The Impact of Crop Minimum Support Prices on Crop Selection and Farmer Welfare in the presence of Strategic Farmers

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In many developing countries, governments often use *Minimum support prices* (MSPs) as interventions to (i) safeguard farmers' income against crop price falls, and (ii) ensure sufficient and balanced production of different crops. In this paper, we examine two questions: (1) What is the impact of MSPs on the farmers' crop selection and production decisions, future crop availabilities, and farmers' expected profits? (2) What is the impact of strategic farmers on crop selection and production decisions, future crop availabilities, and farmers' expected profits? To explore these questions, we present a model in which the market consists of two types of farmers (with heterogeneous production costs): myopic farmers (who make their crop selection and production decisions based on recent market prices) and strategic farmers (who make their decisions by taking all other farmers' decisions into consideration). By examining the dynamic interactions among these farmers for the case when there are two (complementary or substitutable) crops for each farmer to select to grow, we obtain the following results. First, we show that, regardless of the values of the MSPs offered to the crops, the price disparity between the crops worsens as the complementarity between the crops increases. Second, we find that MSP is not always beneficial. In fact, offering MSP for a crop can hurt the profit of those farmers who grow that crop especially when the proportion of strategic farmers is sufficiently small. Third, a bad choice of MSPs can cause the expected quantity disparity between crops to worsen. By taking these two drawbacks of MSPs into consideration, we discuss ways to select effective MSPs that can improve farmers' expected profit and reduce quantity disparity between crops.

Key words: Minimum support prices, subsidies, agricultural supply chains, government and public policy
History:

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1. Introduction

In many developing countries, the agricultural sector is important because: (1) it offers a source of income to a large number of small rural households, and (2) it provides a stable food supply for the country. As such, developing efficient and effective agro-policies to improve farmers' earnings and to stabilize crop availabilities and prices are critical (Thorbecke 1982). While governments in developing countries design and develop a wide variety of agro-policies ranging from input subsidies (for seeds and fertilizers, power, etc.) to output subsidies (for storage and transportation), in this paper, we shall focus on a particular type of output subsidies that is called the *Minimum Support Price* (MSPs). MSPs for different crops are offered by governments in many developing countries like Bangladesh, Brazil, China, India, Pakistan, and Thailand. For example, in 2017, the Indian government offers MSPs for 23 crops, which comprise 7 cereals, 5 pulses (grain seeds of legumes), 7 oil seeds, and 4 commercial crops. Essentially, MSP of a crop serves as a form of "contingent subsidy" to farmers who grow that crop: when the market price of a crop falls below its MSP, government purchases the crop from the farmers at the pre-announced MSP of the crop by absorbing the price shortfall (i.e., the difference between the market price and the MSP). By guaranteeing minimum prices for certain crops, a government intends to provide incentives for farmers to grow a more balanced mixture of crops.

This paper examines the implications of MSPs on: (1) farmers' earnings, and (2) quantity disparity between two crops. Our model is based on the setting of a developing country. To motivate our research questions involving MSPs, let us consider the role played by MSPs in the Indian agricultural sector. MSPs have been introduced as a part of the Green Revolution in 1965 when India's cereal imports reached an alarming stage. This event has triggered the Indian government to establish the Commission for Agricultural Costs & Prices (CACP) with the mandate to develop crop-price policies. As a part of these reforms, MSPs were introduced as incentives to benefit Indian farmers and consumers by increasing food supply at affordable prices (Chand 2003, Malamasuri et al. 2013).[1] With an efficient MSP scheme developed by CACP over the years, India evolved from a grain "deficient" country in mid-1960's to a grain "surplus" country by early 1980's. However, due to the fact that MSPs in India were geared towards rice and wheat production, there was a severe shortage of coarse cereals and oil seeds (Chand 2003, Parikh and Chandrashekhar 2007) and an over-production of rice and wheat. Such an imbalance in the availability of agricultural commodities can lead to micro-nutrient malnutrition (or hidden hunger) (Byerlee et al. 2007). This observation suggests that the efficacy of MSPs should be measured in terms of the availability of different crops and the farmers' expected earnings.

[1] When determining MSPs, CACP takes into account six factors, namely (i) demand and supply, (ii) cost of production, (iii) market price trends, (iv) inter-crop price parity, (v) terms of trade between agriculture and non-agriculture, and (vi) likely implications of MSP on consumers of *that* product (Commission for Agricultural Costs & Prices 2017). In our analysis we take into account the factors (i), (ii), (iii), (iv) and (vi). We account for the supply through considering the response of the farmers' sowing decisions towards the MSPs announced. We account for the demand through the inverse demand functions of the crops. By assuming that the farmers are heterogeneous in their production costs for the two crops, we account for (ii). Based on his type – strategic or myopic – each farmer considers the past price of the crops in a specific way. Thus, we account for (iii) through farmers' perceptions of past prices. We account for (iv) through farmers' individual rationality and their choice of the crops in the light of the past prices and the MSPs of the crops. Finally, we account for (vi) by analyzing the impact of MSP, in confluence with the past market prices, on the future market prices. Thus, we make an attempt to develop a unified framework using a parsimonious model with two crops to comprehensively capture the main features of an MSP scheme.

In this paper we develop a parsimonious model to analyze the impact of MSPs on farmers' earnings, crop availabilities, and crop prices by considering a setting in which there are two (complementary or substitutable) crops from which each farmer can choose one crop to cultivate. In addition to heterogeneous production costs for each crop, we also consider the case when the market is comprised of myopic farmers (who make their crop selection and production decisions based on recent market pries) and strategic farmers (who make their decisions by taking all other farmers' decisions into consideration). By examining the dynamic interactions among myopic and strategic farmers, our model enables us to examine two research questions:

1. What is the impact of MSPs on the farmers' crop selection and production decisions, future crop availabilities, and farmers' expected revenues?
2. What is the impact of strategic farmers on crop selection and production decisions, future crop availabilities, and farmers' expected revenues?

Our equilibrium analysis enables us to obtain the following results. First, we find in Corollaries 1 and 6 that, regardless of the values of MSPs, the price disparity between the crops worsens as the complementarity between the crops increases. Second, we show in Proposition 5 that MSP is not always beneficial. In Proposition 5, specifically, we show that moderately low MSP for a crop will degrade the expected profits of the farmers growing the crop if the number of strategic farmers is very small. Thus, choosing an inappropriate MSP for a crop, especially when there are very few strategic farmers, can actually defeat the intended goal of offering MSP for the crop, which is to benefit the farmers growing the crop. Also, we show in Proposition 4 that when the proportion of strategic farmers is small, offering moderately low MSP for a crop can actually cause fewer strategic farmers to grow that crop. Third, in Proposition 3, we find that the total production of a crop is increasing in the MSP offered for the crop. Therefore, a bad choice of MSPs can cause the production quantity disparity between crops to worsen. Hence, to reduce quantity disparity between crops, a carefully designed MSP policy is critical. Finally, through formulating an optimization problem for a policy-maker to choose crop MSPs in order to maximize social welfare, we illustrate that offering MSPs to complementary (i.e., dissimilar) crops has the potential to achieve higher social welfare at a lower expected expenditure to the policy-maker.

The paper is organized as follows. Section 2 reviews literature related to MSPs. In Section 3 we introduce the model and discuss various assumptions. To explicate our analysis about myopic and strategic farmers' crop selection and production decisions, we examine the case when MSPs are not present in Section 4. Section 5 extends our analysis to the case when MSPs are present. In Section 6 we formulate and discuss the optimization problem of the government whose objective is to set MSPs in order to improve farmers' welfare and crop balance. We conclude in Section 7.

2. Literature Review

Our research pertains to agro-policies that affect both crop selection and crop production by myopic and strategic farmers. The literature on MSPs is vast in the agricultural economics discipline and the reader is referred to Tripathi et al. (2013) and the references therein for a good synopsis on MSPs in developing countries. Without accounting for the price interactions between crops with MSP support and those crops without MSP support, Fox (1956) develops macro-economics analysis to evaluate the impact of MSPs and finds that MSPs can mitigate the fall in GNP during a recession. Dantwala (1967) finds that in spite of the increasing MSPs, the crop market prices continue to rise. More recently, Subbarao et al. (2011) shows evidence that the increase in market price is caused by the increase in MSPs. In the same vein, Chand (2003) presents qualitative assessment of the ill-effects of the wheat- and-rice-centric MSPs on the Indian economy. Chhatre et al. (2016) point out that many farmers in India moved to cultivating high-yield varieties of rice and wheat due to the wheat- and-rice-centric MSPs offered by the Indian government. The authors also identify the various socio-economic and environmental problems associated with an improper choice of MSPs. Besides the Indian context, Spitze (1978) analyzes the impact of federal policy (The Food and Agriculture Act of 1977) on agriculture in the United States. The author states that continuous improvement in gathering and analyzing information is a prerequisite for the design of effective MSPs.

Recent papers on agricultural operations in OM literature include: (i) Tang et al. (2015), Chen and Tang (2015), Parker et al. (2016), Liao et al. (2017) focus on the economic value of disseminating agricultural information to the farmers, (ii) Kazaz and Webster (2011), Dawande et al. (2013), Huh and Lall (2013) examine the issue of resource and inventory management, (iii) Huh et al. (2012), Federgruen et al. (2015), An et al. (2015) focus on contract farming and farmer aggregation, and (iv) Hu et al. (2016), Alizamir et al. (2015), Guda et al. (2016) examine social responsibility and public policy issues arising from the agricultural sector.

While our paper is related to group (iv), it differs from the these papers in the following manner. First, Hu et al. (2016) focus on the value of strategic farmers in the context of a single crop with a deterministic demand function. They show that a tiny fraction of strategic farmers can stabilize the steady state prices. They also extend their analysis to two crops with independent market prices. In contrast, our goal is to evaluate the impact of MSPs on farmers' crop selection and production decisions, and on the market prices of two crops with dependent and yet stochastic market price.

Second, Alizamir et al. (2015) focus on the impact of federal policy on agriculture industry in the United States. They compare two schemes (Price Loss Coverage (PLC) and the Agriculture Risk Coverage (ARC) programs) with respect to (i) farmers' welfare, (ii) federal expenditure, and (iii) consumer welfare. While PLC is akin to MSP, our paper differs from Alizamir et al. (2015) in three aspects. First, they assume there are finite number of farmers, and the production of each farmer can affect the market price (i.e., farmers are price setters). In contrast, our context is that of developing countries, and we consider infinitesimally small farmers whose individual decisions do not affect the market price (i.e., farmers are price-takers). Second, they analyze the case of only one crop, while we consider two crops that can be substitutes or complements. Hence, by capturing the interaction between two crops in our model, we analyze the simultaneous impact of the MSP of each crop on the production of both the crops. Third, they do not consider the existence of myopic and strategic farmers, while we consider a mixture of both myopic and strategic farmers in our model. Our model fits well in the context of developing countries where a large portion of the farming communities are smallholders who are myopic: their crop selection and production decisions are purely based on the most recently oberved market price.

Finally, Guda et al. (2016) examine the role of MSPs in emerging economies, but there are two fundamental differences between our paper and theirs. The first difference is that we assume heterogeneity in farmers' production costs, while they assume homogeneous production costs. In general, the cost of cultivating a crop can vary across farmers depending on the local soil, the climatic conditions, and the farming practices they employ. Second, they consider a single crop and relegate the case of multiple crops as future research due to the inherent complexity. As such, our paper attempts to examine the impact of the MSPs of two crops on the availabilities of one another.

3. Model Preliminaries

We consider two crops (A and B) to be produced by heterogeneous farmers whose production costs are uniformly distributed over the interval $[-0.5, 0.5]$ as in the Hotelling's model. These two crops can be substitutes (e.g., rice and wheat) or complements (e.g., rice and pulses/lentils). For a farmer located at $x \in [-0.5, 0.5]$, his costs of producing crops A and B are given by $c_A(x) = x + 0.5$ and $c_B(x) = 0.5 - x$, respectively. We assume that the farmers are infinitesimally small so that each farmer can produce 1 unit of a crop and each farmer is a price taker.

In our model, the market price of a crop depends on the available quantity of the crop. Let q_t^{kT} denote the "total" availability of crop $k \in \{A, B\}$ in period t and let p_t^k denote the market price of crop $k \in \{A, B\}$ in period t. For ease of exposition, we normalize the size of markets to 1 so that $q_t^{kT} \leqslant 1$ for $k \in \{A, B\}$. Throughout this paper, we assume that the market price p_k^t for crop $k \in \{A, B\}$ in period t satisfies:

$$\begin{aligned} p_t^A &= a - \rho q_t^{AT} - \alpha q_t^{BT} + \epsilon_t^A = \mathbb{E}[p_t^A] + \epsilon_t^A, \quad \text{and} \\ p_t^B &= a - \alpha q_t^{AT} - \rho q_t^{BT} + \epsilon_t^B = \mathbb{E}[p_t^B] + \epsilon_t^B, \end{aligned} \tag{1}$$

where ρ (> 0) is the price sensitivity, and α is a measure of substitutability (if $\alpha > 0$) or complementarity (if $\alpha < 0$) between the two crops. As commonly assumed in the literature for substitutable/complementary products, we shall assume that $\alpha < \rho$. The random variables ϵ_t^k ($k \in \{A, B\}$) denote the market uncertainty in period t. We assume ϵ_t^k are iid (across t and k) with mean 0, variance σ^2 and with distribution and density functions $F(\cdot)$ and $f(\cdot)$, respectively.[2] We also assume that the distribution $F(\cdot)$ has support over a range of value so that the market price p_t^k in non-negative. Let $\overline{F}(\cdot) = (1 - F(\cdot))$ denote the complementary cumulative distribution of ϵ_t^k. The expected profit of a farmer at location x who grows crop $k \in \{A, B\}$ is given by:

$$\Pi_t^k(x) = \mathbb{E}[p_t^k] - c_k(x) = a - \rho q_t^{kT} - \alpha q_t^{jT} - c_k(x), \ j \neq k. \tag{2}$$

[2] To keep the notation simple, we assume that ϵ_t^A and ϵ_t^B follow the same distribution. However, our analysis can be extended to the case of different distributions.

For ease of exposition, we define $r \equiv \rho - \alpha$ (> 0), so that r measures the "dissimilarity" between the two crops, and $\phi \equiv a - \frac{\rho+\alpha}{2}$, which corresponds to the expected market price when half of the market grows A (grows B) (i.e., when $q_t^{AT} = q_t^{BT} = 0.5$). Finally, wherever applicable, we denote the price vector in period t by $\mathbf{P}_t = [p_t^A, p_t^B]^T$. To simplify our exposition and our analysis (e.g., by ruling out the boundary equilibrium solution), we shall make the following assumptions:

Assumption 1. *In each period, each farmer will not be idle and will select exactly one crop to grow.*

First, the non-idling assumption is reasonable especially when the farmer's production cost is lower than the market price p_t^k in general. Second, due to economies of scale, small land-holders in emerging markets cannot afford to grow multiple crops.

Next, let Δp_t be the price disparity between crops A and B in period t. By applying (1) and the fact that $r = \rho - \alpha$, we obtain:

$$\Delta p_t = p_t^A - p_t^B = -r(q_t^{AT} - q_t^{BT}) + \xi_t, \ \forall t,$$

where $\xi_t = \epsilon_t^A - \epsilon_t^B$. To ensure that the price disparity Δp_t is stable over time so that we can rule out boundary equilibrium solution, we shall make the following assumption.

Assumption 2. *The dissimilarity between two crops r satisfies: $0 < r \equiv (\rho - \alpha) < 1$. Also, the variance of the market uncertainty is sufficiently less than 1 (i.e., $\sigma^2 << 1$).*

Since, r measures the "dissimilarity" between two crops, we can treat the crops to be substitutes if r is small and to be complements if r is large. Furthermore, because $0 < r < 1$, $|q_t^A - q_t^B| \leqslant 1$, and $\mathbb{E}[\xi_t] = 0$, it is easy to check that $|\mathbb{E}[\Delta p_t]| \leqslant r < 1$, for all t. Furthermore, when $|\mathbb{E}[\Delta p_t]| \leqslant 1$ and $\sigma^2 << 1$, we can ascertain that $|\Delta p_t| < 1$ nearly always holds so that we can effectively assume $\mathbb{P}(|\Delta p_t| < 1) \approxeq 1$.[3]

[3] We formalize this finding in the lemma below.

Lemma 1. *Let the random variable $X \sim U[-\beta, \beta]$ denote the type of the farmer so that the production costs of crops A and B for farmer who is located at $X = x$ are given by $x + \beta$ and $\beta - x$ respectively. Then,*

$$P\left(|p_t^A - p_t^B| > 2\beta\right) \leqslant P\left(|\xi_t| > 2\beta(1-r)\right) \leqslant \frac{\sigma^2}{2\beta^2(1-r)^2}, \ \text{where } \xi_t = \epsilon_t^A - \epsilon_t^B.$$

Hence, for a given $r \in (0,1)$, we have $P\left(|\Delta p_t| \geqslant 2\beta\right) \to 0$ if $\beta >> \sigma$.

Without loss of generality, we scale β to 1 in our model and assume that σ is sufficiently small (i.e., $\sigma << 1$). Hence, by virtue of Lemma 1, there will be a positive production of each crop in every period.

Assumption 3. *There are two types of farmers in the market: myopic and strategic. Also, the proportion of strategic farmers is $\theta \in [0,1]$.*

In our model, we assume that myopic farmers are those who make their crop selection and production decisions purely based on recent market prices. However, strategic farmers are forward looking, and they make their decisions by taking all other farmers' decisions into consideration. For the convenience of notation, we define $z^+ = \max\{z, 0\}$ and let $\overline{\theta} \equiv (1-\theta)$ throughout this paper.

4. Model Analysis: In the Absence of MSPs

To explicate the analysis that involves crop selection and crop production by myopic and strategic farmers with heterogeneous production costs, we first examine the case when MSPs are absent. (We shall extend our analysis to the case when MSPs are present in Section 5.) By considering different decision making mechanisms adopted by different types of farmers, we now determine their crop selection and production decisions in period t for any realized market prices in period $t-1$ (i.e., p_{t-1}^k for $k \in \{A, B\}$).

Myopic farmers' crop selection and production decisions in period t

Let q_t^{km} denote the quantity of crop $k \in \{A, B\}$ to be produced by the myopic farmers in period t, and let p_t^{km} denote the price of crop k in period t as "anticipated" by the myopic farmers. In our model, each myopic farmer anticipates that $p_t^{km} = p_{t-1}^k$, $k \in \{A, B\}$. Hence, a myopic farmer at $x \in [-0.5, 0.5]$ will grow crop A if $p_t^{Am} - c_A(x) \geqslant p_t^{Bm} - c_B(x)$, and will grow crop B, otherwise. Observe that the myopic farmer located in τ^m is indifferent between the two crops, where $\tau^m = \{x : p_t^{Am} - c_A(x) = p_t^{Bm} - c_B(x)\}$. Because $p_t^{km} = p_{t-1}^k$ for $k \in \{A, B\}$, $\tau^m = \frac{p_{t-1}^A - p_{t-1}^B}{2}$. By applying Assumption 2, we can conclude that $\tau^m \in (-0.5, 0.5)$. Given the threshold τ^m, the segment $\{x : -0.5 \leqslant x < \tau^m\}$ of myopic farmers will grow only crop A, while the segment $\{x : \tau^m < x \leqslant 0.5\}$ of myopic farmers will grow only crop B.

Strategic farmers' crop selection and production decisions in period t

Let q_t^{ks} denote the quantity of crop $k \in \{A, B\}$ to be produced by the strategic farmers in period t, and let p_t^{ks} denote the price of crop k in period t as "anticipated" by the strategic farmers. By taking all other farmers' decisions into consideration, we shall show that strategic farmers can actually anticipate the expected market price in equilibrium so that $p_t^{ks} = \mathbb{E}[p_t^k]$. Also, we shall show later that, among the strategic farmers, the segment $\{x : -0.5 \leqslant x < \tau^s\}$ will grow only A and the segment $\{x : \tau^s < x \leqslant 0.5\}$ will grow only B, where $\tau^s \equiv \tau^s(p_t^{As}, p_t^{Bs}) = \{x : p_t^{As} - c_A(x) = p_t^{Bs} - c_B(x)\}$. (We shall determine the threshold τ^s value in Proposition 1).

Let us illustrate the decisions of different types of framers graphically. Without loss of generality, let us consider the case when $p_{t-1}^A > p_{t-1}^B$. Figure 1 depicts the crop selection and production decisions of myopic and strategic farmers. Also, by noting that the market consists of θ strategic and $\overline{\theta} \equiv (1-\theta)$ myopic farmers, the figure depicts the overall crop selection and production. Recall that τ^s and τ^m are the threshold values associated with the myopic and the strategic farmers, respectively. Therefore, the total quantities of crop A produced by the myopic and the strategic farmers are $q_t^{Am} = \overline{\theta}(\tau^m + 0.5)$ and $q_t^{As} = \theta(\tau^s + 0.5)$, respectively. Thus, the total availability of crop A in period t is given by $q_t^{AT} = q_t^{Am} + q_t^{As} = \theta(\tau^s + 0.5) + \overline{\theta}(\tau^m + 0.5) = \tau + 0.5$, where $\tau = \theta\tau^s + \overline{\theta}\tau^m$. (Regarding the

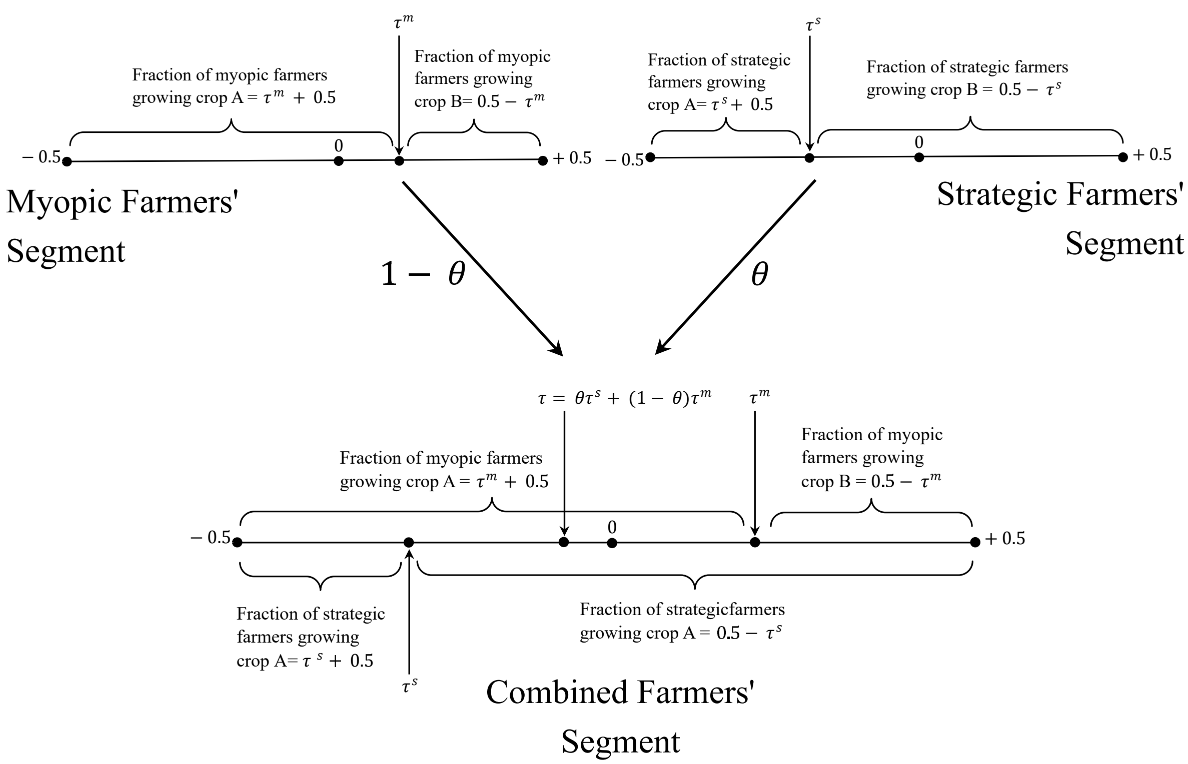


Figure 1 **Total product availability when** $\theta \in [0,1]$ **farmers are strategic and** $p_{t-1}^A > p_{t-1}^B$.

availability of crop B, it is easy to see that the fraction of myopic farmers producing crop B is given by $0.5 - \tau^m$ and that of strategic farmers producing crop B is $0.5 - \tau^s$. Hence, the total availability of crop B is $q_t^{BT} = \theta(0.5 - \tau^s) + \overline{\theta}(0.5 - \tau^m) = 0.5 - \tau$.)

4.1. Farmers' crop selection and production decisions in period t in equilibrium

While the threshold τ^m has been established earlier, the determination of the threshold τ^s is more involved because each strategic farmer takes the crop selection and production decisions of all other farmers into consideration. We now present the following proposition that states the farmers' crop selection and production decisions in period t as depicted in Figure 1. In preparation, let us define a term that will prove useful in our analysis. Let:

$$\hat{r} = \frac{\overline{\theta} r}{1 + r\theta}, \tag{3}$$

where $r \equiv (\rho - \alpha) > 0$ measures the "dissimilarity" between the two crops. Notice that $\hat{r}$ is increasing in r.

PROPOSITION 1. **(Crop selection and production decisions in period t for any realized $\mathbf{P}_{t-1}$)** *For any realized prices $\boldsymbol{P}_{t-1}$, the equilibrium crop selection and production decisions of the farmers in period t can be described as follows:*

1. ***Myopic farmers' decisions:*** *The amount of crop A produced by myopic farmers is given by $q_t^{Am} = \overline{\theta}(\tau^m + 0.5)$, where $\tau^m = \frac{p_{t-1}^A - p_{t-1}^B}{2} = \frac{\Delta p_{t-1}}{2} \in [-0.5, 0.5]$.*
2. ***Strategic farmers' decisions:*** *The amount of crop A produced by strategic farmers is given by $q_t^{As} = \theta(\tau^s + 0.5)$, where $\tau^s = -\hat{r}\tau^m \in [-0.5, 0.5]$.*
3. ***Total production:*** *The total production of crop A is given by $q_t^{AT} = \tau + 0.5$, where $\tau = \theta\tau^s + \overline{\theta}\tau^m = \left(\frac{\hat{r}}{r}\right)\tau^m \in [-0.5, 0.5]$.*

Even though we focus on crop A in the above proposition, the quantity of crop B produced by myopic and strategic farmers can be obtained through symmetry as $q_t^{Bm} = 0.5 - \tau^m$ and $q_t^{Bs} = 0.5 - \tau^s$, respectively. Also, the total production of crop B is $q_t^{BT} = 0.5 - \tau$.

For any given proportion of strategic farmers θ in the market, the first and the second statements of Proposition 1 describe the equilibrium production decisions of the myopic and strategic farmers through the threshold values τ^m and τ^s, respectively. By combining the corresponding production decisions of these two types of farmers, the third statement gives the total availability of each crop in equilibrium. It is interesting to note that, when $\theta = 1$ (i.e., all the farmers are strategic), $\tau = \tau^s = 0$ so that $q_t^{As} = q_t^{Bs} = 0.5$. Hence, when the market consists of strategic farmers only, half of the strategic farmers will grow A and the remaining half will grow B. Also, the realized market price in period $(t-1)$ has no bearing on the strategic farmers' crop selection and production decisions in period t.

Before we proceed, let us calculate the equilibrium expected crop prices as follows. Recall that $\phi = a - \frac{\rho+\alpha}{2}$ and $\Delta p_{t-1} = p^A_{t-1} - p^B_{t-1}$. Also, by recalling from the third statement of Proposition 1 that $q^{AT}_t = \tau + 0.5$ and $q^{BT}_t = 0.5 - \tau$, we can apply (1) to show that:

$$\mathbb{E}[p^A_t] = \phi - r\tau = \phi - \hat{r}\tau^m = \phi - \frac{\hat{r}}{2}\left(p^A_{t-1} - p^B_{t-1}\right) = \phi - \frac{\hat{r}}{2}\Delta p_{t-1}, \text{ and}$$
$$\mathbb{E}[p^B_t] = \phi + r\tau = \phi + \hat{r}\tau^m = \phi + \frac{\hat{r}}{2}\left(p^A_{t-1} - p^B_{t-1}\right) = \phi + \frac{\hat{r}}{2}\Delta p_{t-1}. \tag{4}$$

Also, for any location $x \in [-0.5, 0.5]$, let $\pi^m_t(x)$ and $\pi^s_t(x)$ denote the equilibrium profits of a myopic and a strategic farmers who is located at x, respectively. By using (2) and Proposition 1 that a farmer of type $v \in \{m, s\}$ will grow crop A if $x \leqslant \tau^v$, and will grow crop B, otherwise, we can apply (4) and the production costs $c_A(x) = 0.5 + x$ and $c_B(x) = 0.5 - x$ to show that the profit of a farmer of type $v \in \{m, s\}$ located at x is given as:

$$\pi^v_t(x) = \begin{cases} \Pi^A_t(x) = \mathbb{E}[p^A_t] - c_A(x) = \phi - \frac{\hat{r}}{2}\Delta p_{t-1} - (x + 0.5) & \text{if } x \leqslant \tau^v, \\ \Pi^B_t(x) = \mathbb{E}[p^B_t] - c_B(x) = \phi + \frac{\hat{r}}{2}\Delta p_{t-1} - (0.5 - x) & \text{if } x > \tau^v. \end{cases} \tag{5}$$

4.2. Impact of crop dissimilarity r

Now, let us use the results stated in Proposition 1 to examine the effect of dissimilarity between the crops (i.e., r) on the crop availability disparity (i.e., $\Delta q_t \equiv q^{AT}_t - q^{BT}_t$) and crop price disparity (i.e., $\Delta p_t \equiv p^A_t - p^B_t$) in period t. First, from the third statement of Proposition 1, it is easy to check that $\Delta q_t = q^{AT}_t - q^{BT}_t = 2\tau$, where $\tau = \left(\frac{\hat{r}}{r}\right)\tau^m$. In this case, by considering (3), we can conclude that the crop availability disparity $|\Delta q_t|$ is decreasing in the crop dissimilarity r when $\theta > 0$ and it is independent of r when $\theta = 0$. This result implies that the presence of strategic farmers can improve the balance of crop availability.

Next, let us examine the crop price disparity (i.e., $\Delta p_t \equiv p^A_t - p^B_t$) in period t. From (4) we obtain $|\mathbb{E}[\Delta p_t]| = |\mathbb{E}[p^A_t - p^B_t]| = \hat{r} \cdot | - \Delta p_{t-1}|$. Because the term $\hat{r}$ given in (3) is increasing in r, we can conclude that the expected crop price disparity is increasing in crop dissimilarity r. Moreover, because $\hat{r} < r < 1$, we can conclude that the expected crop price disparity will be dampened over time. The key results can be summarized in the following corollary:

Corollary 1 **(Impact of crop dissimilarity r)**.

1. ***Crop availability disparity:*** *The disparity between the total production quantities of the crops decreases with r if there are strategic farmers. That is $\frac{\partial|\Delta q_t|}{\partial r} < 0$ if $\theta > 0$, where $\Delta q_t = q^{AT}_t - q^{BT}_t$. However, if $\theta = 0$, then $\frac{\partial|\Delta q_t|}{\partial r} = 0$.*

2. ***Crop price disparity:*** *The expected disparity between the two crop prices increases with the crop dissimilarity r. That is $\frac{\partial|\mathbb{E}\Delta p_t|}{\partial r} \geqslant 0$.*

4.3. Impact of recent market prices $\mathbf{P}_{t-1}$

We now use the results stated in Proposition 1 to examine the impact of the realized market prices $\mathbf{P}_{t-1}$ in period $t-1$ on the production decisions of different types of farmers in period t. To avoid repetition, we shall focus on the case when $\Delta p_{t-1} = p^A_{t-1} - p^B_{t-1} > 0$. (The case when $\Delta p_{t-1} = p^A_{t-1} - p^B_{t-1} < 0$ can be analyzed in the exact manner.) By applying the results in Proposition 1 (i.e., $q^{Am}_t = \overline{\theta}(\tau^m + 0.5)$, $q^{As}_t = \theta(\tau^s + 0.5)$, and $q^{AT}_t = \tau + 0.5$), we obtain the following results:

Corollary 2 **(Impact of realized market prices $\mathbf{P}_{t-1}$)**. *Suppose $\Delta p_{t-1} > 0$. Then:*

1. ***Myopic farmers' decisions:*** $\frac{\partial \tau^m}{\partial \Delta p_{t-1}} = \frac{1}{2} > 0$, *and* $\frac{\partial q^{Am}_t}{\partial \Delta p_{t-1}} = \frac{\overline{\theta}}{2} \geqslant 0$.
2. ***Stratetic farmers' decisions:*** $\frac{\partial \tau^s}{\partial \Delta p_{t-1}} = -\frac{\hat{r}}{2} < 0$, *and* $\frac{\partial q^{As}_t}{\partial \Delta p_{t-1}} = -\frac{\theta \hat{r}}{2} \leqslant 0$.
3. ***Total production:*** $\frac{\partial \tau}{\partial \Delta p_{t-1}} = \frac{\hat{r}}{2r} > 0$, *and* $\frac{\partial q^{AT}_t}{\partial \Delta p_{t-1}} = \frac{\hat{r}}{2r} > 0$.
4. ***Expected profit of farmer of type*** $v \in \{m, s\}$***:*** $\frac{\partial \pi^v_t(x)}{\partial \Delta p_{t-1}} = -\frac{\hat{r}}{2} \leqslant 0$ *if* $x < \tau^v$, *and* $\frac{\partial \pi^v_t(x)}{\partial \Delta p_{t-1}} = \frac{\hat{r}}{2} \geqslant 0$ *if* $x > \tau^v$.

Because myopic farmers make their crop selection and production decisions in period t based on the realized market prices $\mathbf{P}_{t-1}$ observed in period $t-1$, more myopic farmers will select to grow the crop that has the higher price in the previous period. This observation explains the first statement of Corollary 2, which stipulates that the larger the price disparity $|\Delta p_{t-1}|$ in period $t-1$, the larger is the disparity in the production quantities of the myopic farmers in period t.

Next, let us consider the second statement. Because each strategic farmer knows the behavior of the myopic farmers and anticipates the behavior of all the other strategic farmers, he anticipates an increase in the production quantity of crop A can cause the price of the crop to go down further. For this reason, fewer strategic farmers will choose to grow A in period t as stated in the second statement.

While the realized market prices $\mathbf{P}_{t-1}$ have opposite effects on the myopic and strategic farmers as shown in the first two statements, the third statement shows that the strategic farmers can never nullify the impact of the decisions of the myopic farmers (and hence the impact of $\mathbf{P}_{t-1}$) on the aggregate product availability in period t. Specifically, the product with higher price in period $t-1$ is always produced more in period t than the product with lower price in period $t-1$. Furthermore, according to the fourth statement of the corollary, a higher value of Δp_{t-1} causes a higher availability of crop A in period t and hurts the expected profits of the farmers (both myopic and strategic) who grow crop A in equilibrium in period t due to the increased production of crop A. Figure 2 pictorially illustrates these three effects that are stated in Corollary 2.

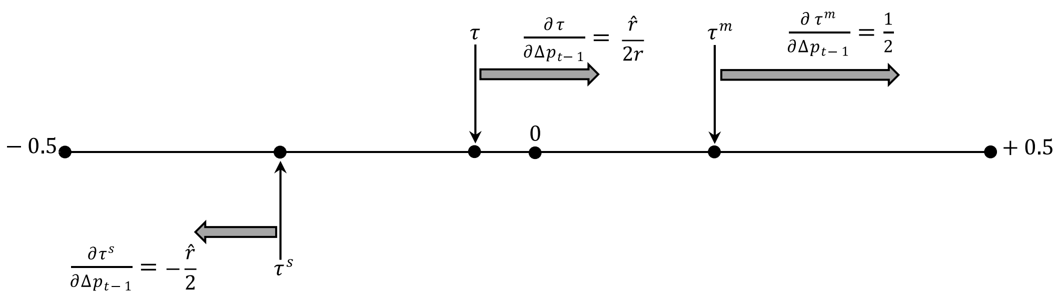


Figure 2 **Sensitivities of τ^m, τ^s and τ in equilibrium to Δp_{t-1}. Note that** $|\frac{\partial \tau^s}{\partial \Delta p_{t-1}}| = |-\frac{\hat{r}}{2}| < \frac{\partial \tau}{\partial \Delta p_{t-1}} = \frac{\hat{r}}{2r} < \frac{\partial \tau^m}{\partial \Delta p_{t-1}} = \frac{1}{2}$.

4.4. Impact of the proportion of strategic farmers θ

Let us examine the impact of the proportion of strategic farmers θ on the farmers' decisions. By considering the equilibrium outcomes as stated in Proposition 1 along with the fact that $\hat{r} = \frac{\bar{\theta} r}{1+r\theta}$ as given in (3), it is easy to show that:

COROLLARY 3 **(Impact of the proportion of strategic farmers θ).** *Suppose $\Delta p_{t-1} > 0$ so that $\tau^m = \frac{\Delta p_{t-1}}{2} > 0$. Then,*

1. ***Myopic farmers' decisions:*** $\frac{\partial \tau^m}{\partial \theta} = 0$ *and* $\frac{\partial q_t^{Am}}{\partial \theta} = -(\tau^m + 0.5) < 0$.
2. ***Strategic farmers' decisions:*** $\frac{\partial \tau^s}{\partial \theta} = \frac{r(1+r)}{(1+r\theta)^2}\tau^m > 0$ *and* $\frac{\partial q_t^{As}}{\partial \theta} > 0$
3. ***Total production:*** $\frac{\partial q_t^{AT}}{\partial \theta} = \frac{\partial \tau}{\partial \theta} = -\frac{(1+r)}{(1+r\theta)^2}\tau^m < 0$ *and* $\frac{\partial^2 q_t^{AT}}{\partial \theta \partial \Delta p_{t-1}} < 0$.
4. ***Expected profit of farmer of type*** $v \in \{m, s\}$***:*** $\frac{\partial \pi_t^v(x)}{\partial \theta} > 0$ *and* $\frac{\partial^2 \pi_t^v(x)}{\partial \theta \partial \Delta p_{t-1}} > 0$ *if* $x < \tau^v$. *Similarly,* $\frac{\partial \pi_t^v(x)}{\partial \theta} < 0$ *and* $\frac{\partial^2 \pi_t^v(x)}{\partial \theta \partial \Delta p_{t-1}} < 0$ *if* $x > \tau^v$.

The first two statements show that the production quantity of crop A produced by the myopic (strategic) farmers is decreasing (increasing) in θ. As stated in statement 3, the submodularity of q_t^{AT}(or τ) in $(\theta, \Delta p_{t-1})$ asserts that the strategic farmers "counteract" the impact of past market prices on the total production quantity q_t^{AT} in period t, and this "counteracting" effect is more pronounced as the proportion of strategic farmers θ increases.

The fourth statement shows that the profit of a farmer (either myopic or strategic) growing crop A (B) in equilibrium is increasing (decreasing) in θ. Moreover, the supermodularity of π_t^v in $(\theta, \Delta p_{t-1})$ for $x < \tau^v$ indicates that the negative impact of past price difference on the farmers growing crop A is mitigated. In summary, the destabilizing effect of past prices on the current expected equilibrium profits of the farmers is mitigated as the proportion of strategic farmers increases.

To summarize, we have shown that past prices will have an impact on the farmers' crop selection and production decisions, product availability, and crops' market prices in the future periods. If a large portion of the farmers are myopic (i.e., θ is small) then the crop with higher price in period $t-1$ (say, crop A) will be grown in abundance in period t. Due to the high availability of crop A, its price in period t is very likely to be low, which hurts the earnings of the those farmers who grow crop A. Consequently, high fluctuations in the past crop prices will destabilize farmers' profits in the current period. To safeguard the earnings of the farmers, many governments in developing countries offer MSPs. However, will MSPs create economic value to farmers? We examine this question in the next section.

5. Minimum Support Prices

We now extend our analysis presented in the last section to incorporate crop MSPs. To begin, let m_t^k denote the MSP associated with crop $k \in \{A, B\}$ in period t.[4] Also, let $\hat{p}_t^{km}$ and $\hat{p}_t^{ks}$ denote the *effective* market prices of crop $k \in \{A, B\}$ in period t as "anticipated" by myopic and strategic farmers, respectively.[5] Because each myopic farmer anticipates that the future selling price is equal to the most recently observed market price p_{t-1}^k, myopic farmers will anticipate that $\hat{p}_t^{km} = \max\{p_{t-1}^k, m_t^k\}$ for crop k. However, because each strategic farmer accounts for the actions of all the other farmers, each strategic farmer can anticipate the *effective* market price in equilibrium based on its expected value so that $\hat{p}_t^{ks} = \mathbb{E}_{\epsilon_t^k} \max\{p_t^k, m_t^k\}$ for crop k, where p_t^k is the actual market price as given in (1).

The decision making process employed by the farmers remains the same as explained in Section 4, except that the anticipated prices p_t^{km} and p_t^{ks} are now replaced by $\hat{p}_t^{km}$ and $\hat{p}_t^{ks}$ for $k \in \{A, B\}$. To ease our exposition and to identify the conditions under which offering

[4] In general, the MSPs are announced before the crop sowing season; the farmers make their sowing decisions with the complete knowledge of the MSPs and the price history of the crops.

[5] To differentiate between the base case and the case when positive MSPs are offered, we use ˆ over the variables of interest in the latter case.

higher MSPs is detrimental to the farmers, we shall assume throughout this section that the difference between the MSPs of two crops is bounded by 1 (i.e., $|m_t^A - m_t^B| < 1$). (However, except Propositions 4 and 5 that discuss possible disadvantages of MSPs, all other results described in this section can be extended to MSPs such that $|m_t^A - m_t^B| > 1$, with additional notation.) We first characterize the unique equilibrium in the presence of MSPs in Proposition 2, which is analogous to Proposition 1.

PROPOSITION 2 **(Equilibrium under MSPs)**. *For any realized prices $\boldsymbol{P}_{t-1}$ and for any given MSPs (m_t^A, m_t^B), the equilibrium crop selection and production decisions of the farmers in period t can be described as follows:*

1. ***Myopic farmers' decisions:*** *The amount of crop A produced by myopic farmers is given by $\hat{q}_t^{Am} = \overline{\theta}(\hat{\tau}^m + 0.5)$, where*
$$\hat{\tau}^m = \frac{\hat{p}_t^{Am} - \hat{p}_t^{Bm}}{2} \in [-0.5, 0.5], \tag{6}$$
$\hat{p}_t^{km} = \max\{p_{t-1}^k, m_t^k\}$, $k \in \{A, B\}$.
2. ***Strategic farmers' decisions:*** *The amount of crop A produced by strategic farmers is given by $\hat{q}_t^{As} = \theta(\hat{\tau}^s + 0.5)$, where*
$$\hat{\tau}^s = -\hat{r}\hat{\tau}^m - \left[\frac{1}{2(1+r\theta)} \int_{m_t^A - \phi + r\hat{\tau}}^{m_t^B - \phi - r\hat{\tau}} F(\epsilon)\, d\epsilon\right] \in [-0.5, 0.5], \tag{7}$$
$\hat{r} = \frac{\overline{\theta} r}{1 + r\theta}$ *and* $\hat{\tau} = \theta\hat{\tau}^s + \overline{\theta}\hat{\tau}^m$.[6]
3. ***Total production:*** *The total production of crop A is given by $\hat{q}_t^{AT} = \hat{\tau} + 0.5$ where $\hat{\tau} = \theta\hat{\tau}^s + \overline{\theta}\hat{\tau}^m \in [-0.5, 0.5]$.*

By using $\hat{\tau}^m$ from (6) and the fact that $\hat{\tau} = \theta\hat{\tau}^s + \overline{\theta}\hat{\tau}^m$, we can obtain $\hat{\tau}^s$ by solving (7) as an equation that involves $\hat{\tau}^s$ as the only variable. Once we determine $\hat{\tau}^s$, we can retrieve $\hat{\tau}$ accordingly. Also, it can be shown that Proposition 2 reduces to Proposition 1 when $m_t^A = m_t^B = 0$.[7]

[6] Note that (7) can be alternatively written as $\hat{\tau}^s = -r\hat{\tau} - \frac{1}{2}\int_{m_t^A - \phi + r\hat{\tau}}^{m_t^B - \phi - r\hat{\tau}} F(\epsilon)\, d\epsilon \in [-0.5, 0.5]$. We will use either of these two definitions of $\hat{\tau}^s$ in our analysis, based on convenience.

[7] First, to ensure that the crop prices are non-negative we require, $p_t^A = \mathbb{E}[p_t^A] + \epsilon_t^A = \phi - r\hat{\tau} + \epsilon_t^A \geqslant 0$, which implies that $\epsilon_t^A \geqslant -\phi + r\hat{\tau}$ for all values of ϵ_t^A. Second, by using the same arguement for crop B, we can conclude that $\epsilon_t^B \geqslant -\phi - r\hat{\tau}$ for all values of ϵ_t^B. Using these two observations and the fact that ϵ_t^A and ϵ_t^B follow the same distribution $F(\cdot)$, we can conclude that $F(\epsilon) = 0$ for all values of $\epsilon \leqslant \max\{-\phi + r\hat{\tau}, -\phi - r\hat{\tau}\}$. Hence, $\int_{m_t^A - \phi + r\hat{\tau}}^{m_t^B - \phi - r\hat{\tau}} F(\epsilon)\, d\epsilon = 0$ so that $\hat{\tau}^s$ is reduced to τ^s when $m_t^A = m_t^B = 0$. Similarly, $\hat{\tau}^m$ is reduced to τ^m and $\hat{\tau}$ is reduced to τ when $m_t^A = m_t^B = 0$. Hence, we can conclude that Proposition 2 reduces to Proposition 1 when $m_t^A = m_t^B = 0$.

Next, consider a special case when all farmers are strategic so that $\theta = 1$. In this case, statement 2 reveals that, when $\theta = 1$, $\hat{r} = 0$, $\hat{\tau} = \hat{\tau}^s$, $\hat{q}_t^{AT} = (0.5 + \hat{\tau}^s)$, $\hat{q}_t^{BT} = (0.5 - \hat{\tau}^s)$, and (7) can be simplified as:

$$\hat{\tau}^s = -\frac{1}{2(1+r)} \int_{m_t^A - \phi + r\hat{\tau}^s}^{m_t^B - \phi - r\hat{\tau}^s} F(\epsilon)\, d\epsilon. \tag{8}$$

By noting that $\hat{\tau}^s$ is independent of $\mathbf{P}_{t-1}$, we can conclude that, when all farmers are strategic, the production quantity of each crop k is increasing in its own MSP m_t^k. Hence, a policy-maker can always select appropriate MSPs to attain a balanced mixture of both crops when all farmers are strategic. However, when the market consists of both myopic and strategic farmers, the selection of proper MSPs is much more complex, and we shall discuss this in Section 6.

Finally, the results stated in Proposition 2 possess the same characteristics as the results stated in Proposition 1. First, observe that the threshold associated with the strategic farmers given in (7) involves two components: (i) the response to the actions of myopic farmers (which is the first term in the RHS of (7), i.e., $-\hat{r}\hat{\tau}^m$, which is analogous to the expression of τ^s given in the second statement of Proposition 1), and (ii) the response to the crop MSPs announced (which is the second term in the RHS of (7)). Thus, MSPs influence the decisions of the strategic farmers in two ways. First, they influence the decisions of strategic farmers via the decisions of the myopic farmers as explained in (i), and we term this effect as the *indirect* effect. Second, the MSPs influence the decisions of strategic farmers directly as explained in (ii), and we term this effect as the *direct* effect. There two effects play an important role in our analysis of the impact of MSPs.

It can be shown that the threshold $\hat{\tau}^s$ for strategic farmers is decreasing and the total product availability threshold $\hat{\tau}$ is increasing in the threshold $\hat{\tau}^m$ for myopic farmers. Specifically, it is easy to observe from Proposition 2 that $\frac{\partial \hat{\tau}^s}{\partial \hat{\tau}^m} < 0$ and $\frac{\partial \hat{\tau}}{\partial \hat{\tau}^m} > 0$. The same characteristics of the thresholds can be observed from Proposition 1 as well. Essentially, these two characteristics of $\hat{\tau}^s$ and $\hat{\tau}$ imply that strategic farmers "counteract" the actions of myopic farmers; however, strategic farmers' counter-actions cannot fully nullify the impact of myopic farmers even when MSPs are offered. Also, it can be shown that the findings made in Corollary 1 regarding the impact of crop disimilarity r continued to hold for any given MSPs of the crops (we refer the reader to Corollary 6 in Appendix A).

In addition to the production quantities as stated in Proposition 2, we can compute the farmers' profits in equilibrium in the presence of MSPs (m_t^A, m_t^B). Analogous to (2), we can express the expected profit of a farmer who is located at x and growing crop $k \in \{A, B\}$ as:

$$\begin{aligned}\hat{\Pi}_t^k(x) &= \mathbb{E}_{\epsilon_t^k}\left[\max\{p_t^k, m_t^k\}\right] - c_k(x) \\ &= \mathbb{E}[p_t^k] + \mathbb{E}_{\epsilon_t^k}\left[\max\{\epsilon_t, m_t^k - \mathbb{E}[p_t^k]\}\right] - c_k(x) \\ &= \mathbb{E}[p_t^k] + \left(m_t^k - \mathbb{E}[p_t^k]\right) F\left(m_t^k - \mathbb{E}[p_t^k]\right) + \int_{m_t^k - \mathbb{E}[p_t^k]}^{\infty} \epsilon f(\epsilon)\, d\epsilon - c_k(x).\end{aligned} \tag{9}$$

By considering (9), we can use the thresholds $\hat{\tau}^m, \hat{\tau}^s$ and $\hat{\tau}$ stated in Proposition 2 along with the production cost $c_A(x) = 0.5 + x$ and $c_B(x) = 0.5 - x$ to determine the expected profit in equilibrium for a farmer of type $v \in \{m, s\}$ and who is located at x in period t as:

$$\hat{\pi}_t^v(x) = \begin{cases} \hat{\Pi}_t^A(x) = \mathbb{E}[p_t^A] + \left(m_t^A - \mathbb{E}[p_t^A]\right) F\left(m_t^A - \mathbb{E}[p_t^A]\right) + \int_{m_t^A - \mathbb{E}[p_t^A]}^{\infty} \epsilon f(\epsilon)\, d\epsilon - (0.5 + x) & \text{if } x \leqslant \hat{\tau}^v, \\ \hat{\Pi}_t^B(x) = \mathbb{E}[p_t^B] + \left(m_t^B - \mathbb{E}[p_t^B]\right) F\left(m_t^B - \mathbb{E}[p_t^B]\right) + \int_{m_t^B - \mathbb{E}[p_t^B]}^{\infty} \epsilon f(\epsilon)\, d\epsilon - (0.5 - x) & \text{if } x > \hat{\tau}^v. \end{cases} \tag{10}$$

Also, by using statement 3 of Proposition 2 stating that $\hat{q}_t^{AT} = 0.5 + \hat{\tau}$ and $\hat{q}_t^{BT} = 0.5 - \hat{\tau}$, we can apply (1) to determine the expected market price $\mathbb{E}[p_t^A]$ and $\mathbb{E}[p_t^B]$ in equilibrium as a function of $\hat{\tau}$, which in turn depends on the MSPs via (7).

5.1. Impact of $\mathbf{P}_{t-1}$ and θ

We now examine the impact of the most recently realized prices $\mathbf{P}_{t-1}$ and the fraction of strategic farmers θ on the equilibrium outcomes, which are as stated in Proposition 2, in the presence of MSPs. Corollary 4, which is an analogue to Corollary 2, explains the impact of $\mathbf{P}_{t-1}$ on the equilibrium. For ease of exposition, we shall focus on crop A only.

COROLLARY 4 **(Impact of the most recently realized prices $\mathbf{P}_{t-1}$ under MSPs)**. *For any given MSPs (m_t^A, m_t^B), the impact of $\boldsymbol{P}_{t-1}$ can be described as follows:*

1. ***Myopic farmers' decisions:*** $\frac{\partial \hat{\tau}^m}{\partial p_{t-1}^A} \geqslant 0$ *and* $\frac{\partial \hat{q}_t^{Am}}{\partial p_{t-1}^A} \geqslant 0$.
2. ***Strategic farmers' decisions:*** $\frac{\partial \hat{\tau}^s}{\partial p_{t-1}^A} \leqslant 0$ *and* $\frac{\partial \hat{q}_t^{As}}{\partial p_{t-1}^A} \leqslant 0$.
3. ***Total production:*** $\frac{\partial \hat{\tau}}{\partial p_{t-1}^A} \geqslant 0$ *and* $\frac{\partial \hat{q}_t^{AT}}{\partial p_{t-1}^A} \geqslant 0$.
4. ***Expected profit of farmer of type*** $v \in \{m, s\}$***:*** $\frac{\partial \hat{\pi}_t^v(x)}{\partial p_{t-1}^A} \leqslant 0$ *if* $x < \hat{\tau}^v$ *and* $\frac{\partial \hat{\pi}_t^v(x)}{\partial p_{t-1}^A} \geqslant 0$ *if* $x > \hat{\tau}^v$.

It is easy to check that Corollary 4 resembles Corollary 2 (for any given p^B_{t-1}) even when MSPs are present; hence, it can be interpreted in the same manner.

Next, we examine the impact of the proportion of strategic farmers θ on the equilibirium outcomes. Corollary 5 is analogous to Corollary 3. However, because of the MSPs, the analysis is more involved in the sense that the result hinges on the comparison between the threshold $\hat{\tau}^m$, as defined in (6), and the threshold $\hat{\tau}^s_0$, where $\hat{\tau}^s_0$ is the value of $\hat{\tau}^s$ (as defined in (7)) evaluated at $\theta = 0$. In other words, $\hat{\tau}^s_0 \equiv \hat{\tau}^s|_{\theta=0} = -2r\hat{\tau}^m - \frac{\int_{m^A_t-\phi+2r\hat{\tau}^m}^{m^B_t-\phi-2r\hat{\tau}^m} F(\epsilon)d\epsilon}{2}$. It can shown that depending on the parameters and the distribution $F(\cdot)$, the difference between $\hat{\tau}^m$ and $\hat{\tau}^s_0$ can be positive or negative, but explicit conditions are not available.

Corollary 5 **(Impact of strategic farmers under MSPs)**. *For any given MSPs (m^A_t, m^B_t), the impact of θ can be described as follows:*

1. ***Myopic farmers' decisions:*** $\frac{\partial \hat{\tau}^m}{\partial \theta} = 0$ *and* $\frac{\partial \hat{q}^{Am}}{\partial \theta} = -(\hat{\tau}^m + 0.5) \leqslant 0$.
2. ***Strategic farmers' decisions:*** $\frac{\partial \hat{\tau}^s}{\partial \theta} \geqslant 0$ *if and only if* $\hat{\tau}^m \geqslant \hat{\tau}^s_0$*, and* $\frac{\partial \hat{q}^{As}_t}{\partial \theta} \geqslant 0$.
3. ***Total production:*** $\frac{\partial \hat{\tau}}{\partial \theta} \leqslant 0$*, and* $\frac{\partial \hat{q}^{AT}_t}{\partial \theta} \leqslant 0$ *if and only if* $\hat{\tau}^m \geqslant \hat{\tau}^s_0$.
4. ***Expected profit of farmer of type*** $v \in \{m, s\}$***:*** *If* $x \leqslant \hat{\tau}^v$*, then* $\frac{\partial \hat{\pi}^v_t(x)}{\partial \theta} \geqslant 0$ *if and only if* $\hat{\tau}^m \geqslant \hat{\tau}^s_0$*. Else, if* $x > \hat{\tau}^v$*, then* $\frac{\partial \hat{\pi}^v_t(x)}{\partial \theta} \leqslant 0$ *if and only if* $\hat{\tau}^m \geqslant \hat{\tau}^s_0$.

When $\hat{\tau}^m \geqslant \hat{\tau}^s_0$, the above corollary exhibits the same characteristics as Corollary 3 (for the case when $\tau^m \geqslant \tau^s$, which holds when the supposition $p^A_{t-1} \geqslant p^B_{t-1}$ holds). Hence, it can be interpreted in the same manner.

However, the above corollary exhibits opposite results when $\hat{\tau}^m < \hat{\tau}^s_0$, where this condition depends on the value of MSPs. This condition is not present in Corollary 3 because, in the absence of MSPs, strategic farmers respond only to myopic farmers' decisions that are determined by the realized prices $\mathbf{P}_{t-1}$. However, in the presence of MSPs, MSPs have a *direct* impact (along with $\mathbf{P}_{t-1}$) on the decisions of myopic farmers as described in (6). Also, MSPs have both *direct* and *indirect* (via the actions of myopic farmers) impacts on strategic farmers as described in (7), which makes the decisions of strategic farmers more intricate. This observation calls for more attention to the analysis of the impact of MSPs on farmers' decisions. We explore this in the following section.

5.2. Impact of MSPs

We now examine the impact of MSPs on the farmer's crop selection and production decisions (again, we focus on crop A alone). In preparation, let us define the following two bounds on the MSP of crop A.

$$\underline{m}_t^A \equiv \underline{m}_t^A(\mathbf{P}_{t-1}, m_t^B) = \max\{p_{t-1}^A, \max\{m_t^B, p_{t-1}^B\} - 1\} \text{ and}$$
$$\overline{m}_t^A \equiv \overline{m}_t^A(\mathbf{P}_{t-1}, m_t^B) = \max\{p_{t-1}^A, \max\{m_t^B, p_{t-1}^B\} + 1\}.$$

With these two bounds, MSP m_t^A is considered to be *low* when $m_t^A < \underline{m}_t^A$, *moderate* when $\underline{m}_t^A \leqslant m_t^A \leqslant \overline{m}_t^A$, and *high* when $m_t^A > \overline{m}_t^A$. The two bounds $\underline{m}_t^A$ and $\overline{m}_t^A$ are intended to establish the necessary and sufficient conditions under which $\hat{\tau}^m$, which represents myopic farmers' crop selection decisions and that is defined in (6) in Proposition 2, is independent of m_t^A, the MSP of crop A. It can be shown that $\hat{\tau}^m$ is independent of m_t^A if and only if either m_t^A is *low* (i.e., $m_t^A \leqslant \underline{m}_t^A$) or m_t^A is *high* (i.e., $m_t^A \geqslant \overline{m}_t^A$). [8] By doing this, we can observe the impact of MSPs when (i) they have only the *direct* effect, and (ii) they have both the *direct* and the *indirect* effects, on the decisions of strategic farmers given by $\hat{\tau}^s$ in (7). By using the two bounds $\underline{m}_t^A$ and $\overline{m}_t^A$, along with the results as stated in Proposition 2, we obtain the following results:

PROPOSITION 3 **(Impact of MSPs on Equilibrium)**. *For any given MSP m_t^B of crop B, the MSP of crop A, m_t^A, affects the production decisions of myopic and strategic farmers as follows:*

1. ***Total production:*** *The total availability of crop A is always increasing in the MSP of A so that $\frac{\partial \hat{q}_t^{AT}}{\partial m_t^A} = \frac{\partial \hat{\tau}}{\partial m_t^A} \geqslant 0$.*
2. ***Low MSP:*** *When $m_t^A \leqslant \underline{m}_t^A$, then: (a) $\hat{q}_t^{Am} = \overline{\theta}\left[\frac{p_{t-1}^A - \max\{m_t^B, p_{t-1}^B\}}{2} + \frac{1}{2}\right]^+$ so that $\frac{\partial \hat{q}_t^{Am}}{\partial m_t^A} = 0$, and (b) $\frac{\partial \hat{q}_t^{As}}{\partial m_t^A} \geqslant 0$.*
3. ***High MSP:*** *When $m_t^A \geqslant \overline{m}_t^A$, then: (a) $\hat{q}_t^{Am} = \overline{\theta}$ so that $\frac{\partial \hat{q}_t^{Am}}{\partial m_t^A} = 0$, and (b) $\frac{\partial \hat{q}_t^{As}}{\partial m_t^A} \geqslant 0$.*
4. ***Moderate MSP:*** *When $\underline{m}_t^A < m_t^A < \overline{m}_t^A$, then: (a) $\hat{q}_t^{Am} \in (0, \overline{\theta})$, and (b) $\frac{\partial \hat{q}_t^{Am}}{\partial m_t^A} = \frac{\overline{\theta}}{2} \geqslant 0$.*

[8] Clearly, when $m_t^A \geqslant \overline{m}_t^A$ then $\hat{p}_t^A = \max\{m_t^A, p_{t-1}^A\} = m_t^A \geqslant \hat{p}_t^B + 1$ so that all the myopic farmers grow crop A and hence $q_t^{Am} = \overline{\theta}(\hat{\tau}^m + 0.5) = \overline{\theta}(0.5 + 0.5) = \overline{\theta}$, which is independent of m_t^A. On the other hand, if $m_t^A \leqslant \underline{m}_t^A$ then, we consider two cases: (i) $p_{t-1}^A \geqslant \max\{m_t^B, p_{t-1}^B\} - 1$ and (ii) $p_{t-1}^A < \max\{m_t^B, p_{t-1}^B\} - 1$. Under case (i), we have $m_t^A \leqslant \underline{m}_t^A = p_{t-1}^A$ and $|p_{t-1}^A - \hat{p}_t^{Bm}| < 1$ because $m_t^A - m_t^B| < 1$ and $|p_{t-1}^A - p_{t-1}^B| < 1$. Hence, $\hat{\tau}^m = \frac{p_{t-1}^A - \hat{p}_t^{Bm}}{2} > -0.5$ so that $q_t^{Am} = \overline{\theta}(\hat{\tau}^m + 0.5)$. Under case (ii), we have $m_t^A \leqslant \underline{m}_t^A = \max\{m_t^B, p_{t-1}^B\} - 1$, hence we have $\hat{\tau}^m = -0.5$ so that $q_t^{Am} = 0$. Therefore, if $m_t^A \leqslant \underline{m}_t^A$, the total production quantity by myopic farmers can we written as $q_t^{Am} = \overline{\theta}\left[\frac{p_{t-1}^A - \max\{m_t^B, p_{t-1}^B\}}{2} + \frac{1}{2}\right]^+$.

The first statement of Proposition 3 shows that the availability of a crop is always increasing in the MSP offered for the crop. Due to this increase in the availability of the crop, its market price drops as its MSP increases. Hence, the equilibrium expected market price of crop A is decreasing in m_t^A (and increasing in m_t^B with details omitted). Therefore, to achieve a better balance of different crops, a policy-maker has to account for the effect of MSP of one crop on the production of the other crop. Further, it is always possible to obtain a desired production-mix of the crops using MSPs.[9]

Now, when MSP m_t^A is *low* (i.e., $m_t^A \leqslant \underline{m}_t^A$), the decisions of myopic farmers are independent of m_t^A (as explained in footnote 8). Hence, when MSP m_t^A is *low*, a slight increase in the MSP m_t^A will not affect the sowing decisions of myopic farmers as stated in part (a) of statement 2. Anticipating the myopic farmers' sowing decisions, more strategic farmers will grow crop A as MSP m_t^A increases. This explains part (b) of statement 2.

Next, when MSP m_t^A is *high* (i.e., $m_t^A \geqslant \overline{m}_t^A$), all myopic farmers will grow crop A (as explained in footnote 8). As such, increasing m_t^A will not increase myopic farmers' production of crop A any further. Anticipating the myopic farmers' sowing decisions, more strategic farmers will grow crop A as MSP m_t^A increases. This explains statement 3. Essentially, the second and the third statements imply that, as long as myopic farmers are "unaffected" by the increase in m_t^A, strategic farmers will increase their production of crop A in order to benefit from the increase in m_t^A.

Finally, let us examine the fourth statement of Proposition 3 in which the MSP m_t^A is *moderate* (i.e., $\underline{m}_t^A < m_t^A < \overline{m}_t^A$). In this case, it can be shown that the production of crop A by the myopic farmers is strictly increasing in m_t^A (and decreasing in m_t^B with details omitted). As shown in the fourth statement, when the MSP is moderate so that $\underline{m}_t^A < m_t^A < \overline{m}_t^A$, more myopic farmers will grow crop A as the MSP m_t^A increases (i.e., $\hat{\tau}^m$ is increasing so that $\hat{q}_t^{Am}$ is increasing in the MSP m_t^A). Anticipating myopic farmers' behavior, strategic farmers make decisions in a more intricate manner, when the MSP m_t^A is *moderate*. However, as it turns out, the amount of crop A produced by strategic farmers q_t^{As} (or equivalently $\hat{\tau}^s$) is not necessarily monotonic in the MSP m_t^A: offering a higher

[9] To see why, suppose $\hat{\tau}_{\text{target}}$ is the targeted production of crop A (so that $1-\hat{\tau}_{\text{target}}$ is the targeted production of crop B). Without loss of generality, assume that initially we set $m_t^A = m_t^B = 0$ so that $\hat{\tau} = \tau$, which is as defined in Proposition 1. If $\hat{\tau} = \tau > \hat{\tau}_{\text{target}}$ then we can set m_t^B sufficiently high so that $\hat{\tau} = \hat{\tau}_{\text{target}}$ is attained. This is possible because from (7) we see that $\lim_{m_t^B \to \infty} \hat{\tau}^s = \max\{-\infty, -0.5\} = -0.5$ and from (6) we see that $\lim_{m_t^B \to p_{t-1}^A + 1} \hat{\tau}^m = -0.5$ so that $\lim_{m_t^B \to \infty} \hat{\tau} = \lim_{m_t^B \to \infty} \{\theta\hat{\tau}^s + \overline{\theta}\hat{\tau}^m\} = -0.5$. Likewise, on the other hand, if $\hat{\tau} = \tau < \hat{\tau}_{\text{target}}$ then we can set m_t^A sufficiently high so that $\hat{\tau} = \hat{\tau}_{\text{target}}$ is attained because it can be shown that $\lim_{m_t^A \to \infty} \hat{\tau} = 0.5$.

MSP for a crop can cause strategic farmers to produce less of the crop. We shall explore this seemingly counter-intuitive result in more detail.

Due to the complexity of the analysis, we shall consider a special case when the market uncertainty $\epsilon_t^k \sim U[-\delta,\delta]$, $k \in \{A,B\}$, instead of a general probability distribution $F(\cdot)$. In preparation, we let:

$$\widetilde{m} = \phi - \delta\left(\frac{1-r}{1+r}\right) < \phi = a - \frac{\rho+\alpha}{2}.$$

Notice that $\widetilde{m} > 0$ when a is sufficiently large and δ is sufficiently small. By considering $\widetilde{m}$, We obtain the following result:

Proposition 4 **(Impact of MSPs on strategic farmers)**. *Suppose the given MSP m_t^B of crop B is such that $m_t^B \geqslant p_{t-1}^B$. Then, for any moderately low MSP of A such that $m_t^A \in \left(\underline{m}_t^A, \min\{m_t^B+1,\widetilde{m}\}\right)$, there exists a threshold $\theta_0 \equiv \theta_0(m_t^A, m_t^B) > 0$ such that $\frac{\partial \hat{\tau}^s}{\partial m_t^A} < 0$ if and only if $0 \leqslant \theta < \theta_0$.*[10]

While Proposition 4 is based on the assumption that the market uncertainty $\epsilon_t^k \sim U[-\delta,\delta]$, $k \in \{A,B\}$, the results stated in the proposition continue to hold for general distribution. (Please see Proposition 6 in Appendix A for details.)

Figure 3 provides a numerical example to verify the results that are stated in Propositions 3 and 4. The parameter values used are $a=1$, $\rho=0.7$, $\alpha=-0.25$, $p_{t-1}^A=0.1$, $p_{t-1}^B=0.5$, $m_t^B=0.55$, $r=0.95$, $\theta=0.1$, and $\epsilon_t^k \sim U[-0.1,0.1]$ (i.e., $\delta=0.1$). As illustrated in the figure, the thresholds $\hat{\tau}^m$ and $\hat{\tau}$ are always increasing in m_t^A. This conforms with the findings as shown in Proposition 3. The threshold $\hat{\tau}^s$ is however not monotonic in m_t^A (it is decreasing in m_t^A until $m_t^A \approx 0.74$), which verifies Proposition 4.

Proposition 4 shows that when m_t^A, the MSP for crop A, is *moderately low* (i.e., $\underline{m}_t^A < m_t^A < \widetilde{m}$), the proportion of strategic farmers producing crop A (i.e., $\hat{\tau}^s$) can be decreasing in m_t^A. Intuitively, one expects that more farmers grow crop A as the MSP of the crop increases. While this is always true in the case of myopic farmers, as shown in Proposition

[10] It can be shown that for any given m_t^B, there exist values of m_t^A that satisfy the conditions listed in Proposition 4 when δ is sufficiently small and crop A is produced more in the previous period. To elaborate, suppose $0 < \delta \leqslant \frac{r(1+r)}{2}(q_{t-1}^{AT} - \frac{1}{2})$ where q_{t-1}^{AT} is the total production of crop A in the previous period. Then $\delta \leqslant \frac{r(1+r)}{2}(q_{t-1}^{AT} - \frac{1}{2}) \Leftrightarrow \delta + \delta\left(\frac{1-r}{1+r}\right) \leqslant r\left(q_{t-1}^{AT} - \frac{1}{2}\right) \Rightarrow \epsilon_{t-1}^A + \delta\left(\frac{1-r}{1+r}\right) \leqslant r\left(q_{t-1}^{AT} - \frac{1}{2}\right)$ (because $\epsilon_{t-1}^A \in [-\delta,\delta]$) $\Leftrightarrow a - \alpha - r q_{t-1}^{AT} + \epsilon_{t-1} < a - \frac{\rho+\alpha}{2} - \delta\left(\frac{1-r}{1+r}\right) \Leftrightarrow p_{t-1}^A < \phi - \delta\left(\frac{1-r}{1+r}\right) = \tilde{m}$. Hence, there exists m_t^A such that $p_{t-1}^A < m_t^A < \tilde{m}$. Next, m_t^B can be chosen sufficiently close to $\tilde{m}$ such that $m_t^B + 1 > \tilde{m}$ so that $\underline{m}_t^A < m_t^A < \min\{m_t^B+1, \widetilde{m}\}$. Note that this provides only a sufficient condition, but not a necessary condition, for the conditions listed in the proposition to hold simultaneously.

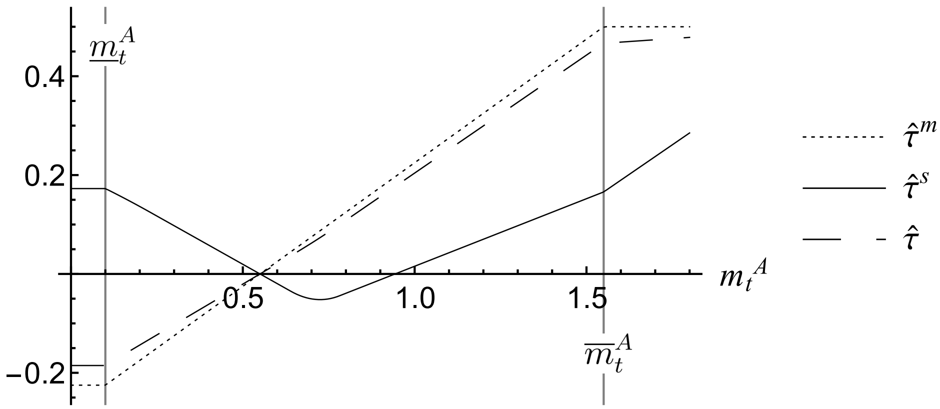


Figure 3 $\hat{\tau}^m$, $\hat{\tau}^s$ and $\hat{\tau}$ as a function of MSP m_t^A.

3 when the MSP is moderately low, it is not true for strategic farmers when $\theta < \theta_0$, as shown in Proposition 4.

The rationale for this counter-intuitive result as stated in Proposition 4 is as follows. Strategic farmers know that, when the MSP of crop A is moderate, more myopic farmers will grow crop A as m_t^A increases. The resulting increase in production of crop A is substantial when θ is small because, by using statement 1 of Proposition 2 (i.e., $\hat{q}^{Am} = \overline{\theta}(\hat{\tau}^m + 0.5)$), it is easy to see that: $\frac{\partial \hat{q}^{Am}}{\partial m_t^A} = \overline{\theta} \frac{\partial \hat{\tau}^m}{\partial m_t^A} = \frac{\overline{\theta}}{2}$ when $m_t^A > p_{t-1}^A$. This substantial increase in the total production quantity q_t^{AT} causes a significant drop in the price of crop A (and causes a steep increase in the price of crop B). By anticipating myopic farmers' behavior, strategic farmers are better off by producing less of crop A and more of crop B. This explains the seemingly counter-intuitive result that is stated in Proposition 4.

To summarize, we find that, when the MSP of crop A is moderately low, increasing the MSP m_t^A can cause fewer strategic farmers to grow crop A (and more strategic farmers to grow crop B). This seemingly counter-intuitive finding offers a hint regarding the condition(s) under which offering MSP for a crop can hurt the earnings of farmers who grow that crop. We shall explore this next.

5.3. Impact of MSPs on farmers' profits

We now examine the impact of the MSP of crop A on the ex-ante expected profits of farmers of each type $v \in \{m, s\}$ as given by (10). By differentiating (10) with respect to m_t^A and by using the fact that the expected market price $\mathbb{E}[p_t^A]$ and $\mathbb{E}[p_t^B]$ in equilibrium depend on the MSPs via $\hat{\tau}$, we obtain:

$$\frac{\partial \hat{\pi}_t^v(x)}{\partial m_t^A} = \begin{cases} F\left(m_t^A - \mathbb{E}[p_t^A]\right) + \overline{F}\left(m_t^A - \mathbb{E}[p_t^A]\right) \cdot \frac{\partial \mathbb{E}[p_t^A]}{\partial m_t^A} & \text{if } x \leqslant \hat{\tau}^v \\ \overline{F}\left(m_t^B - \mathbb{E}[p_t^B]\right) \cdot \frac{\partial \mathbb{E}[p_t^B]}{\partial m_t^A} & \text{if } x > \hat{\tau}^v \end{cases} \tag{11}$$

where $\frac{\partial \mathbb{E}[p_t^A]}{\partial m_t^A} = -r \frac{\partial \hat{\tau}}{\partial m_t^A} \leqslant 0$ and $\frac{\partial \mathbb{E}[p_t^B]}{\partial m_t^A} = r \frac{\partial \hat{\tau}}{\partial m_t^A} \geqslant 0$. As before, we focus on the impact of the MSP of crop A on the expected profits of the farmers. We introduce the following lemma.

Lemma 2 **(Impact of MSPs on farmers' profits)**. *Consider a farmer of type $v \in \{m, s\}$ who is located at $x \in [-0.5, 0.5]$.*

1. ***Farmers growing crop B:*** *If $x > \hat{\tau}^v$, then $\frac{\partial \hat{\pi}_t^v(x)}{\partial m_t^A} \geqslant 0$.*
2. ***Low or high m_t^A on farmers growing crop A:*** *If $x \leqslant \hat{\tau}^v$ and m_t^A is either low or high (i.e., $m_t^A < \underline{m}_t^A$ or $m_t^A > \overline{m}_t^A$), then $\frac{\partial \hat{\pi}_t^v(x)}{\partial m_t^A} \geqslant 0$.*

The lemma explains the *indirect* benefit that m_t^A offers to the farmers growing crop B (i.e., farmers who are located at $x > \hat{\tau}^v$) in equilibrium. When m_t^A is increased, the total availability of crop A (B) increases (decreases) according to statement 1 of Proposition 3. Hence, the expected market price of crop B increases, which will increase the expected profit of those farmers who grow crop B in equilibrium. Furthermore, the lemma proves that, as long as the decisions of the myopic farmers are not "affected" by m_t^A (i.e., m_t^A is low so that $m_t^A \leqslant \underline{m}_t^A$ or m_t^A is high so that $m_t^A \geqslant \overline{m}_t^A$), an increase in m_t^A will always increase the equilibrium expected profit of the farmers who grow crop A. This indicates that, when the myopic farmers are not influenced by the changes in m_t^A, the strategic farmers will make decisions in such a way that the expected profit of all the farmers growing crop A will increase if m_t^A is increased.

It remains to analyze the impact of MSP of crop A on the farmers' expected profits when it is moderate (i.e., $\underline{m}_t^A < m_t^A < \overline{m}_t^A$). To simplify our analysis as before, let us consider a special case when ϵ_t^A and ϵ_t^B are independent random variables that follow $U[-\delta, \delta]$. Further, assume $m_t^k \geqslant p_{t-1}^k$, $k \in \{A, B\}$, so that both the MSPs are effective. Also, we define another threshold that will prove useful in our analysis. Let:

$$\widetilde{m}^A(m_t^B) = \left(\frac{r}{r+2}\right) m_t^B + \frac{2}{r+2}\left[\phi - \delta\left(\frac{2-r}{2+r}\right)\right].$$

Akin to $\tilde{m}$ as defined earlier, $\widetilde{m}^A \equiv \widetilde{m}^A(m_t^B) > 0$ when ϕ is sufficiently large and δ is sufficiently small. The following proposition shows that increasing the MSP of crop A can hurt the expected profits of those farmers who grow crop A in equilibrium.

Proposition 5 **(Impact of moderate m_t^A on farmers' profits)**. *For any given MSP for crop B m_t^B and suppose that the MSP for crop A is moderately low so that $m_t^A \in (\underline{m}_t^A, \widetilde{m}^A)$. Then there exists a threshold $\theta_1 \equiv \theta_1(m_t^A, m_t^B)$ such that $\frac{\partial \hat{\pi}_t^v(x)}{\partial m_t^A} < 0$ for all $\theta < \theta_1$, for each farmer of type $v \in \{m, s\}$ located at $x \leqslant \hat{\tau}^v$. Furthermore, if θ is sufficiently high, then $\frac{\partial \hat{\pi}_t^v(x)}{\partial m_t^A} \geqslant 0$.*

While Proposition 5 is based on the assumption that the market uncertainty $\epsilon_t^k \sim U[-\delta, \delta]$, $k \in \{A, B\}$, the results stated in the proposition continued to hold for general distribution. (Please see Proposition 7 in Appendix A for details.)

In Proposition 5, we identify a scenario in which increasing the MSP of a crop can decrease the expected profits of the farmers who grow that crop. According to the proposition, when the MSP of crop A is moderately low so that $m_t^A \in (\underline{m}_t^A, \widetilde{m}^A)$ and when there are very few strategic farmers (i.e., θ is sufficiently small so that $\theta < \theta_1$), then increasing m_t^A will hurt the expected profits of the farmers who grow crop A (i.e., for farmers who are of type $v \in \{m, s\}$ and located at x with $x \leqslant \hat{\tau}^v$). This is because, even with a small increase in m_t^A, there is a substantial increase in the production of crop A by the myopic farmers (because the proportion of myopic farmers $(1-\theta)$ is large when θ is small). Consequently, there is a drop in the price of crop A. This drop in the market price of crop A, coupled with the moderately low value of m_t^A, will reduce the expected profits of those farmers who grow crop A.

We can conclude that, when θ is sufficiently small, there exists a threshold, say, m_t^{A*} (as shown in Figure 4) such that offering MSP of A in $(\underline{m}_t^A, m_t^{A*})$ is disadvantageous to the farmers who grow crop A. In other words, by choosing m_t^A in the interval $(\underline{m}_t^A, m_t^{A*})$, the policy-maker creates an undesirable frenzy among the myopic farmers who switch to crop A thereby substantially increasing the production of crop A that causes a significant drop in the price of the crop, which overrides the benefit accrued by the increase in m_t^A at moderately low values, thereby hurting the expected profits of the farmers growing crop A in equilibrium.

Figure 4 provides a numerical example when the equilibrium revenue of farmers growing crop A decreases with an increase in m_t^A. The parameter values used for the example are $a = 1$, $\rho = 0.7$, $\alpha = -0.25$, $p_{t-1}^A = 0.1$, $p_{t-1}^B = 0.5$, $m_t^B = 0.55$, $r = 0.95$, $\theta = 0.1$, and $\epsilon_t^k \sim U[-0.1, 0.1]$. Note that it suffices to observe the sensitivities of expected revenues from the crops with respect to the MSP m_t^A, because the expected profits of a farmer from growing the crops are the expected revenue less the production cost of the corresponding crop, where the latter are independent of the MSPs for all $x \in [-0.5, 0.5]$.

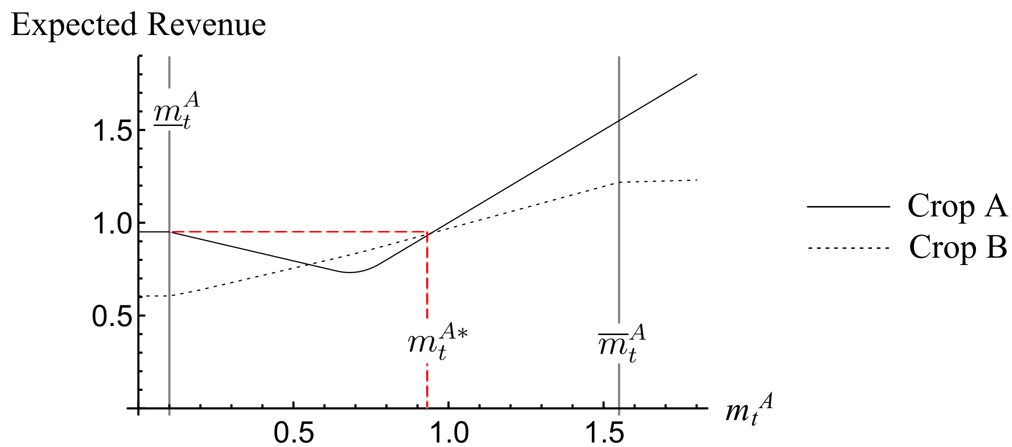


Figure 4 **Expected revenue from crop A and crop B as a function of MSP m_t^A.**

While the expected profit of the farmers who grow crop B is always non-decreasing in m_t^A (as shown in the first statement of Lemma 2), the profit of a farmer who grows crop A is non-monotonic in m_t^A. From Figure 4 we can draw the following conclusions about the value of MSPs. First, relative to the case when MSP is absent, offering a higher MSP that has $m_t^A > m_t^{A*}$ can benefit farmers who grow A as well as those who grow B. Second, relative to the case when MSP is absent, offering a moderately low MSP for a crop, say, crop A, can make those farmers who grow A to become worse off and make those farmers who grow B to become better off. When this happens, the actual impact of MSP for crop A violates the intended goal for offering MSP for crop A (which is intended to benefit farmers who grow crop A). Therefore, selecting an appropriate level of MSP is crucial and a policy-maker has to exercise sufficient care in choosing the right MSPs m_t^k, $k \in \{A, B\}$ to ensure that they: (i) benefit the farmers, especially those who grow crop k, and (ii) balance the crop availabilities for the consumer. We explore this topic further in Section 6.

6. Selection of efficient MSPs

Lastly, in this section, we formulate the optimization problem of a social planner (i.e., the policy-maker or the government) whose objective is to choose crop MSPs such that the farmers and the consumers can be benefited to the largest extent at the lowest possible total expenditure. First, we define *farmer surplus* in period t as follows:

$$\mathscr{F}_t(m_t^A, m_t^B) = \theta \int_{-0.5}^{0.5} \pi_t^s(x)\, dx + \overline{\theta} \int_{-0.5}^{0.5} \pi_t^m(x)\, dx$$

$$
\begin{aligned}
=&\theta \left[\int_{-0.5}^{\tau^s} \left(\mathbb{E}\left[\max\{p_t^A, m_t^A\}\right] - (x+0.5) \right) dx + \int_{\tau^s}^{0.5} \left(\mathbb{E}\left[\max\{p_t^B, m_t^B\}\right] - (0.5-x) \right) dx \right] \\
&+ \overline{\theta} \left[\int_{-0.5}^{\tau^m} \left(\mathbb{E}\left[\max\{p_t^A, m_t^A\}\right] - (x+0.5) \right) dx + \int_{\tau^m}^{0.5} \left(\mathbb{E}\left[\max\{p_t^B, m_t^B\}\right] - (0.5-x) \right) dx \right],
\end{aligned}
$$

where $p_t^A = \mathbb{E}[p_t^A] + \epsilon_t^A = \phi - r\hat{\tau} + \epsilon_t^A$, $p_t^B = \mathbb{E}[p_t^B] + \epsilon_t^B = \phi + r\hat{\tau} + \epsilon_t^B$ and $\hat{\tau} = \theta\hat{\tau}^s + \overline{\theta}\hat{\tau}^m$, as in Proposition 2. Second, We capture the *disutility of the consumers* through the imbalance of crop availability as follows:

$$
\mathscr{C}_t(m_t^A, m_t^B) = -\left(q_t^{AT} - q_t^{BT}\right)^2 = -4\hat{\tau}^2.
$$

Third, the *total expected expenditure* incurred by the policy-maker by setting MSPs m_t^A and m_t^B is given by:

$$
\begin{aligned}
\mathscr{K}_t(m_t^A, m_t^B) =& \hat{q}_t^{AT} \cdot \mathbb{E}[m_t^A - p_t^A]^+ + \hat{q}_t^{BT} \cdot \mathbb{E}[m_t^B - p_t^B]^+ \\
=& (\hat{\tau} + 0.5)\mathbb{E}[m_t^A - \phi + r\hat{\tau} - \epsilon_t^A]^+ + (0.5 - \hat{\tau})\mathbb{E}[m_t^B - \phi - r\hat{\tau} - \epsilon_t^B]^+,
\end{aligned}
$$

because government has to bear an expected expenditure of $\mathbb{E}[m_t^k - p_t^k]^+$ for all the quantity of $\hat{q}_t^{kT}$ of crop $k \in \{A, B\}$ produced. The quantity $\hat{q}_t^{kT}$ is as given in Proposition 2.

Using $\mathscr{F}_t$, $\mathscr{C}_t$ and $\mathscr{K}_t$, we can define the social welfare (maximization) problem ($\mathbf{SWP}_t$) in period t as below:

$$
\begin{aligned}
\mathbf{SWP}_t: \quad & \max_{m_t^A, m_t^B} \mathscr{W}_t(m_t^A, m_t^B) = \{\lambda \mathscr{F}_t(m_t^A, m_t^B) + (1-\lambda)\mathscr{C}_t(m_t^A, m_t^B)\} - \eta \mathscr{K}_t(m_t^A, m_t^B) \\
& \text{such that } 0 \leqslant m_t^k \leqslant M, k \in \{A, B\}, \\
& \qquad \mathscr{K}_t(m_t^A, m_t^B) \leqslant B,
\end{aligned}
$$

where $\lambda \in (0,1)$ and $(1-\lambda) \in (0,1)$ are the exogenous weights associated by the policy-maker to farmers' welfare and consumers' welfare, respectively, η is the sensitivity of the policy-maker (or the government) to its expenditure, M is the maximum limit of the MSP to be awarded to a crop, and B is a bound on the expected expenditure to be incurred (we can consider the constraint $\mathscr{K}_t(m_t^A, m_t^B) \leqslant B$ as a budget constraint).

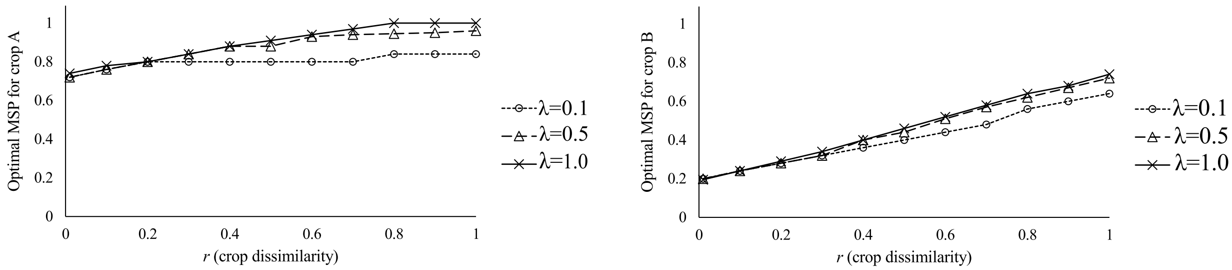
Having analyzed the impact of MSPs chosen by a policy-maker on farmers' crop selection and production decisions in the earlier section, we now focus on the effect of crop dissimilarity r (i.e., substitutability or complementarity) on the optimal choice of crop MSPs and crop balance. Offering crop MSPs without understanding the degree of complementarity (or substitutability) between the crops being supported by the MSPs can destabilize the availability of those crops to the consumers. For instance, MSPs focused on wheat and rice (which are substitutes) caused a severe shortage of coarse cereals and oil seeds and an over-production of rice and wheat in the Indian economy (Chand 2003, Parikh and Chandrashekhar 2007). Hence, we note that it is important to explore the impact of r, which measures the "dissimilarity" between the two crops, on the choice of MSPs and the consequent production decisions of farmers.

Given the complexity of the above problem, we solve it numerically and draw some insights. The parameter values used in our numerical example are $a=1$, $\rho=0.7$, $p_{t-1}^A=0.1$, $p_{t-1}^B=0.9$, $\theta=0.1$, $\eta=0.3$, $B=0.2$, $M=1$ and $\epsilon_t^k \sim U[-0.1, 0.1]$. We take the "weight" $\lambda=0.1, 0.5, 1$, which correspond to low, medium and high values, respectively. In our discussion we focus on the impact of crop dissimilarity (r) on the optimal choice of MSPs. We change r by varying α while retaining ρ constant (i.e., $\rho=0.7$).

As shown in Figure 5, the optimal value of MSP for crop A is higher than that for crop B, for each value of λ, because our example is based on the case when the previous period price of crop A is lower than that of crop B (i.e., $0.1=p_{t-1}^A<p_{t-1}^B=0.9$). Because of this past price differential, more myopic farmers choose to grow crop B and so a larger MSP should be offered for crop A in order to entice a few of these farmers to switch to growing crop A from growing crop B. Furthermore, we notice that the optimal MSPs of the crops are increasing in r, which can be explained as follows. When r increases (i.e., α decreases while ρ is left unchanged), the expected prices of the crops increase, for any given production quantities of the crops.[11] Hence it is less likely that the realized market prices are lower than the crop MSPs. As such, the government can afford to increase the MSPs in order to benefit the farmers. Thus, for any given budget, government will be able to offer higher MSPs for complementary crops (like rice/wheat and pulses/vegetables) than for substitutable crops (like rice and wheat).

Furthermore, when a policy-maker gives higher importance to the welfare of the farmers (i.e., as λ increases), the crop MSPs also increase, because, when appropriately chosen, higher MSPs improve farmers' revenues. The case when $\lambda=1$ corresponds to the extreme case when a policy-maker is concerned only about the welfare of the farmers but not at all about the welfare of the consumers.

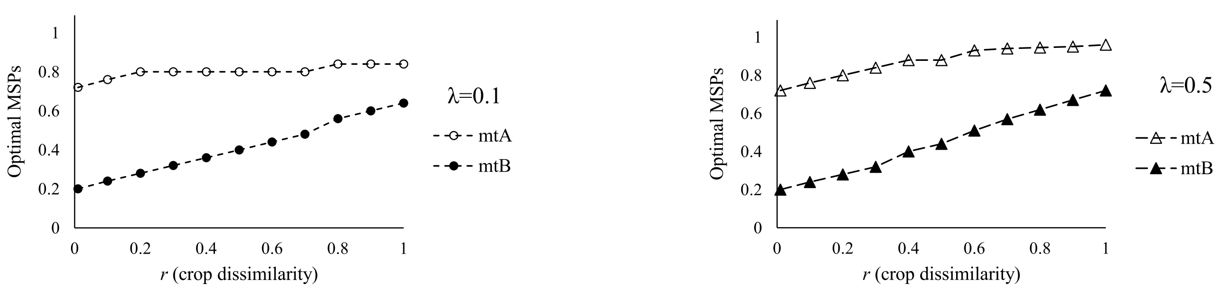
[11] Note that by differentiating (1) with respect to α, we obtain for $k\in\{A,B\}$ that $\frac{\partial \mathbb{E}[p_t^k]}{\partial \alpha}=-q_t^{jT}\leqslant 0$, $j\neq k$.



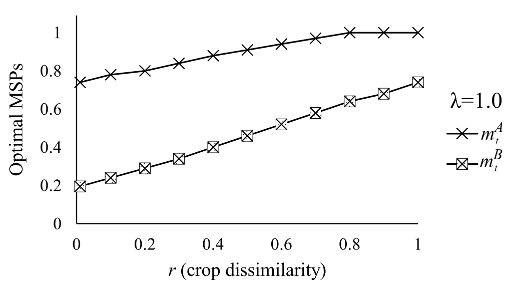
(a) Crop A (b) Crop B

Figure 5 Optimal MSPs for crops A and B for low, medium, and high λ values.

Next, the plots in Figure 6 indicate that the difference between the MSPs of crops A and B is decreasing in r, for any given value of λ. That is, as the complementarity between the crops (i.e., r) increases the crop MSPs have to be set in such a way that the difference between them decreases, in order to maintain a balance in crop production quantities. In other words, to maintain a balance of complementary crops (eg., rice and vegetables), a policy-maker should offer comparable MSPs for both the crops.



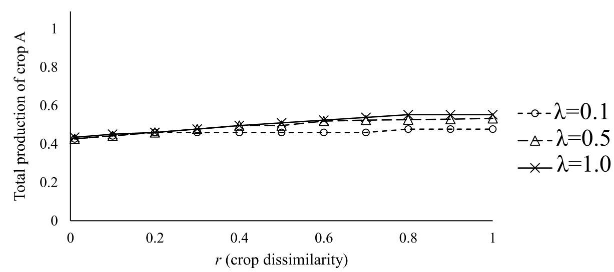
(a) $\lambda = 0.1$ (b) $\lambda = 0.5$



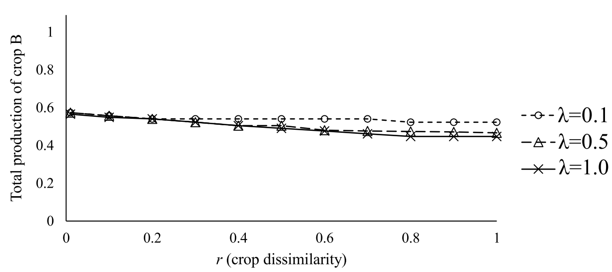
(c) $\lambda = 1$

Figure 6 MSPs of crops A and B for low, medium, and high λ values.

The total crop production quantities for our example are given in Figures 7a and 7b. (Note that the production of each crop is approximately 0.5 so that the production of crops is balanced.) Furthermore, we can observe from Figures 7a and 7b that the crop production quantity disparity decreases as λ decreases because lower values of λ give more importance to consumer welfare, which increases as the production quantity disparity between the crops decreases (we omit separate plots for individual values of λ due to space constraints).



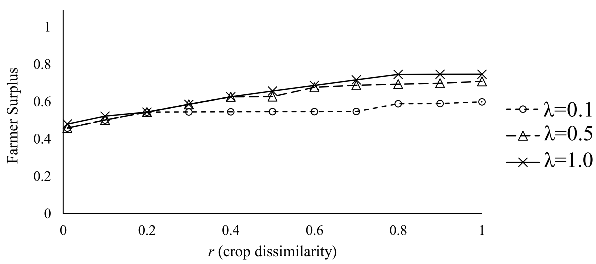
(a) Crop A



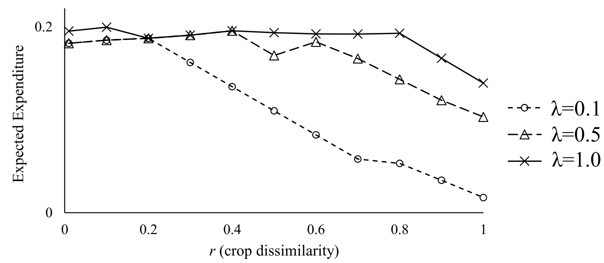
(b) Crop B

Figure 7 Total production of crops A and B for low, medium, and high λ values

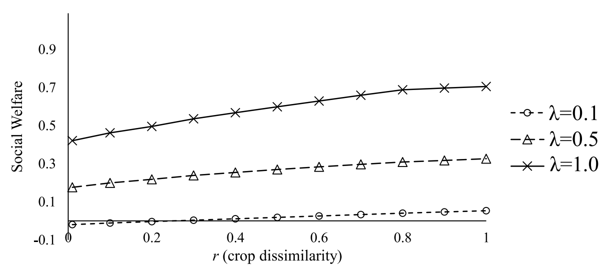
Finally, Figure 8 gives the plots of farmer surplus ($\mathscr{F}$), total expected expenditure incurred by policy-maker ($\mathscr{K}$), and social welfare ($\mathscr{W}$). (We omit the consumer disutility ($\mathscr{C}$) graph due to space constraints. The consumer disutility values can be easily obtained from Figures 7a and 7b by using the fact that $\mathscr{C} = -(q_t^{AT} - q_t^{BT})^2$). It is interesting to observe from Figure 8a that farmer surplus is increasing in crop disparity (r). This is due to the fact that, for any given production quantities of the crops, the expected prices of the crops increase as the complementarity between the crops increases. From Figure 8b we observe that the total expenditure incurred by a policy-maker in administering the MSP program is decreasing in r, when r is sufficiently high. Because the expected market prices of the crops are high when r is high, in many instances the crop market prices tend to be higher than the crop MSPs, which obviates the need for the policy-maker to purchase the crop at MSP, thereby reducing the expected expenditure incurred from the MSP program. Hence, by combining the farmer surplus (Figure 8a) and expected expenditure (Figure 8b) plots, we can infer that a policy-maker will achieve a higher farmer surplus at a lower expense by offering MSPs to diverse crops. Finally, from Figure 8c, we observe that the total social surplus increases as r increases.



(a) Farmer surplus



(b) Expected expenditure



(c) Social welfare

Figure 8 Farmer surplus, expected expenditure and social welfare for low, medium, and high λ values

7. Conclusions

In this paper, we analyzed the role of *minimum support prices* (MSPs), which is a government intervention to safeguard farmers' incomes against crop price falls and, at the same time, to ensure sufficient and balanced production of different crops. First, by considering a mixture of myopic and strategic farmers, we analyzed the behavior of myopic and strategic farmers, and their crop selection and production decisions, in the absence of MSPs. Later, we extended our analysis to incorporate MSPs and to analyze their impact, along with past prices, on farmers' crop selection and production decisions, future crop availabilities, and farmers' expected profits. Second, we discussed the impact of strategic farmers on farmers' crop selection and production decisions, future crop availabilities, and farmers' expected profits. By examining the interactions among a mixed population of myopic and strategic farmers for the case when there are two (complementary or substitutable) crops, we made the following findings.

First, we showed that, regardless of the MSPs offered to the crops, the price disparity between the crops always worsens as the complementarity between the crops increases. Second, we found that MSPs may not always be beneficial to farmers. We proved that when there are very few strategic farmers, an improper choice of MSP of a crop can negatively impact the profits of the farmers, both myopic and strategic, who grow that crop. This defeats the actual goal of MSP for a crop, which is to benefit the farmers who grow that crop. Third, we showed that the total production of a crop is increasing in the MSP offered for the crop and decreasing in the MSP offered for the other crop. Therefore, a carefully chosen MSPs can always be used to balance crop productions. Hence, to reduce quantity disparity between crops, a carefully designed MSP policy is critical.

Finally, we formulated the optimization problem of a policy-maker (i.e., government) with an objective to maximize social welfare (which is the sum of farmers' surplus and consumers' welfare less the policy-maker's expenditure) subject to a budgetary constraint on the expected expenditure incurred by the policy-maker in administering the MSP program. Given the complexity of the problem, we solved it numerically to draw a few practical insights, especially those pertaining to the impact of nature of crops (i.e., crop complementarity or dissimilarity) on the optimal choice of crop MSPs. First, we noted that, even though crop MSPs are increasing in the complementarity (or dissimilarity) between the crops, the difference between the crop MSPs decreases. Second, we observed that offering MSPs to dissimilar crops is efficient in achieving higher farmer surplus and higher social welfare at a lower expected expenditure. Hence, we inferred that it is more advantageous to offer MSPs to complementary crops like rice and pulses (or vegetables) than to offer MSPs to crops that are close substitutes like rice and wheat.

Our paper represents an initial attempt to examine the efficacy of MSPs of two (complementary or substitutable) crops in the presence of market price uncertainty and strategic farmers. However, there are plenty directions for future research. A natural and a challenging extension of our model is to incorporate multiple periods in the presence of hoarding; i.e., each farmer can sell his perishable crop over the next few period periods). In doing so, one can explore the impact of MSPs on the farmers crop planning and selling decisions over time. Another direction of future research is to examine the economic value of agricultural advisory services to farmers. Specifically, one can analyze the impact of long-term farming assistance programs that can enable farmers to take more strategic production decisions. Such a study will provide insights on the design and choice of such long-term programs vis-à-vis short-term (contingent) subsidy programs such as MSPs.

References

Alizamir S, Iravani F, Mamani H (2015) Price vs. revenue protection: An analysis of government subsidies in the agriculture industry. Technical report, Working paper.

An J, Cho SH, Tang CS (2015) Aggregating smallholder farmers in emerging economies. *Production and Operations Management* 24(9):1414–1429.

Byerlee D, Janvry AD, Sadoulet E, Townsend R, Klytchnikova I (2007) World development report 2008 : Agriculture for development. URL `http://documents.worldbank.org/curated/en/587251468175472382/World-development-report-2008-agriculture-for-development`.

Chand R (2003) Minimum support price in agriculture: Changing requirements. *Economic and Political Weekly* 3027–3028.

Chen YJ, Tang CS (2015) The economic value of market information for farmers in developing economies. *Production and Operations Management* 24(9):1441–1452.

Chhatre A, Devalkar S, Seshadri S, Ramaswami B (2016) Crop diversification = Profit and Risk: The Role of Intermediaries (Presented at the 4th ISB – POMS workshop, Indian School of Business, Hyderabad, India).

Commission for Agricultural Costs & Prices (2017) Determinants of MSP. URL `http://cacp.dacnet.nic.in`, Ministry of Agriculture & Farmers Welfare, Government of India, Accessed July 3, 2017.

Dantwala M (1967) Incentives and disincentives in Indian agriculture. *Indian Journal of Agricultural Economics* 22(2):1.

Dawande M, Gavirneni S, Mehrotra M, Mookerjee V (2013) Efficient distribution of water between head-reach and tail-end farms in developing countries. *Manufacturing & Service Operations Management* 15(2):221–238.

Federgruen A, Lall U, Simsek AS (2015) Supply chain analysis of contract farming. Technical report, Working paper.

Fox KA (1956) The contribution of farm price support programs to general economic stability. *Policies to Combat Depression*, 295–356 (NBER).

Guda H, Rajapakshe T, Dawande M, Janakiraman G (2016) Agricultural support prices in developing economies: Operational analysis and its use in policy making. Technical report, Working paper.

Hu M, Liu Y, Wang W (2016) Altruistic rationality: The value of strategic farmers, social entrepreneurs and for-profit firms in crop planting decisions .

Huh WT, Athanassoglou S, Lall U (2012) Contract farming with possible reneging in a developing country: Can it work? *IIMB Management Review* 24(4):187–202.

Huh WT, Lall U (2013) Optimal crop choice, irrigation allocation, and the impact of contract farming. *Production and Operations Management* 22(5):1126–1143.

Kazaz B, Webster S (2011) The impact of yield-dependent trading costs on pricing and production planning under supply uncertainty. *Manufacturing & Service Operations Management* 13(3):404–417.

Liao CN, Chen YJ, Tang CS (2017) Information provision policies for improving farmer welfare in developing countries: Heterogeneous farmers and market selection. *Manufacturing & Service Operations Management* .

Malamasuri K, B S, Parmar P, Malve S (2013) A historical perspective for minimum support price of agriculture crops. *Kisan World – A Journal of Agriculture and Rural Development* 40(12):46–48.

Parikh J, Chandrashekhar S (2007) Extension of msp: Fiscal and welfare implications. *A Study for the Planning Commission [of India] by Integrated Action and Research for Development* .

Parker C, Ramdas K, Savva N (2016) Is it enough? evidence from a natural experiment in India's agriculture markets. *Management Science* 62(9):2481–2503.

Spitze R (1978) The food and agriculture act of 1977: issues and decisions. *American Journal of Agricultural Economics* 60(2):225–235.

Subbarao D, et al. (2011) The challenge of food inflation. Technical report.

Tang CS, Wang Y, Zhao M (2015) The implications of utilizing market information and adopting agricultural advice for farmers in developing economies. *Production and Operations Management* 24(8):1197–1215.

Thorbecke E (1982) *Agricultural sector analysis and models in developing countries*. Number 5 (Food & Agriculture Org.).

Tripathi AK, et al. (2013) Agricultural price policy, output, and farm profitabilityexamining linkages during post-reform period in India. *Asian Journal of Agriculture and Development* 10(1):91–111.

Appendix A: Supplementary and Additional Results

Corollary 6 **(Impact of crop dissimilarity)**. *For a given pair of MSPs* (m_t^A, m_t^B) *the following statements hold:*

1. ***Crop availability disparity:*** *The disparity between the total production quantities of the crops decreases with* r *if there are strategic farmers. That is* $\frac{\partial |\Delta q_t|}{\partial r} < 0$ *if* $\theta > 0$ *where* $\Delta q_t = q_t^{AT} - q_t^{BT}$. *If* $\theta = 0$, *then* $\frac{\partial |\Delta q_t|}{\partial r} = 0$.
2. ***Crop price disparity:*** *However, the expected disparity between the two crop prices increases with the crop dissimilarity* r. *That is* $\frac{\partial |\mathbb{E}\Delta p_t|}{\partial r} \geqslant 0$.

Proposition 6 **(Some strategic farmers may forgo low MSPs)**. *Let $\tilde{\tilde{m}}$ be the unique value of m_t^A satisfying the equation $F(m_t^A - \phi + r\hat{\tau}^m) = \frac{r}{2+r}$. Then for each m_t^A such that $\underline{m}_t^A < m_t^A < \min\{\overline{m}_t^A, \tilde{\tilde{m}}\}$*[12]*, there exists a θ_0 such that $\frac{\partial \hat{\tau}^s}{\partial m_t^A} \leqslant 0$ for all $\theta < \theta_0$. Further, if θ is sufficiently high then $\frac{\partial \hat{\tau}^s}{\partial m_t^A} \geqslant 0$ always (i.e., $\lim_{\theta\to 1} \frac{\partial \hat{\tau}^s}{\partial m_t^A} \geqslant 0$ always).*

Proposition 7 **(Effect of moderate MSPs)**. *Let $\tilde{\tilde{m}}$ be the unique value of m_t^A satisfying the equation $F(m_t^A - \phi + r\hat{\tau}^m) = \frac{r}{2+r}$. Then for each m_t^A such that $\underline{m}_t^A < m_t^A < \min\{\overline{m}_t^A, \tilde{\tilde{m}}\}$*[13]*, there exists a threshold θ_1 such that $\frac{\partial \hat{\pi}_t^v(x)}{\partial m_t^A} \leqslant 0$ for all $\theta < \theta_1$, for each farmer of type $v \in \{m, s\}$ located at $x \leqslant \hat{\tau}^v$. Further, if θ is sufficiently high then $\frac{\partial \hat{\pi}_t^v(x)}{\partial m_t^A} \geqslant 0$ always (i.e., $\lim_{\theta\to 1} \frac{\partial \hat{\pi}_t^A}{\partial m_t^A} \geqslant 0$ always).*

Appendix B: Proofs

Proof of Lemma 1: First, we note that

$$|\xi_t| \leqslant 2\beta(1-r) \Rightarrow -2\beta(1-r) \leqslant \xi_t \leqslant 2\beta(1-r) \Rightarrow \{-2\beta \leqslant \xi_t - 2r\beta\} \wedge \{\xi_t + 2r\beta \leqslant 2\beta\}$$
$$\Rightarrow -2\beta \leqslant -2r\beta + \xi_t \leqslant p_t^A - p_t^B \leqslant 2r\beta + \xi_t \leqslant 2\beta \Rightarrow |p_t^A - p_t^B| \leqslant 2\beta.$$

Hence,

$$|p_t^A - p_t^B| > 2\beta \Rightarrow |\xi_t| > 2\beta(1-r)$$
$$\Leftrightarrow \mathbb{P}\left(|p_t^A - p_t^B| > 2\beta\right) \leqslant \mathbb{P}\left(|\xi_t| > 2\beta(1-r)\right) \leqslant \left(\frac{\sigma}{\sqrt{2}\beta(1-r)}\right)^2,$$

where the last inequality is obtained by using Chebyshev's inequality. ■

Proof of Lemma 2: The first statement is proved by using (11) and the fact that $\frac{\partial \mathbb{E}[p_t^B]}{\partial m_t^A} = r\frac{\partial \hat{\tau}}{\partial m_t^A} > 0$.

For the second statement, by substituting (23) in (11) and simplifying, we obtain that for every $x \leqslant \hat{\tau}^v$, $v \in \{m, s\}$,

$$\frac{\partial \hat{\pi}_t^v(x)}{\partial m_t^A} = \frac{2F(m_t^A - \phi + r\hat{\tau}) + r\theta F(m_t^A - \phi + r\hat{\tau})\overline{F}(m_t^B - \phi - r\hat{\tau}) - 2r\overline{\theta}\,\overline{F}(m_t^A - \phi + r\hat{\tau})\frac{\partial \hat{\tau}^m}{\partial m_t^A}}{2 + r\theta\left(\overline{F}(m_t^A - \phi + r\hat{\tau}) + \overline{F}(m_t^B - \phi - r\hat{\tau})\right)} \tag{12}$$

which is non-negative if $\frac{\partial \hat{\tau}^m}{\partial m_t^A} = 0$. Hence, by using Proposition 3 we obtain the desired result. ■

Proof of Proposition 1:

1. The myopic farmers anticipate the price in period t to be the same as the price in period $t-1$. A farmer produces crop A as long as the anticipated benefit from crop A is more than that from crop B, otherwise the farmer produces crop B (by Assumption 1). Therefore, the fraction of myopic farmers growing crop A is then given by:

$$\mathbb{P}\left(p_{t-1}^{Am} - c_A(x) \geqslant p_{t-1}^{Bm} - c_B(x)\right) = \mathbb{P}\left(p_{t-1}^{Am} - \left(x + \frac{1}{2}\right) \geqslant p_{t-1}^{Bm} - \left(\frac{1}{2} - x\right)\right)$$
$$= \mathbb{P}\left(x \leqslant \frac{p_{t-1}^{Am} - p_{t-1}^{Bm}}{2}\right) = \left(\frac{p_{t-1}^{Am} - p_{t-1}^{Bm}}{2}\right) + 0.5 \tag{13}$$

since $\frac{p_{t-1}^{Am} - p_{t-1}^{Bm}}{2} \in [-0.5, 0.5]$ (by Assumption 2). Thus, we obtain the threshold value as $\tau^m = \frac{p_{t-1}^{Am} - p_{t-1}^{Bm}}{2}$.

[12] Note that if $\tilde{\tilde{m}} < \underline{m}_t^A$ then the range of interest is empty. Hence, this condition is likely to be encountered when p_{t-1}^A is sufficiently low.

[13] Note that if $\tilde{\tilde{m}} < \underline{m}_t^A$ then the range of interest is empty. Hence, this condition is likely to be encountered when p_{t-1}^A is sufficiently low.

2. The strategic farmers on the other hand are forward-looking and hence anticipate the market price in period t by taking into account the total availability of the crops, which takes into account the behaviors of the myopic farmers and the other strategic farmers. Hence, by using the principle of rational expectations, the fraction of the strategic farmers growing crop A is given by:
$$\mathbb{P}\left(\mathbb{E}[p_t^A] - c_A(x) \geqslant \mathbb{E}[p_t^B] - c_B(x)\right) = \mathbb{P}\left(x \leqslant \frac{\mathbb{E}[p_t^A] - \mathbb{E}[p_t^B]}{2}\right). \tag{14}$$
From (1), and the fact that $q_t^{BT} = 1 - q_t^{AT}$ we obtain
$$\mathbb{E}[p_t^A] = a - \alpha - r q_t^{AT} \text{ and } \mathbb{E}[p_t^B] = a - \rho + r q_t^{AT} \Rightarrow \mathbb{E}[p_t^A] - \mathbb{E}[p_t^B] = r(1 - 2q_t^{AT}) \in (-r, r) \subset (-1, 1),$$
where $q_t^{AT} \in [0, 1]$ is the total production quantity of crop A. Therefore, from (14) we obtain the threshold τ^s as:
$$\tau^s = \frac{\mathbb{E}[p_t^A] - \mathbb{E}[p_t^B]}{2} = \frac{r(1 - 2q_t^{AT})}{2}. \tag{15}$$
and the total production quantity of crop A by strategic farmers is $q_t^{As} = \theta(\tau^s + 0.5)$. Further, using the fact that $q_t^{AT} = q_t^{As} + q_t^{Am} = \theta\tau^s + \overline{\theta}\tau^m + 0.5$ and substituting it in (15) we obtain:
$$\tau^s = \left(\frac{-r\overline{\theta}}{1 + r\theta}\right)\tau^m = -\hat{r}\tau^m.$$
Note that $|\tau^s| = |\hat{r}||\tau^m| < |\tau^m| \Rightarrow \tau^s \in [-0.5, 0.5]$ since $r < 1$ by Assumption 2.

3. The total availability of crop A is given by
$$q_t^{AT} = q_t^{As} + q_t^{Am} = \theta\tau^s + \overline{\theta}\tau^m + 0.5 = \tau + 0.5, \tag{16}$$
and by using (15), we obtain $\tau = \frac{\overline{\theta}\tau^m}{1 + r\theta}$. Note that $|\tau| = |\frac{\hat{r}}{r}||\tau^m| < |\tau^m| \Rightarrow \tau \in [-0.5, 0.5]$ since $\hat{r} < r$.

Proof of Proposition 2:

1. The fraction of myopic farmers sowing crop A is given by:
$$\mathbb{P}\left(\hat{p}_t^{Am} - c_A(x) \geqslant \hat{p}_t^{Bm} - c_B(x)\right) = \mathbb{P}\left(x \leqslant \frac{\hat{p}_t^{Am} - \hat{p}_t^{Bm}}{2}\right).$$
Since $|p_t^A - p_t^B| < 1$ and $|m_t^A - m_t^B| < 1$ by assumptions, we obtain $|\hat{p}_t^{Am} - \hat{p}_t^{Bm}| < 1$ so that the threshold value $\hat{\tau}^m$ is given by $\hat{\tau}^m = \frac{\hat{p}_t^{Am} - \hat{p}_t^{Bm}}{2} \in [-0.5, 0.5]$. The total quantity of crop A produced by myopic farmers is given by $\hat{q}_t^{Am} = \overline{\theta}(\hat{\tau}^m + 0.5)$.

2. On the other hand, the price anticipated by the strategic farmers for crop $k \in \{A, B\}$ is given by $\hat{p}_t^{ks} = \mathbb{E}_{\epsilon_t} \max\{p_t^k, m_t^k\}$ where $p_t^k = \mathbb{E}[p_t^k] + \epsilon_t$. Hence, the fraction of strategic farmers growing crop A is given by:
$$\mathbb{P}\left(\hat{p}_t^{As} - c_A(x) \geqslant \hat{p}_t^{Bs} - c_B(x)\right) = \mathbb{P}\left(x \leqslant \frac{\hat{p}_t^{As} - \hat{p}_t^{Bs}}{2}\right). \tag{17}$$
Using the fact that $\hat{p}_t^{ks} = \mathbb{E}[p_t^k] + \mathbb{E}_{\epsilon_t} \max\{\epsilon_t, m_t^k - \mathbb{E}[p_t^k]\}$, we can write
$$\begin{aligned}\hat{p}_t^{As} - \hat{p}_t^{Bs} &= \mathbb{E}[p_t^A] - \mathbb{E}[p_t^B] - \int_{m_t^A - \mathbb{E}[p_t^A]}^{\infty} F(\epsilon)\, d\epsilon + \int_{m_t^B - \mathbb{E}[p_t^B]}^{\infty} F(\epsilon)\, d\epsilon \\ &= \mathbb{E}[p_t^A] - \mathbb{E}[p_t^B] - \int_{m_t^A - \mathbb{E}[p_t^A]}^{m_t^B - \mathbb{E}[p_t^B]} F(\epsilon)\, d\epsilon. \end{aligned} \tag{18}$$

We know that $\mathbb{E}[p_t^A]-\mathbb{E}[p_t^B]=r(1-2\hat{q}_t^{AT})\in(-1,1)$, where $\hat{q}_t^{AT}$ is the total availability of crop A. Hence, (18) can we written as:

$$\hat{p}_t^{As}-\hat{p}_t^{Bs}=\mathbb{E}[p_t^A]-\mathbb{E}[p_t^B]-\int_{m_t^A-\mathbb{E}[p_t^A]}^{m_t^B-\mathbb{E}[p_t^B]}F(\epsilon)\,d\epsilon. \tag{19}$$

For any given set of MSPs (m_t^A,m_t^B), we have exactly one of these three cases to hold: (i) $\mathbb{E}[p_t^A]-\mathbb{E}[p_t^B]>m_t^A-m_t^B$, or (ii) $\mathbb{E}[p_t^A]-\mathbb{E}[p_t^B]<m_t^A-m_t^B$, or (iii) $\mathbb{E}[p_t^A]-\mathbb{E}[p_t^B]=m_t^A-m_t^B$. When (iii)holds, then trivially $\hat{p}_t^{As}-\hat{p}_t^{Bs}=\mathbb{E}[p_t^A]-\mathbb{E}[p_t^B]\in(-1,1)$. Hence, it remains to check if $\hat{p}_t^{As}-\hat{p}_t^{Bs}\in(-1,1)$ for cases (i) and (ii).

In case (i), we note that

$$\begin{aligned}
\hat{p}_t^{As}-\hat{p}_t^{Bs}&=\mathbb{E}[p_t^A]-\mathbb{E}[p_t^B]-\int_{m_t^A-\mathbb{E}[p_t^A]}^{m_t^B-\mathbb{E}[p_t^B]}F(\epsilon)\,d\epsilon\leqslant\mathbb{E}[p_t^A]-\mathbb{E}[p_t^B]<1, \text{ and}\\
\hat{p}_t^{As}-\hat{p}_t^{Bs}&=\mathbb{E}[p_t^A]-\mathbb{E}[p_t^B]-\int_{m_t^A-\mathbb{E}[p_t^A]}^{m_t^B-\mathbb{E}[p_t^B]}F(\epsilon)\,d\epsilon\geqslant\mathbb{E}[p_t^A]-\mathbb{E}[p_t^B]-\{m_t^B-\mathbb{E}[p_t^B]-(m_t^A-\mathbb{E}[p_t^B]+2r\hat{\tau})\}\\
&=m_t^A-m_t^B>-1.
\end{aligned}$$

In case (ii), we get

$$\begin{aligned}
\hat{p}_t^{As}-\hat{p}_t^{Bs}&=\mathbb{E}[p_t^A]-\mathbb{E}[p_t^B]+\int_{m_t^B-\mathbb{E}[p_t^B]}^{m_t^A-\mathbb{E}[p_t^A]}F(\epsilon)\,d\epsilon\geqslant\mathbb{E}[p_t^A]-\mathbb{E}[p_t^B]>-1, \text{ and}\\
\hat{p}_t^{As}-\hat{p}_t^{Bs}&=\mathbb{E}[p_t^A]-\mathbb{E}[p_t^B]+\int_{m_t^B-\mathbb{E}[p_t^B]}^{m_t^A-\mathbb{E}[p_t^A]}F(\epsilon)\,d\epsilon\leqslant-2r\hat{\tau}+\{m_t^A-\mathbb{E}[p_t^B]+2r\hat{\tau}-(m_t^B-\mathbb{E}[p_t^B])\}\\
&=m_t^A-m_t^B<1.
\end{aligned}$$

Hence, if $|m_t^A-m_t^B|<1$ then $\hat{p}_t^{As}-\hat{p}_t^{Bs}\in(-1,1)$ always and so we obtain the threshold τ^s as

$$\hat{\tau}^s=\frac{\hat{p}_t^{As}-\hat{p}_t^{Bs}}{2}=\frac{\mathbb{E}[p_t^A]-\mathbb{E}[p_t^B]}{2}-\frac{1}{2}\int_{m_t^A-\mathbb{E}[p_t^A]}^{m_t^B-\mathbb{E}[p_t^B]}F(\epsilon)\,d\epsilon\in(-0.5,0.5) \tag{20}$$

and the total production quantity of crop A by strategic farmers as $\hat{q}_t^{As}=\theta(\hat{\tau}^s+0.5)\in(0,1)$.

The total production of crops A and B are then given by $\hat{q}_t^{AT}=\hat{q}_t^{Am}+\hat{q}_t^{As}=\hat{\tau}+0.5$ and $\hat{q}_t^{BT}=0.5-\hat{\tau}$. Hence, we obtain $\mathbb{E}[p_t^A]=a-\alpha-r\hat{q}_t^{AT}=\phi-r\hat{\tau}$ and $\mathbb{E}[p_t^B]=a-\rho+r\hat{q}_t^{AT}=\phi+r\hat{\tau}$. Substituting these values in (20), we obtain

$$\hat{\tau}^s=-r\hat{\tau}-\frac{1}{2}\int_{m_t^A-\phi+r\hat{\tau}}^{m_t^B-\phi-r\hat{\tau}}F(\epsilon)\,d\epsilon\in[-0.5,0.5]. \tag{21}$$

By substituting $\hat{\tau}=\theta\hat{\tau}^s+\overline{\theta}\hat{\tau}^m$ in (21) we obtain (7). Note that the above equation is an implicit definition of $\hat{\tau}^s$ so that $\hat{\tau}^m$, $\hat{\tau}^s$ and $\hat{\tau}$ are all functions of of p_{t-1}^A, p_{t-1}^B, m_t^A and m_t^B. Hence, it is important to check the existence of equilibrium and, if possible, show that (7) is satisfied by a unique value of $\hat{\tau}$ in order to prove uniqueness of the equilibrium.

Proof of Uniqueness of $\hat{\tau}^s$: Let $RHS_{(21)}$ and $LHS_{(21)}$ denote the right-hand side and the left-hand side of (21), respectively. We note that $\frac{\partial LHS_{(21)}}{\partial\hat{\tau}^s}=1$, $LHS_{(21)}|_{\hat{\tau}^s=-0.5}=-0.5$ and $LHS_{(21)}|_{\hat{\tau}^s=0.5}=0.5$.

Next, we proved that $RHS_{(21)} \in [-0.5, 0.5]$. Hence, $RHS_{(21)}|_{\hat{\tau}^s=-0.5} \geqslant -0.5$ and $RHS_{(21)}|_{\hat{\tau}^s=0.5} \leqslant 0.5$. Further,

$$\begin{aligned}\frac{\partial RHS_{(21)}}{\partial \hat{\tau}^s} &= -r\theta - \frac{1}{2}\left[-r\theta F\left(m_t^B - \phi - r\hat{\tau}\right) - r\theta F\left(m_t^A - \phi + r\hat{\tau}\right)\right] \\ &= -\frac{r\theta}{2}\left[\overline{F}\left(m_t^B - \phi - r\hat{\tau}\right) + \overline{F}\left(m_t^A - \phi + r\hat{\tau}\right)\right] < 0\end{aligned}$$

Hence, by intermediate value theorem, there exists a unique solution to (21).

3. By definition, $q_t^{As} = \theta(\hat{\tau}^s + 0.5)$, $q_t^{Am} = \overline{\theta}(\hat{\tau}^m + 0.5)$ and $q_t^{AT} = q_t^{Am} + q_t^{As} = \hat{\tau} + 0.5$. ■

Proof of Proposition 3:

1. First, we note that $\frac{\partial \tau^m}{\partial m_t^A} \geqslant 0$ and $\frac{\partial \tau^m}{\partial m_t^B} \leqslant 0$ by its definition given in (6). Next, after differentiating (7) implicitly with respect to m_t^A and simplifying by using $\hat{r} = \frac{r\overline{\theta}}{1+r\theta}$, we obtain

$$2(1+r\theta)\frac{\partial \hat{\tau}^s}{\partial m_t^A} = -2r\overline{\theta}\frac{\partial \hat{\tau}^m}{\partial m_t^A} + F\left(m_t^A - \phi + r\hat{\tau}\right) + r\left[F\left(m_t^A - \phi + r\hat{\tau}\right) + F\left(m_t^B - \phi - r\hat{\tau}\right)\right]\frac{\partial \hat{\tau}}{\partial m_t^A}. \quad (22)$$

However, by definition of $\hat{\tau}$, we obtain $\frac{\partial \hat{\tau}^s}{\partial m_t^A} = \frac{1}{\theta}\left[\frac{\partial \hat{\tau}}{\partial m_t^A} - \overline{\theta}\frac{\partial \hat{\tau}^m}{\partial m_t^A}\right]$. Hence, we obtain:

$$\frac{\partial \hat{\tau}}{\partial m_t^A} = \frac{\theta F\left(m_t^A - \phi + r\hat{\tau}\right) + 2\overline{\theta}\frac{\partial \hat{\tau}^m}{\partial m_t^A}}{2 + r\theta\left[\overline{F}\left(m_t^A - \phi + r\hat{\tau}\right) + \overline{F}\left(m_t^B - \phi - r\hat{\tau}\right)\right]} \geqslant 0. \quad (23)$$

Similarly, we obtain

$$\frac{\partial \hat{\tau}}{\partial m_t^B} = -\frac{\theta F\left(m_t^B - \phi - r\hat{\tau}\right) - 2\overline{\theta}\frac{\partial \hat{\tau}^m}{\partial m_t^B}}{2 + r\theta\left[\overline{F}\left(m_t^A - \phi + r\hat{\tau}\right) + \overline{F}\left(m_t^B - \phi - r\hat{\tau}\right)\right]} \leqslant 0. \quad (24)$$

This proves the first statement. Further, by using the equation (22) for $\frac{\partial \hat{\tau}^s}{\partial m_t^A}$, we obtain

$$\frac{\partial \hat{\tau}^s}{\partial m_t^A} = \frac{F\left(m_t^A - \phi + r\hat{\tau}\right) - r\overline{\theta}\left[\overline{F}\left(m_t^A - \phi + r\hat{\tau}\right) + \overline{F}\left(m_t^B - \phi - r\hat{\tau}\right)\right]\frac{\partial \hat{\tau}^m}{\partial m_t^A}}{2 + r\theta\left[\overline{F}\left(m_t^A - \phi + r\hat{\tau}\right) + \overline{F}\left(m_t^B - \phi - r\hat{\tau}\right)\right]}. \quad (25)$$

Similarly, we obtain

$$\frac{\partial \hat{\tau}^s}{\partial m_t^B} = -\frac{F\left(m_t^B - \phi - r\hat{\tau}\right) + r\overline{\theta}\left[\overline{F}\left(m_t^A - \phi + r\hat{\tau}\right) + \overline{F}\left(m_t^B - \phi - r\hat{\tau}\right)\right]\frac{\partial \hat{\tau}^m}{\partial m_t^B}}{2 + r\theta\left[\overline{F}\left(m_t^A - \phi + r\hat{\tau}\right) + \overline{F}\left(m_t^B - \phi - r\hat{\tau}\right)\right]}. \quad (26)$$

Note that the signs of $\frac{\partial \hat{\tau}^s}{\partial m_t^A}$ and $\frac{\partial \hat{\tau}^s}{\partial m_t^B}$ cannot be ascertained easily.

2. By the definition of $\underline{m}_t^A$, there are two values of $\underline{m}_t^A$ that are possible: (i) $\underline{m}_t^A = p_{t-1}^A > (\max\{m_t^B, p_{t-1}^B\} - 1)$ and (ii) $\underline{m}_t^A = (\max\{m_t^B, p_{t-1}^B\} - 1) \geqslant p_{t-1}^A$. In case (i) we have $m_t^A \leqslant \underline{m}_t^A = p_{t-1}^A$ then $\hat{\tau}^m = \frac{p_{t-1}^A - \max\{m_t^B, p_{t-1}^B\}}{2}$ because $|m_t^A - m_t^B| < 1$. In case (ii) we have $m_t^A < (\max\{m_t^B, p_{t-1}^B\} - 1)$ which implies that $\hat{\tau}^m = -0.5$. Hence, $q_t^{Am} = \left[\frac{p_{t-1}^A - \max\{m_t^B, p_{t-1}^B\}}{2} + 0.5\right]^+$, which is independent of m_t^A. Hence, $\frac{\partial q_t^{Am}}{m_t^A} = 0 = \frac{\partial \hat{\tau}^m}{m_t^A}$. Using $q_t^{As} = \theta(\hat{\tau}^s + 0.5)$ we obtain $\frac{\partial q_t^{As}}{\partial m_t^A} = \theta\frac{\partial \hat{\tau}^s}{\partial m_t^A} \geqslant 0$ by noting from (25) that $\frac{\partial \hat{\tau}^s}{\partial m_t^A} \geqslant 0$ if $\frac{\partial \hat{\tau}^m}{m_t^A} = 0$.

3. The fact that $\hat{\tau}^m = +\frac{1}{2}$ when $m_t^A \geqslant \overline{m}_t^A$ follows directly from the fact that all the farmers produce crop A when $m_t^A \geqslant \overline{m}_t^A$. The proof of the remaining results follows as in part 2.

4. If $\underline{m}_t^A < m_t^A < \overline{m}_t^A$, then $\hat{\tau}^m = \frac{m_t^A - \max\{p_{t-1}^B, m_t^B\}}{2} \in (-0.5, 0.5) \Rightarrow q_t^{Am} \in (0, \overline{\theta})$. Hence, $\frac{\partial \hat{\tau}^m}{\partial m_t^A} = \frac{1}{2} \Rightarrow \frac{\partial q_t^{Am}}{\partial m_t^A} = \frac{\overline{\theta}}{2}$. Further, from (25) we obtain, $\frac{\partial \hat{\tau}^s}{\partial m_t^A} = \frac{F\left(m_t^A - \phi + r\hat{\tau}\right) - \frac{r\overline{\theta}}{2}\left[\overline{F}\left(m_t^A - \phi + r\hat{\tau}\right) + \overline{F}\left(m_t^B - \phi - r\hat{\tau}\right)\right]}{2 + r\theta\left[\overline{F}\left(m_t^A - \phi + r\hat{\tau}\right) + \overline{F}\left(m_t^B - \phi - r\hat{\tau}\right)\right]}$, whose sign cannot be ascertained. ■

Proof of Proposition 4: From (7), we have

$$2\hat{\tau}^s = -2r\hat{\tau} - \int_{m_t^A-\phi+r\hat{\tau}}^{m_t^B-\phi-r\hat{\tau}} F(\epsilon)\,d\epsilon = -2r\hat{\tau} - \int_{m_t^A-\phi+r\hat{\tau}}^{m_t^B-\phi-r\hat{\tau}} \frac{\epsilon+\delta}{2\delta}\,d\epsilon.$$

By substituting $\hat{\tau} = \theta\hat{\tau}^s + \overline{\theta}\hat{\tau}^m$ and $\hat{\tau}^m \equiv \hat{\tau}^m(m_t^A, m_t^B) = \frac{m_t^A - m_t^B}{2}$, we obtain

$$\hat{\tau}^s(m_t^A, m_t^B) = \left(\frac{m_t^A - m_t^B}{2}\right)\left[\frac{2\delta - 2\phi + m_t^A + m_t^B - r\overline{\theta}\,(2\delta + 2\phi - m_t^A - m_t^B)}{4\delta + r\theta\,(2\delta + 2\phi - m_t^A - m_t^B)}\right] \tag{27}$$

$$\Rightarrow \frac{\partial\hat{\tau}^s}{\partial m_t^A} = \frac{1}{2} + \frac{2\delta(1+r)(m_t^A - m_t^B)}{(4\delta + r\theta(2\delta + 2\phi - m_t^A - m_t^B))^2} - \frac{(1+r)(2\delta + 2\phi - m_t^A - m_t^B)}{2\,(4\delta + r\theta(2\delta + 2\phi - m_t^A - m_t^B))}$$

$$= \frac{V(\theta)}{2\,(4\delta + r\theta(2\delta + 2\phi - m_t^A - m_t^B))^2} \tag{28}$$

where

$$\begin{aligned} V(\theta) =& r^2(2\delta + 2\phi - m_t^A - m_t^B)^2\theta^2 \\ &+ r(2\delta + 2\phi - m_t^A - m_t^B)\left(8\delta - (1+r)(2\delta + 2\phi - m_t^A - m_t^B)\right)\theta \\ &- 8\delta\left((1+r)(\phi - m_t^A) - (1-r)\delta\right), \end{aligned}$$

which is a convex quadratic in θ and $V(0) < 0$ if $\underline{m}_t^A < m_t^A \leqslant \phi - \delta\left(\frac{1-r}{1+r}\right) (< \phi)$, that is if the MSP of A is moderately small. Hence, there exists a $\theta_0 > 0$ such that $V(\theta) < 0$ if and only if $\theta < \theta_0$, that is $\frac{\partial\hat{\tau}^s}{\partial m_t^A} < 0$ if and only if $\theta < \theta_0$. ■

Proof of Proposition 5: First, we obtain $\hat{\tau}^s(m_t^A, m_t^B)$ as given in (27). Second, since $|m_t^A - m_t^B| < 1$, we have $\hat{\tau}^m(m_t^A, m_t^B) = \frac{m_t^A - m_t^B}{2}$. Using these values of $\hat{\tau}^s(m_t^A, m_t^B)$ and $\hat{\tau}^m(m_t^A, m_t^B)$, we obtain

$$\hat{\tau}(m_t^A, m_t^B) = \theta\hat{\tau}^s(m_t^A, m_t^B) + \overline{\theta}\hat{\tau}^m(m_t^A, m_t^B) = \left(\frac{m_t^A - m_t^B}{2}\right)\left[\frac{4\delta - \theta\,(2\delta + 2\phi - m_t^A - m_t^B)}{4\delta + r\theta\,(2\delta + 2\phi - m_t^A - m_t^B)}\right].$$

On substituting the value of $\hat{\tau}(m_t^A, m_t^B)$ in (11) we obtain

$$\lim_{\theta\to 0} \frac{\partial\hat{\pi}_t^v(x)}{\partial m_t^A} = \frac{(2+r)^2}{8\delta}\left[m_t^A - \widetilde{m}^A\right] < 0$$

for $v \in \{m, s\}$ and $x \leqslant \hat{\tau}^v$. Hence the result.

Further, from (23), we have $\lim_{\theta\to 1} \frac{\partial\hat{\tau}}{\partial m_t^A} = \frac{F\left(m_t^A - \phi + r\hat{\tau}^s\right)}{2 + r\left[\overline{F}\left(m_t^A - \phi + r\hat{\tau}^s\right) + \overline{F}\left(m_t^B - \phi - r\hat{\tau}^s\right)\right]}$ so that, from (11), we obtain

$$\begin{aligned} \lim_{\theta\to 1} \frac{\partial\hat{\pi}_t^v(x)}{\partial m_t^A} &= F\left(m_t^A - \phi + r\hat{\tau}^s\right) - r\frac{F\left(m_t^A - \phi + r\hat{\tau}^s\right)}{2 + r\left[\overline{F}\left(m_t^A - \phi + r\hat{\tau}^s\right) + \overline{F}\left(m_t^B - \phi - r\hat{\tau}^s\right)\right]} \\ &= \frac{2F\left(m_t^A - \phi + r\hat{\tau}^s\right) + rF\left(m_t^A - \phi + r\hat{\tau}^s\right)\overline{F}\left(m_t^B - \phi - r\hat{\tau}^s\right)}{2 + r\left[\overline{F}\left(m_t^A - \phi + r\hat{\tau}^s\right) + \overline{F}\left(m_t^B - \phi - r\hat{\tau}^s\right)\right]} \geqslant 0\;\forall x \leqslant \hat{\tau}^v, v \in \{A, B\}. \end{aligned}$$

$\frac{\partial\hat{\pi}_t^v(x)}{\partial m_t^A} \geqslant 0$ for all $x > \tau^v$ by Lemma 2. Hence, $\lim_{\theta\to 1} \frac{\partial\hat{\pi}_t^v(x)}{\partial m_t^A} \geqslant 0$ for all $x \in (-0.5, 0.5)$ because of continuity of $\hat{\pi}_t^v(x)$ at $x = \hat{\tau}^v$, for $v \in \{A, B\}$. ■

Proof of Proposition 6: If $\underline{m}_t^A < m_t^A < \overline{m}_t^A$, then from Proposition 3, we have $\frac{\partial\hat{\tau}^m}{\partial m_t^A} = \frac{1}{2}$. Hence, from (25) we find that if

$$F\left(m_t^A - \phi + r\hat{\tau}\right) < \frac{r\overline{\theta}}{2 + r\overline{\theta}}\left[1 + \overline{F}\left(m_t^B - \phi - r\hat{\tau}\right)\right] \tag{29}$$

then $\frac{\partial \hat{\tau}^s}{\partial m_t^A} \leqslant 0$. Note that $m_t^A - \phi + r\hat{\tau}^m$ is increasing in m_t^A and hence the equation $F\left(m_t^A - \phi + r\hat{\tau}^m\right) = \frac{r}{2+r}$ has a unique solution (which we denote by $\tilde{\tilde{m}}$). Hence, for m_t^A such that $\underline{m}_t^A < m_t^A < \min\left\{\overline{m}_t^A, \tilde{\tilde{m}}\right\}$ we have

$$\lim_{\theta\to 0} F\left(m_t^A - \phi + r\hat{\tau}\right) = F\left(m_t^A - \phi + r\hat{\tau}^m\right) < \frac{r}{2+r} = \lim_{\theta\to 0} \frac{r\overline{\theta}}{2+r\overline{\theta}} < \lim_{\theta\to 0} \frac{r\overline{\theta}}{2+r\overline{\theta}}\left[1+\overline{F}\left(m_t^B - \phi - r\hat{\tau}\right)\right]$$

Hence, there exists θ_0 (sufficiently close to 0) such that $F\left(m_t^A - \phi + r\hat{\tau}\right) < \frac{r\overline{\theta}}{2+r\overline{\theta}}\left[1+\overline{F}\left(m_t^B - \phi - r\hat{\tau}\right)\right]$ for all $\theta \in [0, \theta_0)$. The proof is completed by using (29). ■

Proof of Proposition 7: If $\underline{m}_t^A < m_t^A < \overline{m}_t^A$, then from Proposition 3, we have $\frac{\partial \hat{\tau}^m}{\partial m_t^A} = \frac{1}{2}$. Hence, from (12), we observe that if

$$F\left(m_t^A - \phi + r\hat{\tau}\right) \leqslant \frac{r\overline{\theta}}{2+r\overline{\theta}+r\theta\overline{F}\left(m_t^B - \phi - r\hat{\tau}\right)} \tag{30}$$

then $\frac{\partial \hat{\pi}_t^A}{\partial m_t^A} \leqslant 0$. Hence, we have

$$\lim_{\theta\to 0} F\left(m_t^A - \phi + r\hat{\tau}\right) = F\left(m_t^A - \phi + r\hat{\tau}^m\right) < \frac{r}{2+r} = \lim_{\theta\to 0}\left\{\frac{r\overline{\theta}}{2+r\overline{\theta}+r\theta\overline{F}\left(m_t^B - \phi - r\hat{\tau}\right)}\right\}$$

Hence, there exists θ_1 (sufficiently close to 0) such that $F\left(m_t^A - \phi + r\hat{\tau}\right) < \frac{r\overline{\theta}}{2+r\overline{\theta}+r\theta\overline{F}\left(m_t^B - \phi - r\hat{\tau}\right)}$ for all $\theta \in [0, \theta_1)$. The proof is completed by using (30). Further, $\lim_{\theta\to 1} \frac{\partial \hat{\pi}_t^v(x)}{\partial m_t^A} \geqslant 0$ is shown the same way as in Proposition 5. ■

Proof of Corollary 1:

1. $|\Delta q_t| = |q_t^{AT} - q_t^{BT}| = 2|\tau| = \frac{2\overline{\theta}}{1+r\theta}|\tau^m| \Rightarrow \frac{\partial|\Delta q_t|}{\partial r} < 0$ if $\theta > 0$. Clearly, if $\theta = 0$ then $|\Delta q_t|$ is independent of r.
2. $|\mathbb{E}\Delta p_t| = |\Delta p_{t-1}|\hat{r} \Rightarrow \frac{\partial|\mathbb{E}\Delta p_t|}{\partial r} = |\Delta p_{t-1}|\frac{\partial \hat{r}}{\partial r} > 0$. ■

Proof of Corollary 2: The proof follows directly from the expressions derived in Proposition 1 and (5). ■

Proof of Corollary 3:

1. The proof follows from the definition of τ^m and q_t^{Am} given in the first statement of Proposition 1.
2. From the second statement of Proposition 1 we obtain:

$$\tau^s = -\hat{r}\tau^m \Rightarrow \frac{\partial \tau^s}{\partial \theta} = -\tau^m \frac{\partial \hat{r}}{\partial \theta} = \frac{r(1+r)}{(1+r\theta)^2}\tau^m \text{ and} \tag{31}$$

$$q_t^{As} = \theta(\tau^s + \frac{1}{2}) \Rightarrow \frac{\partial q_t^{As}}{\partial \theta} = \tau^s + \frac{1}{2} + \theta\frac{\partial \tau^s}{\partial \theta} = -\hat{r}\tau^m + \frac{1}{2} + \theta\frac{r(1+r)}{(1+r\theta)^2}\tau^m, \tag{32}$$

   which gives the desired result on simplification.
3. The expression for $\frac{\partial \tau}{\partial \theta}$ (or equivalently $\frac{\partial q_t^{AT}}{\partial \theta}$) is obtained by differentiating the expression for τ (or equivalently q_t^{AT}) given in the third statement of Proposition 1. Next, from the third statement of Corollary 2 we obtain $\frac{\partial^2 \tau}{\partial\theta\partial\Delta p_{t-1}} = \frac{\partial^2 q_t^{AT}}{\partial\theta\partial\Delta p_{t-1}} = -\frac{r+1}{2(1+r\theta)^2} < 0$.
4. The result is obtained by successively differentiating (5) with respect to θ followed by Δp_{t-1}. ■

Proof of Corollary 4:

1. By differentiating (6) with respect to p_{t-1}^A, p_{t-1}^B and Δp_{t-1}, we get

$$\frac{\partial \hat{\tau}^m}{\partial p_{t-1}^A} = \frac{1}{2}\cdot \mathbb{I}_{\{m_t^A < p_{t-1}^A\}} \geqslant 0, \text{ and } \frac{\partial \hat{\tau}^m}{\partial p_{t-1}^B} = -\frac{1}{2}\cdot \mathbb{I}_{\{m_t^B < p_{t-1}^B\}} \leqslant 0,$$

   where $\hat{p}^{km} = \max\{m_t^k, p_{t-1}^k\},\ k \in \{A, B\}$. The expressions for $\frac{\partial q_t^{Am}}{\partial p_{t-1}^k}$ can be obtained by using $q_t^{Am} = \overline{\theta}(\tau^m + 0.5)$.

2. By differentiating (7) with respect to p_{t-1}^A, we get

$$\frac{\partial\hat{\tau}^s}{\partial p_{t-1}^A} = -\frac{r}{2}\cdot\left[\overline{F}\left(m_t^A-\phi+r\hat{\tau}\right)+\overline{F}\left(m_t^B-\phi-r\hat{\tau}\right)\right]\frac{\partial\hat{\tau}}{\partial p_{t-1}^A}$$
$$\Rightarrow \frac{\partial\hat{\tau}^s}{\partial p_{t-1}^A} = -\left[\frac{\overline{\theta}r\left[\overline{F}\left(m_t^A-\phi+r\hat{\tau}\right)+\overline{F}\left(m_t^B-\phi-r\hat{\tau}\right)\right]}{2+\theta r\left[\overline{F}\left(m_t^A-\phi+r\hat{\tau}\right)+\overline{F}\left(m_t^B-\phi-r\hat{\tau}\right)\right]}\right]\frac{\partial\hat{\tau}^m}{\partial p_{t-1}^A}\leqslant 0$$

where the second equation is obtained by using $\hat{\tau}=\theta\hat{\tau}^s+\overline{\theta}\hat{\tau}^m$. Similarly, we obtain

$$\frac{\partial\hat{\tau}^s}{\partial p_{t-1}^B} = -\left[\frac{\overline{\theta}r\left[\overline{F}\left(m_t^A-\phi+r\hat{\tau}\right)+\overline{F}\left(m_t^B-\phi-r\hat{\tau}\right)\right]}{2+\theta r\left[\overline{F}\left(m_t^A-\phi+r\hat{\tau}\right)+\overline{F}\left(m_t^B-\phi-r\hat{\tau}\right)\right]}\right]\frac{\partial\hat{\tau}^m}{\partial p_{t-1}^B}\geqslant 0.$$

The expressions for $\frac{\partial q_t^{As}}{\partial p_{t-1}^k}$ can be obtained by using $q_t^{As}=\theta(\tau^s+0.5)$.

3. By using the fact that $\hat{\tau}=\theta\hat{\tau}^s+\overline{\theta}\hat{\tau}^m$ and $q_t^{AT}=\tau+0.5$, we obtain

$$\frac{\partial\hat{\tau}}{\partial p_{t-1}^A}=\frac{\partial q_t^{AT}}{\partial p_{t-1}^A}=\left[\frac{2\overline{\theta}}{2+r\theta\left[\overline{F}\left(m_t^A-\phi+r\hat{\tau}\right)+\overline{F}\left(m_t^B-\phi-r\hat{\tau}\right)\right]}\right]\frac{\partial\hat{\tau}^m}{\partial p_{t-1}^A}\geqslant 0$$
$$\frac{\partial\hat{\tau}}{\partial p_{t-1}^B}=\frac{\partial q_t^{AT}}{\partial p_{t-1}^B}=\left[\frac{2\overline{\theta}}{2+r\theta\left[\overline{F}\left(m_t^A-\phi+r\hat{\tau}\right)+\overline{F}\left(m_t^B-\phi-r\hat{\tau}\right)\right]}\right]\frac{\partial\hat{\tau}^m}{\partial p_{t-1}^B}\leqslant 0.$$

4. If $x\leqslant\hat{\tau}^v$, $v\in\{m,s\}$, then the farmer at x produces crop A. Hence, from (10) we obtain

$$\frac{\partial\hat{\pi}_t^v(x)}{p_{t-1}^A}=\frac{\partial\hat{\Pi}_t^A(x)}{p_{t-1}^A}=-r\overline{F}\left(m_t^A-\phi+r\hat{\tau}\right)\frac{\partial\hat{\tau}}{\partial p_{t-1}^A}\leqslant 0 \text{ and}$$
$$\frac{\partial\hat{\pi}_t^v(x)}{p_{t-1}^B}=\frac{\partial\hat{\Pi}_t^A(x)}{p_{t-1}^B}=-r\overline{F}\left(m_t^A-\phi+r\hat{\tau}\right)\frac{\partial\hat{\tau}}{\partial p_{t-1}^B}\geqslant 0.$$

Similarly, when $x>\hat{\tau}^v$, $v\in\{m,s\}$, then the farmer at x produces crop B. Hence, from (10) we obtain

$$\frac{\partial\hat{\pi}_t^v(x)}{p_{t-1}^A}=\frac{\partial\hat{\Pi}_t^B(x)}{p_{t-1}^A}=r\overline{F}\left(m_t^B-\phi-r\hat{\tau}\right)\frac{\partial\hat{\tau}}{\partial p_{t-1}^A}\geqslant 0 \text{ and}$$
$$\frac{\partial\hat{\pi}_t^v(x)}{p_{t-1}^B}=\frac{\partial\hat{\Pi}_t^A(x)}{p_{t-1}^B}=r\overline{F}\left(m_t^B-\phi-r\hat{\tau}\right)\frac{\partial\hat{\tau}}{\partial p_{t-1}^B}\leqslant 0. \quad\blacksquare$$

Proof of Corollary 5 : For exposition, we define the sign function as: $sgn[x]=-1$ if $x<0$, $sgn[x]=+1$ if $x>0$ and $sgn[0]=0$.

1. From (6) we obtain $\frac{\partial\hat{\tau}^m}{\partial\theta}=0$. Since $\hat{q}_t^{Am}=\overline{\theta}(\hat{\tau}^m+0.5)$, we obtain $\frac{\partial\hat{q}^{Am}}{\partial\theta}=-(\hat{\tau}^m+0.5)\leqslant 0$.

2. From (7) we obtain

$$\frac{\partial\hat{\tau}^s}{\partial\theta}=-2r\frac{\partial\hat{\tau}}{\partial\theta}+r\frac{\partial\hat{\tau}}{\partial\theta}\left(F(m_t^A-\phi-r\hat{\tau})+F(m_t^B-\phi+r\hat{\tau})\right)=-r\left(\overline{F}(m_t^A-\phi+r\hat{\tau})+\overline{F}(m_t^B-\phi-r\hat{\tau})\right)\frac{\partial\hat{\tau}}{\partial\theta},$$

and $\hat{\tau}=\theta\hat{\tau}^s+\overline{\theta}\hat{\tau}^m\Rightarrow\frac{\partial\hat{\tau}}{\partial\theta}=\hat{\tau}^s+\theta\frac{\partial\hat{\tau}^s}{\partial\theta}-\hat{\tau}^m$ so that

$$\frac{\partial\hat{\tau}^s}{\partial\theta}=\frac{r\left(\overline{F}(m_t^A-\phi+r\hat{\tau})+\overline{F}(m_t^B-\phi-r\hat{\tau})\right)}{1+r\theta\left(\overline{F}(m_t^A-\phi+r\hat{\tau})+\overline{F}(m_t^B-\phi-r\hat{\tau})\right)}\cdot(\hat{\tau}^m-\hat{\tau}^s)\Rightarrow\text{sgn}\left[\frac{\partial\hat{\tau}^s}{\partial\theta}\right]=\text{sgn}\left[\hat{\tau}^m-\hat{\tau}^s\right]\geqslant 0\Leftrightarrow\hat{\tau}^m\geqslant\hat{\tau}_0^s.$$

Since $q_t^{As}=\theta(\hat{\tau}^s+0.5)$ we obtain

$$\frac{\partial q_t^{As}}{\partial\theta}=\hat{\tau}^s+0.5+\theta\frac{\partial\hat{\tau}^s}{\partial\theta}=\frac{(\hat{\tau}^s+0.5)+r\theta\left(\overline{F}(m_t^A-\phi+r\hat{\tau})+\overline{F}(m_t^B-\phi-r\hat{\tau})\right)(\hat{\tau}^m+0.5)}{1+r\theta\left(\overline{F}(m_t^A-\phi+r\hat{\tau})+\overline{F}(m_t^B-\phi-r\hat{\tau})\right)}\geqslant 0$$

because $\hat{\tau}^m,\hat{\tau}^s\in[-0.5,0.5]$.

3. As shown in part 2, $\frac{\partial\hat{\tau}}{\partial\theta} = \hat{\tau}^s + \theta\frac{\partial\hat{\tau}^s}{\partial\theta} - \hat{\tau}^m$. Hence,
$$\frac{\partial\hat{\tau}}{\partial\theta} = \frac{(\hat{\tau}^s - \hat{\tau}^m)}{1 + r\theta\left(\overline{F}(m_t^A - \phi + r\hat{\tau}) + \overline{F}(m_t^B - \phi - r\hat{\tau})\right)} \Rightarrow \operatorname{sgn}\left[\frac{\partial\hat{\tau}}{\partial\theta}\right] = \operatorname{sgn}[\hat{\tau}^s - \hat{\tau}^m] \leqslant 0 \Leftrightarrow \hat{\tau}^m \geqslant \hat{\tau}_0^s.$$
4. The result is obtained by differentiating (10) with respect to θ. ■

Proof of Corollary 6: By differentiating (7) implicitly by r we obtain
$$\begin{aligned}\frac{\partial\hat{\tau}^s}{\partial r} &= -2\hat{\tau} - 2r\frac{\partial\hat{\tau}}{\partial r} + \left(F(m_t^A - \phi + r\hat{\tau}) + F(m_t^B - \phi - r\hat{\tau})\right)\frac{\partial}{\partial r}(r\hat{\tau}) \\ &= -\left(\overline{F}(m_t^A - \phi + r\hat{\tau}) + \overline{F}(m_t^B - \phi - r\hat{\tau})\right)\left(\hat{\tau} + r\frac{\partial\hat{\tau}}{\partial r}\right).\end{aligned}$$
Further, by definition of $\hat{\tau}$ we obtain $\frac{\partial\hat{\tau}}{\partial r} = \theta\frac{\partial\hat{\tau}^s}{\partial r}$. Hence, we obtain
$$\begin{aligned}\frac{1}{\theta}\frac{\partial\hat{\tau}}{\partial r} &= -\left(\overline{F}(m_t^A - \phi + r\hat{\tau}) + \overline{F}(m_t^B - \phi - r\hat{\tau})\right)\left(\hat{\tau} + r\frac{\partial\hat{\tau}}{\partial r}\right) \\ \Rightarrow \frac{\partial\hat{\tau}}{\partial r} &= -\frac{\theta\left(\overline{F}(m_t^A - \phi + r\hat{\tau}) + \overline{F}(m_t^B - \phi - r\hat{\tau})\right)\hat{\tau}}{1 + r\theta\left(\overline{F}(m_t^A - \phi + r\hat{\tau}) + \overline{F}(m_t^B - \phi - r\hat{\tau})\right)}.\end{aligned}$$
Therefore, we have the following:

1.
$$\Delta q_t = q_t^{AT} - q_t^{BT} = 2q_t^{AT} - 1 = 2\hat{\tau} \Rightarrow \frac{\partial\Delta q_t}{\partial r} = 2\frac{\partial\hat{\tau}}{\partial r} = -\frac{2\theta\hat{\tau}\left(\overline{F}(m_t^A - \phi + r\hat{\tau}) + \overline{F}(m_t^B - \phi - r\hat{\tau})\right)}{1 + r\theta\left(\overline{F}(m_t^A - \phi + r\hat{\tau}) + \overline{F}(m_t^B - \phi - r\hat{\tau})\right)}.$$
Now, consider the value of $\hat{\tau}$ at $r = 0$. We consider two cases: (i) $\hat{\tau}|_{r=0} > 0$ and (ii) $\hat{\tau}|_{r=0} < 0$. It is easy to see that when $\hat{\tau}|_{r=0} = 0$ then $\frac{\partial\Delta q_t}{\partial r} = 0$. When (i) $\hat{\tau}|_{r=0} > 0$, then crop A is produced more than crop B at $r = 0$ (i.e.,$\Delta q_t|_{r=0} > 0$) and hence, since strategic farmers are present (i.e., $\theta > 0$), $\frac{\partial\Delta q_t}{\partial r} < 0$. That is the disparity between the quantities of crops A and B decreases because of the strategic farmers. When (ii) $\hat{\tau}|_{r=0} < 0$, then crop A is produced less than crop B at $r = 0$ (i.e.,$\Delta q_t|_{r=0} < 0$) and hence, since strategic farmers are present (i.e., $\theta > 0$), $\frac{\partial\Delta q_t}{\partial r} > 0$. That is the disparity between the quantities of crops A and B decreases because of the strategic farmers. Hence,

2.
$$\begin{aligned}&\mathbb{E}[\Delta p_t] = \mathbb{E}[p_t^A - p_t^B] = \mathbb{E}[p_t^A] - \mathbb{E}[p_t^B] = -2r\hat{\tau} \Rightarrow \frac{\partial\mathbb{E}[\Delta p_t]}{\partial r} = -2\left(\hat{\tau} + r\frac{\partial\hat{\tau}}{\partial r}\right) \\ &\Rightarrow \frac{\partial\mathbb{E}[\Delta p_t]}{\partial r} = -2\left(\hat{\tau} + r\frac{\partial\hat{\tau}}{\partial r}\right) = \frac{-2\hat{\tau}}{1 + r\theta\left(\overline{F}(m_t^A - \phi + r\hat{\tau}) + \overline{F}(m_t^B - \phi - r\hat{\tau})\right)}\end{aligned}$$
Now, consider the value of $\hat{\tau}$ at $r = 0$. We consider two cases: (i) $\hat{\tau}|_{r=0} > 0$ and (ii) $\hat{\tau}|_{r=0} < 0$. It is easy to see that when $\hat{\tau}|_{r=0} = 0$ then $\frac{\partial\mathbb{E}\Delta p_t}{\partial r} = 0$. When (i) $\hat{\tau}|_{r=0} > 0$, then crop A is produced more than crop B at $r = 0$ and hence $\mathbb{E}[\Delta p_t]|_{r=0} < 0$. Hence, $\frac{\partial\mathbb{E}\Delta p_t}{\partial r} < 0$ (i.e., as r increases the price of crop A goes further down while that of crop B goes up, thus widening the gap between the two prices). When (ii) $\hat{\tau}|_{r=0} < 0$, then crop A is produced less than crop B at $r = 0$ and hence $\mathbb{E}[\Delta p_t]|_{r=0} > 0$. Hence, $\frac{\partial\mathbb{E}\Delta p_t}{\partial r} > 0$ (i.e., as r increases the price of crop A goes further up while that of crop B goes down, thus widening the gap between the two prices). Hence, $\frac{\partial|\mathbb{E}\Delta p_t|}{\partial r} \geqslant 0$. ■