m.$$

Then, the optimal expected profit of the gender-i subsystem is

$$\Pi_i(p_i^*) = p_i^* \left(\mu N_i - \frac{c}{R - p_i^*}\right) - rN_i = \left(\sqrt{R\mu N_i} - \sqrt{c}\right)^2 - rN_i.$$

To ensure the strictly positive optimal profit, the parameter values must satisfy

$$\sqrt{R\mu N_i} - \sqrt{c} > \sqrt{rN_i},$$

or equivalently,

$$\sqrt{R\mu} - \frac{\sqrt{c}}{\sqrt{N_i}} > \sqrt{r} \text{ and } \sqrt{R\mu} - \sqrt{r} > \frac{\sqrt{c}}{\sqrt{N_i}} > 0.$$

Therefore,

$$\frac{d\Pi_i(p_i^*)}{dN_i} = \sqrt{R\mu}\left(\sqrt{R\mu} - \frac{\sqrt{c}}{\sqrt{N_i}}\right) - r > \sqrt{R\mu r} - r = \sqrt{r}(\sqrt{R\mu} - \sqrt{r}) > 0.$$

That is, $\Pi_i(p_i^*)$ increases in N_i. Consequently, when $(\sqrt{R\mu N_i}-\sqrt{c})^2 - rN_i > 0$, or equivalently, when $N_i > \widetilde{N}_i = \frac{c}{(\sqrt{R\mu}-\sqrt{r})^2}$, the subsystem of gender i obtains a strictly positive profit. This completes the proof.

Proof of Proposition 3: First, we prove that $\widehat{N}_f > \widetilde{N}_f$. Note that the dedicated system $\mathcal{DE}$ is a special case of the dedicated system $\mathcal{DU}$ because the latter's gender-based pricing and wage policy relaxes the former's gender neutral equal requirements on price and wage. When both female and male subsystems are operating in $\mathcal{DE}$ and $\mathcal{DU}$, we must have the total profit of $\mathcal{DU}$ system, $\Pi_m(p_m^*) + \Pi_f(p_f^*)$ (as stated in Proposition 2) is higher than that of $\mathcal{DE}$ system, $\Pi_{\mathcal{DE}}^h(p_{\mathcal{DE}}^{h*})$ (as stated in Proposition 1); i.e, $\Pi_m(p_m^*)+\Pi_f(p_f^*) > \Pi_{\mathcal{DE}}^h(p_{\mathcal{DE}}^{h*})$. When $N_f = \widehat{N}_f$, based on Proposition 1, we have $\Pi_{\mathcal{DE}}^* = \Pi_{\mathcal{DE}}^h(p_{\mathcal{DE}}^{h*}) = \Pi_{\mathcal{DE}}^l(p_{\mathcal{DE}}^{l*})$. Consequently, when $N_f = \widehat{N}_f$,

$$\Pi_m(p_m^*) + \Pi_f(p_f^*) > \Pi_{\mathcal{DE}}^h(p_{\mathcal{DE}}^{h*}) = \Pi_{\mathcal{DE}}^l(p_{\mathcal{DE}}^{l*}) = \Pi_m(p_m^*).$$

That is, $\Pi_f(p_f^*) > 0$ when $N_f = \widehat{N}_f$. Recall from the proof of Proposition 2 that $\Pi_f(p_f^*) = 0$ when $N_f = \widetilde{N}_f$ and $\Pi_f(p_f^*)$ increases in N_f. Thus, $\widehat{N}_f > \widetilde{N}_f$.

When $N_f > \widehat{N}_f$, for the $\mathcal{DE}$ system, in the proof of Proposition 1, we have shown that the platform sets the price

$$p_{\mathcal{DE}}^{h*} \in \left(R - \sqrt{\frac{cR}{\mu N_f}}, \min\left\{R - \frac{c}{\mu N_f}, R - \sqrt{\frac{cR}{\mu N_m}}\right\}\right).$$

and drivers of both genders receive a wage $w_{\mathcal{DE}}^{h*} = rN_f \Big/ \left(\mu N_f - \frac{c}{R-p_{\mathcal{DE}}^{h*}}\right)$. The effective arrival rate of female riders $\lambda_{f,\mathcal{DE}}^* = \mu N_f - \frac{c}{R-p_{\mathcal{DE}}^{h*}}$. For the $\mathcal{DU}$ system, from Proposition 2 and based on the above inequality relationship, we know that the price charged to the female riders

$$p_{f,\mathcal{DU}}^* = R - \sqrt{\frac{cR}{\mu N_f}} < p_{\mathcal{DE}}^{h*} < R - \sqrt{\frac{cR}{\mu N_m}} = p_{m,\mathcal{DU}}^*,$$

and the wage paid to the female drivers $w_{f,\mathcal{DU}}^* = rN_f \Big/ \left(\mu N_f - \frac{c}{R-p_{f,\mathcal{DU}}^*}\right)$. The effective arrival rate of female riders $\lambda_{f,\mathcal{DU}}^* = \mu N_f - \frac{c}{R-p_{f,\mathcal{DU}}^*}$. As $p_{f,\mathcal{DU}}^* < p_{\mathcal{DE}}^{h*}$, $w_{f,\mathcal{DU}}^* < w_{\mathcal{DE}}^{h*}$ and $\lambda_{f,\mathcal{DE}}^* < \lambda_{f,\mathcal{DU}}^*$.

Proof of Proposition 4: Here, we focus on the equilibrium outcome that male drivers "all participate" and male riders "all join"; that is, $\lambda_m^e = \Lambda_m$ and $n_m^e = N_m$. We now analyze the joining and participating behavior of female riders and drivers.

Given the number of participating female drivers n_f, to ensure that female riders are willing to join, we should have

$$U_f(0+\Lambda_m, n_f+N_m) = \frac{\alpha N_m + n_f}{N_m + n_f} R - p - cW(0+\Lambda_m, n_f+N_m) \geq 0, \quad (12)$$

where $W(0+\Lambda_m, n_f+N_m) = \frac{1}{(N_m+n_f)\mu-\Lambda_m}$. Otherwise, female rides would not join. Recall that we have assumed $\mu N_i < \Lambda_i$, $i = f, m$, and thus, $\mu N < \Lambda$. In equilibrium, some female riders must balk the system. The equilibrium effective arrival rate of female riders can be obtained by solving

$$U_f(\lambda_f+\Lambda_m, n_f+N_m) = \frac{\alpha N_m + n_f}{N_m+n_f}R - p - cW(\lambda_f+\Lambda_m, n_f+N_m) = 0,$$

where $W(\lambda_f+\Lambda_m, n_f+N_m) = \frac{1}{(N_m+n_f)\mu-(\lambda_f+\Lambda_m)}$. It can be shown that female riders' equilibrium arrival rate

$$\lambda_f^e(p, n_f, N_m) = \mu(N_m+n_f) - \Lambda_m - \frac{c}{(\alpha N_m+n_f)R/(N_m+n_f)-p}.$$

Then, we have

$$d_f(n_f, N_m) = \frac{\beta\Lambda_m + \lambda_f^e(p, n_f, N_m)}{n_f+N_m} = \frac{\beta\Lambda_m + \mu(N_m+n_f) - \Lambda_m - \frac{c}{(\alpha N_m+n_f)R/(N_m+n_f)-p}}{N_m+n_f}. \quad (13)$$

Taking the first order derivative with respect to n_f, we get

$$\frac{\partial d_f(n_f, N_m)}{\partial n_f} = \frac{(1-\beta)\Lambda_m}{(N_m+n_f)^2} + \frac{c(R-p)}{((\alpha N_m+n_f)R - p(N_m+n_f))^2} > 0$$

due to $p < R$ (otherwise, no rider is willing to join). That is, the safety-adjusted demand rate $d_f(n_f, N_m)$ is increasing in the number of participating female drivers n_f.

Recall that a female driver is willing to participate if and only if her net utility given in (7),

$$S_f(\lambda_f^e(p, n_f, N_m) + \Lambda_m, n_f+N_m) = \frac{\beta\Lambda_m + \lambda_f^e(p, n_f, N_m)}{n_f+N_m}w - r \geq 0.$$

And we just show that $d_f(n_f, N_m)$ increases in n_f. Following the same logic stated in the proof of Lemma 1, we can get the following result:

$$n_f^e = \begin{cases} N_f, & \text{if and only if} \quad w \geq \frac{r(N_m+N_f)}{\beta\Lambda_m+\lambda_f^e(p,N_f,N_m)}, \\ 0, & \text{if and only if} \quad w < \frac{r(N_m+1)}{\beta\Lambda_m+\lambda_f^e(p,1,N_m)}. \end{cases}$$

Note that when $n_f^e = 0$, no female drivers in the system, which is not our focus here. Hereafter, we restrict our attention to the case that $n_f^e = N_f$. Hence, $n_i^e = N_i$, $i \in \{f, m\}$, and the corresponding equilibrium effective arrival rate of female riders for any given price p is $\lambda_f^e(p) = \mu N - \Lambda_m - \frac{c}{\phi_\alpha R - p}$. Based on the inequality (12), we then have the following requirement over p: $0 < p \leq \phi_\alpha R - \frac{c}{\mu N - \Lambda_m}$, and also, the wage should satisfy $w \geq \frac{r(N_m+N_f)}{\beta\Lambda_m+\lambda_f^e(p)} = \frac{rN}{(\beta-1)\Lambda_m+\mu N - c/(\phi_\alpha R-p)}$. Moreover, it can be further shown that

$$\frac{\partial\left(\frac{\partial d_f(n_f,N_m)}{\partial n_f}\right)}{\partial\alpha} = \frac{-2c(R-p)RN_m}{((\alpha N_m+n_f)R-p(N_m+n_f))^3} < 0, \text{ and } \frac{\partial\left(\frac{\partial d_f(n_f,N_m)}{\partial n_f}\right)}{\partial\beta} = \frac{-\Lambda_m}{(N_m+n_f)^2} < 0.$$

That is, $\frac{\partial d_f(n_f,N_m)}{\partial n_f}$ decreases in both α and β. From (13), we then get

$$\frac{\partial d_f(n_f,N_m)}{\partial N_m} = \frac{(1-\beta)\Lambda_m}{(N_m+n_f)^2} + \frac{c(\alpha R - p)}{((\alpha N_m + n_f)R - p(N_m+n_f))^2}.$$

Note that when $\beta \to 1$ and α is small such that $\alpha < p/R$, $\frac{\partial d_f(n_f,N_m)}{\partial N_m} < 0$.

Proof of Proposition 5: Plugging $\lambda_f^e(p) = \mu N - \Lambda_m - \frac{c}{\phi_\alpha R - p}$ and equation (9) into $\Pi_{\mathcal{P}}(p)$, we get

$$\Pi_{\mathcal{P}}(p) = (p - w(p))(\Lambda_m + \lambda_f^e(p)) = p\left(\mu N - \frac{c}{\phi_\alpha R - p}\right) - \frac{rN}{1 + \frac{(\beta-1)\Lambda_m}{\mu N - \frac{c}{\phi_\alpha R - p}}}. \tag{14}$$

Then we can derive that

$$\frac{d\Pi_{\mathcal{P}}(p)}{dp} = \mu N - \frac{c\phi_\alpha R}{(\phi_\alpha R - p)^2} + \frac{crN(\beta-1)\Lambda_m}{((\mu N + (\beta-1)\Lambda_m)(\phi_\alpha R - p) - c)^2}, \tag{15}$$

and

$$\frac{d^2\Pi_{\mathcal{P}}(p)}{dp^2} = -\frac{2c\phi_\alpha R}{(\phi_\alpha R - p)^3} + \frac{2crN(\beta-1)\Lambda_m(\mu N + (\beta-1)\Lambda_m)}{((\mu N + (\beta-1)\Lambda_m)(\phi_\alpha R - p) - c)^3}. \tag{16}$$

Since $p \le \bar{p} = \phi_\alpha R - \frac{c}{\mu N - \Lambda_m}$, we have $p < \phi_\alpha R - \frac{c}{\mu N - \Lambda_m + \beta\Lambda_m}$, which is equivalent to $(\mu N + (\beta - 1)\Lambda_m)(\phi_\alpha R - p) > c$. Besides, $\beta \le 1$ and $\mu N > \Lambda_m$ (the requirement to ensure that the female riders join the system). Hence, $\frac{d^2\Pi_{\mathcal{P}}(p)}{dp^2} < 0$, which implies that $\Pi_{\mathcal{P}}(p)$ is concave in p. Therefore, there must have an interior optimal solution in the range $(0, \bar{p})$ if and only if

$$\frac{d\Pi_{\mathcal{P}}(p)}{dp}\ |_{p\to 0} > 0 \quad \text{and} \quad \frac{d\Pi_{\mathcal{P}}(p)}{dp}\ |_{p\to\bar{p}} < 0.$$

For ease of notation, let

$$\kappa_1(\alpha,\beta) = \frac{d\Pi_{\mathcal{P}}(p)}{dp}\ |_{p\to 0} \quad = \quad \mu N - \frac{c}{\phi_\alpha R} + \frac{crN(\beta-1)\Lambda_m}{((\mu N + (\beta-1)\Lambda_m)\phi_\alpha R - c)^2}, \tag{17}$$

$$\kappa_2(\alpha,\beta) = \frac{d\Pi_{\mathcal{P}}(p)}{dp}\ |_{p\to\bar{p}} \quad = \quad \mu N - \frac{\phi_\alpha R(\mu N - \Lambda_m)^2}{c} + \frac{Nr(\beta-1)(N\mu - \Lambda_m)^2}{c\beta^2\Lambda_m}. \tag{18}$$

Recall that $\phi_\alpha = \frac{\alpha N_m + N_f}{N_m + N_f} = \frac{\alpha N_m + N_f}{N}$ and $\phi'_\alpha = \frac{d}{d\alpha}\left(\frac{\alpha N_m + N_f}{N}\right) = \frac{N_m}{N} > 0$. Again, applying $(\mu N + (\beta-1)\Lambda_m)(\phi_\alpha R - p) > c$, $\beta \le 1$ and $\mu N > \Lambda_m$, for any $\alpha \le 1$ and $\beta \le 1$, we have

$$\frac{\partial\kappa_1(\alpha,\beta)}{\partial\alpha} = \frac{cN_m}{NR\phi_\alpha^2} + \frac{-2crN_m(\beta-1)\Lambda_m(\mu N + (\beta-1)\Lambda_m)R}{((\mu N + (\beta-1)\Lambda_m)\phi_\alpha R - c)^3} > 0,$$

$$\frac{\partial\kappa_1(\alpha,\beta)}{\partial\beta} = crN\Lambda_m\left(\frac{1}{((\mu N + (\beta-1)\Lambda_m)\phi_\alpha R - c)^2} + \frac{2(1-\beta)\phi_\alpha R\Lambda_m}{((\mu N + (\beta-1)\Lambda_m)\phi_\alpha R - c)^3}\right) > 0,$$

$$\frac{\partial\kappa_2(\alpha,\beta)}{\partial\alpha} = -\frac{R(\mu N - \Lambda_m)^2 N_m}{Nc} < 0; \frac{\partial\kappa_2(\alpha,\beta)}{\partial\beta} = \frac{Nr(N\mu - \Lambda_m)^2}{c\Lambda_m}\cdot\frac{2-\beta}{\beta^3} > 0.$$

That is, $\kappa_1(\alpha,\beta)$ increases in both α and β, and $\kappa_2(\alpha,\beta)$ decreases in α but increases in β. When $\alpha=\beta=1$, we have $\phi_\alpha=\frac{\alpha N_m+N_f}{N}=1$. Besides, to ensure that there exists a rider joining the empty system, we must have $R>\frac{c}{N\mu}$. Hence,

$$\kappa_1(1,1)=\mu N-\frac{c}{R}>0 \text{ and } \kappa_2(1,1)=\mu N-\frac{R(N\mu-\Lambda_m)^2}{c}.$$

Note that $\kappa_2(1,1)<0$ when $R>\frac{c\mu N}{(\mu N-\Lambda_m)^2}$.

Proof of Proposition 6 : Recall that $p^*_{\mathcal{P}}$ satisfies equation $\frac{d\Pi_{\mathcal{P}}(p)}{dp}=0$, that is,

$$\frac{d\Pi_{\mathcal{P}}(p)}{dp}\Big|_{p=p^*_{\mathcal{P}}}=\mu N-\frac{c\phi_\alpha R}{(\phi_\alpha R-p^*_{\mathcal{P}})^2}+\frac{crN(\beta-1)\Lambda_m}{((\mu N+(\beta-1)\Lambda_m)(\phi_\alpha R-p^*_{\mathcal{P}})-c)^2}=0. \tag{19}$$

Let $f_1(p^*_{\mathcal{P}}(\alpha,\beta),\alpha,\beta)=\frac{d\Pi_{\mathcal{P}}(p)}{dp}\Big|_{p=p^*_{\mathcal{P}}}$. Hence, according to the implicit function theorem, we have

$$\begin{aligned}
\frac{\partial p^*_{\mathcal{P}}(\alpha,\beta)}{\partial\alpha} &= -\frac{\partial f_1(p^*_{\mathcal{P}}(\alpha,\beta),\alpha,\beta)/\partial\alpha}{\partial f_1(p^*_{\mathcal{P}}(\alpha,\beta),\alpha,\beta)/\partial p^*_{\mathcal{P}}}=-\frac{cR\phi_\alpha^{'}\left(\frac{p^*_{\mathcal{P}}+\phi_\alpha R}{(\phi_\alpha R-p^*_{\mathcal{P}})^3}+\frac{2rN(1-\beta)\Lambda_m(\mu N+(\beta-1)\Lambda_m)}{((\mu N+(\beta-1)\Lambda_m)(\phi_\alpha R-p^*_{\mathcal{P}})-c)^3}\right)}{\frac{d^2\Pi_{\mathcal{P}}(p)}{dp^2}\,|_{p=p^*_{\mathcal{P}}}}>0, \text{and}\\
\frac{\partial p^*_{\mathcal{P}}(\alpha,\beta)}{\partial\beta} &= -\frac{\partial f_1(p^*_{\mathcal{P}}(\alpha,\beta),\alpha,\beta)/\partial\beta}{\partial f_1(p^*_{\mathcal{P}}(\alpha,\beta),\alpha,\beta)/\partial p^*_{\mathcal{P}}}=-\frac{crN\Lambda_m\frac{(\mu N+(1-\beta)\Lambda_m)(\phi_\alpha R-p^*_{\mathcal{P}})-c}{((\mu N+(\beta-1)\Lambda_m)(\phi_\alpha R-p^*_{\mathcal{P}})-c)^3}}{\frac{d^2\Pi_{\mathcal{P}}(p)}{dp^2}\,|_{p=p^*_{\mathcal{P}}}}>0,
\end{aligned}$$

because it has been shown in the proof of Proposition 5 that $\frac{d^2\Pi_{\mathcal{P}}(p)}{dp^2}\,|_{p=p^*_{\mathcal{P}}}<0$ and $(\mu N+(\beta-1))(\phi_\alpha R-p^*_{\mathcal{P}})-c>0$ (as $p^*_{\mathcal{P}}<\bar{p}<\phi_\alpha R-\frac{c}{\mu N+(\beta-1)\Lambda_m}$). Thus, $p^*_{\mathcal{P}}$ increases in both α and β.

Since $p^*_{\mathcal{P}}<\bar{p}<\phi_\alpha R$, we have

$$-\frac{\partial f_1(p^*_{\mathcal{P}}(\alpha,\beta),\alpha,\beta)}{\partial\alpha}>-R\phi_\alpha^{'}\left(\frac{2c\phi_\alpha R}{(\phi_\alpha R-p^*_{\mathcal{P}})^3}+\frac{2crN(1-\beta)\Lambda_m(\mu N+(\beta-1)\Lambda_m)}{((\mu N+(\beta-1)\Lambda_m)(\phi_\alpha R-p^*_{\mathcal{P}})-c)^3}\right)=\phi_\alpha^{'}R\frac{d^2\Pi_{\mathcal{P}}(p)}{dp^2}\Big|_{p=p^*_{\mathcal{P}}},$$

where the last equality comes from equation (16) (stated in the proof of Proposition 5). Recall from the proof of Proposition 5 that

$$\frac{d^2\Pi_{\mathcal{P}}(p)}{dp^2}\Big|_{p=p^*_{\mathcal{P}}}<0.$$

Consequently,

$$\frac{\partial p^*_{\mathcal{P}}(\alpha,\beta)}{\partial\alpha}=-\frac{\partial f_1(p^*_{\mathcal{P}}(\alpha,\beta),\alpha,\beta)/\partial\alpha}{\partial f_1(p^*_{\mathcal{P}}(\alpha,\beta),\alpha,\beta)/\partial p^*_{\mathcal{P}}}=-\frac{\partial f_1(p^*_{\mathcal{P}}(\alpha,\beta),\alpha,\beta)/\partial\alpha}{\frac{d^2\Pi_{\mathcal{P}}(p)}{dp^2}\Big|_{p=p^*_{\mathcal{P}}}}<\phi_\alpha^{'}R.$$

Recall that $\lambda^*_{f,\mathcal{P}}=\mu N-\Lambda_m-\frac{c}{\phi_\alpha R-p^*_{\mathcal{P}}}$. We can easily show that

$$\frac{d\lambda^*_{f,\mathcal{P}}}{d\alpha}=\frac{c(\phi_\alpha^{'}R-\frac{\partial p^*_{\mathcal{P}}}{\partial\alpha})}{(\phi_\alpha R-p^*_{\mathcal{P}})^2}>0 \quad\text{and}\quad \frac{d\lambda^*_{f,\mathcal{P}}}{d\beta}=\frac{-c\frac{\partial p^*_{\mathcal{P}}}{\partial\beta}}{(\phi_\alpha R-p^*_{\mathcal{P}})^2}<0.$$

Thus, $\lambda^*_{f,\mathcal{P}}$ increases in α but decreases in β.

Recall that $w^*_{\mathcal{P}} = \frac{rN}{(\beta-1)\Lambda_m+\mu N-c/(\phi_\alpha R-p^*_{\mathcal{P}})} = \frac{rN}{\beta\Lambda_m+\lambda^*_{f,\mathcal{P}}}$. We can show that

$$\frac{dw^*_{\mathcal{P}}}{d\alpha} = -\frac{rNc\frac{(\phi'_\alpha R-\frac{\partial p^*_{\mathcal{P}}}{d\alpha})}{(\phi_\alpha R-p^*_{\mathcal{P}})^2}}{\left((\beta-1)\Lambda_m+\mu N-c/(\phi_\alpha R-p^*_{\mathcal{P}})\right)^2} < 0,$$

since $\frac{\partial p^*_{\mathcal{P}}(\alpha,\beta)}{\partial\alpha} < \phi'_\alpha R$. Thus, $w^*_{\mathcal{P}}$ is decreasing in α.

Recall that $\lambda^e_f(p) = \mu N - \Lambda_m - \frac{c}{\phi_\alpha R-p}$ and $w = \frac{rN}{\beta\Lambda_m+\lambda^e_f(p)}$ (see the proof of Proposition 4). Through one-to-one mapping, we get

$$p = \phi_\alpha R - \frac{c}{\mu N-\Lambda_m-\lambda^e_f}.$$

Then, we can rewrite the platform's objective function (14) to be a function of any given effective arrival rate λ^e_f, which is

$$\Pi_{\mathcal{P}}(\lambda^e_f) = (p-w)(\Lambda_m+\lambda^e_f) = \left(\phi_\alpha R - \frac{c}{\mu N-\Lambda_m-\lambda^e_f} - \frac{rN}{\beta\Lambda_m+\lambda^e_f}\right)(\Lambda_m+\lambda^e_f). \tag{20}$$

The optimal effective arrival rate $\lambda^*_{f,\mathcal{P}}$ then must satisfy

$$\begin{aligned}\frac{d\Pi_{\mathcal{P}}(\lambda^e_f)}{d\lambda^e_f}\,|_{\lambda^e_f=\lambda^*_{f,\mathcal{P}}} \;=\;& (\Lambda_m+\lambda^*_{f,\mathcal{P}})\left(\frac{rN}{(\beta\Lambda_m+\lambda^*_{f,\mathcal{P}})^2} - \frac{c}{(\mu N-\Lambda_m-\lambda^*_{f,\mathcal{P}})^2}\right)\\ &+\phi_\alpha R - \frac{c}{\mu N-\Lambda_m-\lambda^*_{f,\mathcal{P}}} - \frac{rN}{\beta\Lambda_m+\lambda^*_{f,\mathcal{P}}} = 0.\end{aligned} \tag{21}$$

Because the profit margin must be strictly positive profit, it is required that

$$p^*_{\mathcal{P}} - w^*_{\mathcal{P}} = \phi_\alpha R - \frac{c}{\mu N-\Lambda_m-\lambda^*_{f,\mathcal{P}}} - \frac{rN}{\beta\Lambda_m+\lambda^*_{f,\mathcal{P}}} > 0.$$

Then, from (21), we get

$$\frac{rN}{(\beta\Lambda_m+\lambda^*_{f,\mathcal{P}})^2} < \frac{c}{(\mu N-\Lambda_m-\lambda^*_{f,\mathcal{P}})^2} < \frac{2c\mu N}{(\mu N-\Lambda_m-\lambda^*_{f,\mathcal{P}})^3}. \tag{22}$$

Furthermore, we can further simplify (21) to the following equation:

$$\frac{d\Pi_{\mathcal{P}}(\lambda^e_f)}{d\lambda^e_f}\,|_{\lambda^e_f=\lambda^*_{f,\mathcal{P}}} = \phi_\alpha R - \frac{c\mu N}{(\mu N-\Lambda_m-\lambda^*_{f,\mathcal{P}})^2} + \frac{rN(1-\beta)\Lambda_m}{(\beta\Lambda_m+\lambda^*_{f,\mathcal{P}})^2} = 0.$$

Then, we have

$$\frac{\partial}{\partial\beta}\left(\frac{d\Pi_{\mathcal{P}}(\lambda^e_f)}{d\lambda^e_f}\,|_{\lambda^e_f=\lambda^*_{f,\mathcal{P}}}\right) = -\frac{2c\mu N\cdot\frac{d\lambda^*_{f,\mathcal{P}}}{d\beta}}{(\mu N-\Lambda_m-\lambda^*_{f,\mathcal{P}})^3} - \frac{rN\Lambda_m}{(\beta\Lambda_m+\lambda^*_{f,\mathcal{P}})^2} - \frac{2rN\Lambda_m(1-\beta)\left(\Lambda_m+\frac{d\lambda^*_{f,\mathcal{P}}}{d\beta}\right)}{(\beta\Lambda_m+\lambda^*_{f,\mathcal{P}})^3} = 0,$$

from which we get

$$-\frac{d\lambda^*_{f,\mathcal{P}}}{d\beta} = \frac{\frac{rN}{(\beta\Lambda_m+\lambda^*_{f,\mathcal{P}})^2} + \frac{2rN(1-\beta)\Lambda_m}{(\beta\Lambda_m+\lambda^*_{f,\mathcal{P}})^3}}{\frac{2c\mu N}{(\mu N-\Lambda_m-\lambda^*_{f,\mathcal{P}})^3} + \frac{2rN(1-\beta)\Lambda_m}{(\beta\Lambda_m+\lambda^*_{f,\mathcal{P}})^3}}\Lambda_m < \Lambda_m,$$

where the last inequality comes from (22). Thus,

$$\frac{d\lambda^*_{f,\mathcal{P}}}{d\beta} + \Lambda_m > 0. \tag{23}$$

Then, we can show that

$$\frac{dw^*_{\mathcal{P}}}{d\beta} = \frac{d(\frac{rN}{\beta\Lambda_m+\lambda^*_{f,\mathcal{P}}})}{d\beta} = \frac{-rN}{(\beta\Lambda_m+\lambda^*_{f,\mathcal{P}})^2}(\Lambda_m + \frac{d\lambda^*_{f,\mathcal{P}}}{d\beta}) < 0.$$

That is, $w^*_{\mathcal{P}}$ is decreasing in β.

Based on (20), by applying the envelope theorem, we have

$$\frac{d\Pi^*_{\mathcal{P}}}{d\alpha} = \frac{\partial\Pi_{\mathcal{P}}(\lambda^e_f,\alpha)}{\partial\alpha}\Big|_{\lambda^e_f=\lambda^*_{f,\mathcal{P}}} = \phi'_\alpha R(\Lambda_m + \lambda^*_{f,\mathcal{P}}) > 0$$

since $\phi'_\alpha = \frac{d}{d\alpha}\left(\frac{\alpha N_m+N_f}{N_m+N_f}\right) = \frac{N_m}{N_m+N_f} > 0$. We can also show that

$$\frac{d\Pi^*_{\mathcal{P}}}{d\beta} = \frac{\partial\Pi_{\mathcal{P}}(\lambda^e_f,\alpha)}{\partial\beta}\Big|_{\lambda^e_f=\lambda^*_{f,\mathcal{P}}} = \frac{rN\Lambda_m(\Lambda_m+\lambda^*_{f,\mathcal{P}})}{(\beta\Lambda_m+\lambda^*_{f,\mathcal{P}})^2} > 0.$$

Furthermore, since $\frac{d\lambda^*_{f,\mathcal{P}}}{d\alpha} > 0$, $\frac{d\lambda^*_{f,\mathcal{P}}}{d\beta} < 0$ and $\frac{d\lambda^*_{f,\mathcal{P}}}{d\beta} + \Lambda_m > 0$, we obtain

$$\frac{d^2\Pi^*_{\mathcal{P}}}{d\alpha^2} = \phi'_\alpha R\frac{d\lambda^*_{f,\mathcal{P}}}{d\alpha} > 0 \quad \text{and} \quad \frac{d^2\Pi^*_{\mathcal{P}}}{d\beta^2} = \frac{rN\Lambda_m\left((\beta\Lambda_m+\lambda^*_{f,\mathcal{P}})\frac{d\lambda^*_{f,\mathcal{P}}}{d\beta} - 2(\Lambda_m+\lambda^*_{f,\mathcal{P}})(\Lambda_m+\frac{d\lambda^*_{f,\mathcal{P}}}{d\beta})\right)}{(\beta\Lambda_m+\lambda^*_{f,\mathcal{P}})^3} < 0.$$

Thus, $\Pi^*_{\mathcal{P}}$ is increasing and convex in α but increasing and concave in β.

Proof of Proposition 7: Based on Proposition 5, by setting $\alpha = \beta = 1$, we get $\phi_\alpha = \frac{\alpha N_m+N_f}{N} = 1$, the first order condition becomes

$$\frac{d\Pi_{\mathcal{P}}(p)}{dp} = \mu N - \frac{cR}{(R-p)^2} = 0.$$

It can be easily shown that the optimal price $p^*_{\mathcal{P}} = R - \sqrt{\frac{cR}{\mu N}}$. Next, from Proposition 4, we can get the total effective arrival rate $\lambda^*_{f,\mathcal{P}} + \lambda^*_{m,\mathcal{P}} = \mu N - \frac{c}{R-p^*_{\mathcal{P}}} = \mu N - \sqrt{\frac{c\mu N}{R}}$. Recall from Proposition 2 that the gender-based optimal price in the dedicated system $\mathcal{DU}$ is $p^*_i = R - \sqrt{\frac{cR}{\mu N_i}}$, $i = f, m$. The corresponding equilibrium arrival rate in subsystem i is $\lambda^*_i = \mu N_i - \sqrt{\frac{c\mu N_i}{R}}$.

Since $\sqrt{N} = \sqrt{N_m+N_f} < \sqrt{N_m} + \sqrt{N_f}$, we have

$$\lambda^*_{f,\mathcal{P}}+\lambda^*_{m,\mathcal{P}}-(\lambda^*_m+\lambda^*_f) = \mu N-\sqrt{\frac{c\mu N}{R}}-\left(\mu N_m - \sqrt{\frac{c\mu N_m}{R}} + \mu N_f - \sqrt{\frac{c\mu N_f}{R}}\right) = \sqrt{\frac{c\mu}{R}}(\sqrt{N_m}+\sqrt{N_f}-\sqrt{N}) > 0.$$

This reveals that the total effective arrival rate of riders in the pooling system is higher than that in the dedicated system $\mathcal{DU}$. We now compare the platform's expected profits in the pooling system and the dedicated system $\mathcal{DU}$. We can show that

$$\begin{aligned}\Pi_{\mathcal{P}}(p^*_{\mathcal{P}}) - \Pi_m(p^*_m) - \Pi_f(p^*_f) &= p^*_{\mathcal{P}}(\lambda^*_{f,\mathcal{P}} + \lambda^*_{m,\mathcal{P}}) - rN - (p^*_m\lambda^*_m - rN_m + p^*_f\lambda^*_f - rN_f) \\ &> p^*_{\mathcal{P}}(\lambda^*_m + \lambda^*_f) - p^*_m\lambda^*_m - p^*_f\lambda^*_f > 0,\end{aligned}$$

because $N = N_m + N_f$. Thus, $\Pi^*_{\mathcal{P}} > \Pi^*_{\mathcal{DU}}$. This completes the proof.