# The Common-Probability Auction Puzzle<sup>\*</sup>

M. Kathleen Ngangoué<sup>†</sup> Andrew Schotter<sup>†</sup>

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#### Abstract

This paper presents a puzzle in the behavior of experimental subjects in what we call common-probability auctions. In common-value auctions, uncertainty is defined over values while, in common-probability auctions, uncertainty is defined over probabilities. We find that in contrast to the substantial overbidding found in common-value auctions, bidding in strategically equivalent common-probability auctions is consistent with Nash-equilibrium. Additional treatments reveal that subjects valued the auctioned items equally, implying that differences in bidding behavior originate in the strategic uncertainty of the auction.

JEL-Classification: D44, D81, C70, C90

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<sup>&</sup>lt;sup>†</sup>New York University, 19 West 4th St., 6th floor, NY NY 10012. Schotter: and drew.schotter@nyu.edu, Ngangoue: kn44@nyu.edu.

# 1 Introduction

In the typical common-value auction a good is sold whose unknown value is common to all bidders. Each bidder receives a signal about the value drawn from a commonly known distribution and, based on the signal received, makes a bid. Hence, in such auctions the value of the good being auctioned is known only probabilistically – its value is uncertain. This way of modeling auctions has been motivated, for instance, with the famous example of bidding for oil rights among oil companies. In this paper we call such auctions with common uncertainty about a value "common-value (CV)" auctions.

Models of common-value auctions generally assume that bidders adjust their beliefs about the item's expected value by updating the relevant range of values. Another possibility that, so far in the experimental literature, has been neglected is that bidders may also update their expectations by revising probabilities. That is the case, for instance, if uncertainty stems primarily from not knowing a probability rather than a value.

Suppose an investment project can either succeed or fail. In case of a success the project's value is precisely known, as well as in case of a failure. For such investments with binary outcomes forming beliefs directly about the success probability (rather than an expected value) seems natural, and uncertainty may then be modeled in probabilities. Consider, for instance, firms bidding for bonds issued by a corporation under financial stress. Here the value of the bond at maturity is known but what is uncertain is the probability of default by the corporation. If investors do their due diligence they will receive a signal about the common default risk drawn from a commonly known distribution and, based on this probability signal, make a bid for the bond. In such situations the uncertainty involved in the auction comes from not knowing the common default risk or, more generally, a common probability. We call such auctions "common-probability (CP)" auctions.

The question we ask in this paper is whether bidders process these two types of auctions in the same way. In other words, do bidders when facing two strategically equivalent common-value and common-probability auctions submit identical bids or does the fact that one auction exhibits uncertainty in the value domain while the other exhibits it in the probability domain lead to differences in bidding behavior?

We answer this question by comparing the two cases with uncertainty

in values versus probabilities only. Of course, most markets correspond to a hybrid setting where both values and probabilities are uncertain. Yet, we presume that especially in markets with binary outcomes agents may naturally reflect about probabilities. Other examples include auctions for non-performing loans where interested bidders inquire the underlying default risk before bidding. Similarly, in auctions for artwork with dubious provenance collectors might have a precise assessment of the object's value provided its provenance is good, but the same object is worthless if it is counterfeit or stolen. A major uncertainty arise then from not knowing the odds of an immaculate provenance.

We design our experiments such that in both auction formats subjects should form identical beliefs about the item's expected value, but they revise their beliefs for different reasons: uncertainty is resolved either in values or in probabilities. Concretely, in the different treatments of our experiment subjects face either CV or CP auctions and bid for equivalent lotteries. In one case we present subjects with a random asset (a bond) whose (face) value is known but whose default risk is not, while in the other we face our subjects with a different asset (e.g. mineral rights) whose failure risk is known but whose positive value is uncertain. These assets define lotteries for which our subjects bid and are strategically equivalent in that bidders should have identical expectations conditional on equivalent probability and value signals.

What we find is interesting. First, in contrast to the prediction of the risk-neutral Nash equilibrium theory, our subjects' bids in CP auctions are significantly lower than their bids in equivalent CV auctions. More specifically, while our subjects in CV auctions tend to bid above the naïve bidding function (i.e., bidding the expected value given the signal), subjects in CP auctions tend to bid below the even lower risk-neutral Nash equilibrium bid function. As a result, while winning bidders overbid in both auctions, they are less vulnerable to the winners' curse in the CP than in the CV auction due to their less aggressive bidding.

The difference between these two observed bidding behaviors may emanate from two distinct sources. First, subjects could possibly value the lotteries with uncertain values differently than those with uncertain probabilities. If so, differences in valuation could be captured by modeling individual preferences with, for instance, different curvatures of utility functions or with non-expected utility theories. Second, perhaps strategic reasoning differs across the two types of uncertainty.

To sort out whether the observed difference between CV and CP auctions already originate in individual, subjective valuations, we ran an additional experiment that dispenses with strategic incentives but leaves the lotteries intact. In this experiment, subjects are asked to simply evaluate (price) the same lotteries underlying our CV and CP auctions. It therefore provides us with the opportunity to identify whether the difference in bids across our auction formats derived from an underlying difficulty in lottery evaluation or strategic behavior. We call these treatments the CVL and CPL treatments, indicating that they involve lottery valuations as opposed to auctions.

We find that neither the perception nor the valuation of lotteries underlie the wedge between the bids for CP and CV lotteries. Stripping the auction game of its strategic elements suggests that the difference in bids observed in our auctions does not come from our subjects evaluating the underlying lotteries differently since, in a non-strategic setting, pricing of CV and CP lotteries exhibit negligible differences. This finding points to an interaction between the type of uncertainty our subjects face and its implications for strategic uncertainty in the auctions. How exactly this interaction works remains an intriguing puzzle worth studying in future research.

Our paper is connected to a number of different literatures. First, there is the obvious connection to the literature on common-value auctions and the extensive evidence on the winners' curse (Kagel and Levin (1986); Kagel et al. (1989); Charness and Levin (2009); Charness et al. (2014), i.e.; see also Kagel and Levin (2002) for an excellent review). So far, the winner's curse effect was found to be connected to the information structure (Grosskopf et al., 2018) and to decline only with sufficient experience in the laboratory (Dyer et al., 1989; Kagel and Richard, 2001; Casari et al., 2007) or familiarity with the task in the field (Harrison and List, 2008).<sup>1</sup> Our contribution here is to investigate the extent to which the winners' curse is robust to the introduction of common probabilities as opposed to common values.

The main drivers of the winner's curse phenomenon are still subject to a debate. The experimental literature provides mixed evidence on the im-

<sup>&</sup>lt;sup>1</sup>Relatedly, overbidding in independent private value auctions has been attributed to misperception of winning probabilities (to some extent) (Dorsey and Razzolini, 2003), learning dynamics (feedback information) (Neugebauer and Selten, 2006) and imperfect best response combined with risk aversion (Goeree et al., 2002).

portance of emotions like the thrill of winning (Cox et al., 1992; Holt and Sherman, 1994; Van Den Bos et al., 2008; Astor et al., 2013) or the fear of losing (Delgado et al., 2008). Other explanations offered relate more directly to strategic uncertainty. For instance, subjects possibly misidentify the connection between other bidders' actions and their private signals (Eyster and Rabin, 2005; Crawford and Iriberri, 2007; Eyster, 2019). Alternatively, subjects might have difficulties to perform the type of contingent reasoning involved in equilibrium behavior. More precisely, in order to avoid overbidding, subjects should bid conditional on their private signal being the highest among all signals and should shave their bid downward. Anticipating the informational content of winning is, however, a difficult task. It requires a sophisticated level of contingent reasoning, that, in general, most bidders struggle with. Besides common-value auctions difficulties related to contingent reasoning extend to other settings like "Acquiring-A-company" games (Bazerman and Samuelson, 1983; Charness and Levin, 2009; Martínez-Marquina et al., 2019, in an individual decision game variant), voting games (Esponda and Vespa, 2014) or asset markets (Carrillo and Palfrey, 2011; Ngangoue and Weizsäcker, 2019). In all these settings, uncertainty appears to be a crucial factor in impeding contingent reasoning (Martínez-Marquina et al., 2019; Koch and Penczynski, 2018; Ngangoue and Weizsäcker, 2019; Moser, 2019). Note, however, that none of the behavioral explanations mentioned above hinge on a specific definition of uncertainty. Our finding that uncertain probabilities mitigate the winner's curse effect points to the limits of the explanations offered and demands further investigations in this direction.

Finally, there is a growing literature in economics and psychology on how people view lotteries with uncertain outcomes versus those with uncertain probabilities. The main findings appear to be that when asked to choose between lotteries involving uncertain outcomes or uncertain probabilities, subjects appear to have no strong preference (Kuhn and Budescu, 1996; González-Vallejo et al., 1996; Du and Budescu, 2005; Eliaz and Ortoleva, 2016) while when asked to price these same lotteries subjects appear to value the uncertain outcome lotteries above those with uncertain probabilities (Schoemaker, 1991; Du and Budescu, 2005). The results of our individualdecision making experiment II are consistent with this existing literature on one-person decision problems, but our auction setting brings attention to the finding that these small differences observed in individual decision-making are substantially magnified in a strategic environment.

In this paper we will proceed as follows. In Section 2 we present our experimental design. Section 3 presents the theory related to our experiment and some hypotheses that we test in Section 4. Section 5 presents an additional experiment that allows us to narrow down possible explanations. We then discuss various decision models and their limitations in explaining our results in Section 6. We finally conclude in Section 7.

# 2 Experimental Design

A total of 212 students from New York University participated in the experiment, which consisted of two auction treatments, called CVA and CPA, and two lottery treatments, called CVL and CPL.<sup>2</sup> The two lottery treatments CVL and CPL shed some light on how, in a non-strategic setting, bidders value common-value versus common-probability objects. In this section, however, we focus on presenting the two main auction treatments CVA and CPA, and relegate the description of the lottery treatments CVL and CPL to Section 5.

The auction treatments were conducted with approximately half of the subjects, of which 55 were assigned to treatment CVA and 52 to the other treatment CPA. Sessions for treatments CVA and CPA lasted approximately 90 minutes and subjects earned, on average, \$23.78. The currency used in the experiment were credits (C) with C 6 corresponding to \$1.

Both treatments had identical procedures. The experiment was computerized with oTree (Chen et al., 2016) and consisted of two parts. Subjects needed to pass a comprehension test before they could start the first part of the experiment. In the first part, subjects participated in a set of firstprice auctions. In the second part, attitudes toward risk, compound risk and ambiguity were elicited (see Appendix B for a detailed description). At the end of the experiment, subjects learned their payoffs in the first and second parts and answered a small, unincentivized questionnaire. In the questionnaire, they provided some information on their socio-demographic background, about their general approach to the auction game, and took

 $<sup>^{2}</sup>$ The experiment was organized and recruited with the software hroot (Bock et al., 2012).

Frederick's cognitive reflection test (Frederick, 2005).

In the first part of both treatments, subjects engaged in 8 different auction environments with 10 separate auctions each. At the beginning of every auction, subjects were randomly matched into groups of four bidders (i = 1, ..., 4). Subjects in our auctions bid for lotteries described as either common value (CV) or common probability (CP) lotteries. Both lotteries are defined by two parameters v and p where v is a non-zero payoff of the lottery and p is the percentage probability of receiving that payoff (with (100 - p) defining the percentage probability of receiving 0).<sup>3</sup> In a CP lottery the two outcomes  $\{v, 0\}$  are known but p is uncertain while in a CV lottery, the opposite is true.<sup>4</sup>

We define by k that aspect of the lottery that is known to the bidder (either k =: p in the CV auction or k =: v in the CP auction). Analogously, we define  $\tilde{u}$  as the unknown component of the lottery, that is a random variable uniformly drawn from an interval  $[\gamma_l, \gamma_h]$  (we use in the following tildes to denote random variables). Hence, in the CV lottery where p is known for sure, we define  $\tilde{u} =: \tilde{v} \in [\gamma_l, \gamma_h]$  as the unknown aspect of the lottery while in CP, where v is known,  $\tilde{u} = \tilde{p} \in [\gamma_l, \gamma_h]$  is unknown.

For an example of a CP lottery consider a bond whose only risk is a default risk but whose face value is known. Here the lottery defined by this bond has a known value if no default occurs (v), but the default probability is uncertain  $(\tilde{p})$ . On the other hand, a CV lottery corresponds to the standard example of an oil field, whose economic value could be positive or zero. Here, the non-zero value of this investment,  $\tilde{v}$ , is unknown while the probability of receiving it, p, is known.

We will now define the two main treatments in our experiment.

 $<sup>^{3}</sup>$ We deliberately focus on binary zero-outcome lotteries to keep the cognitive costs of computing expected values comparable.

 $<sup>^{4}</sup>$ In auctions with affiliated values a prize in form of a lottery ticket may generate some precautionary bidding if subjects have decreasing absolute risk aversion (Eso and White, 2004; Kocher et al., 2015). In the instructions, we do not specifically frame the lottery as an *ex post* risk but subjects probably perceive it that way. The observed bids in our CV treatment are, if at all, too high and do not suggest that the lottery ticket introduced a precautionary premium by lowering bids. Even though the general direction of a corresponding DARA effect in common-value auctions is not clear, if bids exhibit a precautionary premium, there are no apparent reasons for premia to drastically differ across treatments.

#### 2.1 Common-Value Auctions

In a common-value auction subjects bid for CV lotteries, that pay off either a positive value  $\tilde{v}$  or zero credits. Subjects know p but have incomplete information about the positive value  $\tilde{v}$ , which is uniformly distributed between  $\gamma_l$ and  $\gamma_h$ . In other words, they learn that  $\tilde{v} \sim U[\gamma_l, \gamma_h]$  with  $0 < \gamma_l < \gamma_h < 100$ .

The computer determines the exact lottery by randomly drawing a value  $\tilde{v}$ . Subjects, however, do not observe this value at the moment of decisionmaking. Instead, each of the four bidders in the auction receives a private signal  $s_i$  independently from each other. The signal is informative about the true lottery in that it is drawn from an interval that is symmetric around the true value  $\tilde{v}$ . More precisely,  $s_i \sim U[\tilde{v} - \varepsilon, \tilde{v} + \varepsilon], \varepsilon > 0$ . Signals become more informative with a smaller support, that is with decreasing  $\varepsilon$ .

A Bayesian bidder would infer from observing a specific signal  $s_i$  that the unknown value  $\tilde{v}$  must lie within  $[s_i - \varepsilon; s_i + \varepsilon]$ . To help the subjects we provide this information to them before they bid. Given this information, subjects place a bid for the lottery at the bottom of the decision screen (see, Figure 1 for an example).



Figure 1: Example of Decision Screen in Treatment CVA

At the end of an auction, the auction winner is determined and the true lottery value, v is revealed. The lottery is played, the winner receives the lottery's outcome, either 0 or v, and pays her bid. Except for their own profit calculation, the feedback is the same for all bidders: Every bidder observes the true lottery, the lottery outcome and the highest bid.<sup>5</sup>

#### 2.2 Common-Probability Auctions

In a common-probability auction subjects bid for CP lotteries, that pay off a positive value v with a percentage probability  $\tilde{p}$  and zero with the complementary probability  $(100 - \tilde{p})$ . Here, subjects know v (the known component of the auction) but have incomplete information about  $\tilde{p}$ , the probability of receiving v, which is uniformly distributed between  $\gamma_l$  and  $\gamma_h$ . In other words, they learn that  $\tilde{p} \sim U[\gamma_l, \gamma_h]$  with  $0 < \gamma_l < \gamma_h < 100$ .

The computer determines the exact lottery by randomly drawing a probability  $\tilde{p}$  and each of the four bidders receives independently from each other a private signal  $s_i$ . The signal is informative about the true probability in that it is drawn from an interval that is symmetric around  $\tilde{p}$ . More precisely,  $s_i \sim U[\tilde{p} - \varepsilon, \tilde{p} + \varepsilon], \varepsilon > 0$ , implying that  $\tilde{p}$  lies within  $[s_i - \varepsilon; s_i + \varepsilon]$ .

Subjects then place a bid for the lottery, whereupon the auction winner is determined and the true probability,  $\tilde{p}$ , is revealed. The lottery is played, the winner receives the lottery's outcome, either 0 or v, and pays her bid. Every bidder, here too, observes the true lottery, the lottery outcome and the highest bid.

#### 2.3 Parameters

In both treatments, subjects engage in 8 different auction environments and within each environment they participated in 10 auctions. The lotteries they bid for in each of the eight environments are defined by an n-tuple  $(k, E[\tilde{u}], \varepsilon)$ , where k is either the known probability p (presented to the subjects as a percentage) or the known value v in the CP auction,  $E[\tilde{u}]$ identifies the interval (of fixed length) from which the uncertain component (either  $\tilde{v}$  or  $\tilde{p}$ ) is drawn, and  $\varepsilon$  defines the signal precision  $(\frac{1}{3}\varepsilon^2)^{-1}$ . With a 2x2x2 factorial design, we obtain 8 different parameter combinations by varying these 3 components across 2 sets of parameters:  $k \in \{40, 60\}, E[\tilde{u}] \in$  $\{40, 60\}, \varepsilon \in \{4, 8\}$ . Table 1 presents the exact 8 auction environments that our subjects engaged in in either the CP or the CV treatments:

<sup>&</sup>lt;sup>5</sup>The computer breaks ties between maximum bids randomly.

 Table 1: LOTTERY PARAMETERS

Lottery type	k	$\gamma_l^*$	$\gamma_h^*$	ε	
1	60	30	90	4	
2	40	10	70	4	
3	40	30	90	4	
4	60	10	70	4	
5	60	30	90	8	
6	40	10	70	8	
7	40	30	90	8	
8	60	10	70	8	
*: $E[\tilde{u}] = 40$ and $E[\tilde{u}] = 60$ correspond to $[\gamma_l, \gamma_h] = [10, 70]$ and $[\gamma_l, \gamma_h] = [30, 90]$ , respectively.					

Hence, the auctions presented to our subjects differ with respect to the known value k (column 2), the support of the unknown parameter  $[\gamma_l, \gamma_h]$  (columns 3 and 4), as well as the signal precision given by  $\varepsilon$  (column 5). In choosing our parameters we faced a set of constraints, the most important of which was to choose our values of k such that the CP and CV auctions were strategically equivalent. As one can see in Table 1, our design allows us to separately identify the sources of variation in our subjects' bid functions as we vary the different parameters of our auction in a ceteris paribus fashion. For example, we are able to hold the known parameter k (probability or value) constant and see how bids vary as we change the precision of the signal distribution or alternatively, the support,  $[\gamma_l, \gamma_h]$ , of the unknown parameter  $\tilde{u}$ . Detecting differences in subjects' bid functions as a result of these ceteris paribus changes, if they exist, allows us to infer how subjects process the different auctions they face.

Subjects then play 10 different auctions of each in these eight auction environments. For each of these 10 auctions, the computer randomly selects a true lottery on the basis of the environment's parameters. More precisely, the exact lottery (i.e., the true  $\tilde{v}$  of the CV or  $\tilde{p}$  of the CP lottery) and the corresponding signals could differ from auction to auction within an environment.

## **3** Predictions Under Linear Expected Utility

In this section, we discuss the standard benchmarks under risk-neutral expected utility to which we will compare the results.

The CV and the CP auctions are strategically equivalent under the assumption that bidders are risk-neutral expected utility maximizers. To make the analogy more salient, we will henceforth use the letters k and  $\tilde{u}$ . In the CV auction, the probability is known (k =: p) but the value is unknown ( $\tilde{v} =: \tilde{u}$ ). To facilitate comparison between treatments we use percentage values for p and write the *ex-ante* expected value of the CV lottery as:

$$100 \cdot E[L^{CV}] = p \cdot E(\tilde{v}) = k \cdot E(\tilde{u}).$$

In the CP auction the notation is reversed as the high value is now the known parameter, v =: k, and the probability is unknown,  $\tilde{p} =: \tilde{u}$ . Thus, the *ex ante* expected value of the lottery is similarly denoted with

$$100 \cdot E[L^{CP}] = v \cdot E(\tilde{p}) = k \cdot E(\tilde{u}).$$

There are three bidding functions to which we can compare empirical bids:

Naive bid: 
$$100 \cdot E[L|s_i] = E[v|s_i] = k \cdot s_i$$
 (1a)

Break-Even bid:  $100 \cdot E[L|s_i = \max_{\forall j} \{s_j\}] = k \cdot \left(s_i - \varepsilon \frac{n-1}{n+1}\right), \ j = 1, ..., 4$ (1b)

RNNE bid: 
$$100 \cdot b^*(s_i) = k \cdot \left[s_i - \varepsilon + \frac{2\varepsilon}{n+1}e^{-(\frac{n}{2\varepsilon})[s_i - (\gamma_l + \varepsilon)]}\right]$$
 (1c)

A naive bidder will bid the expected value of the lottery given her private signal (see Equation 1a). A more sophisticated bidder will take into account the winner's curse effect and will bid the expected value assuming her signal is the highest. She will therefore shave her bid downwards to make, on average, zero profits with a break-even bid (see Equation 1b). A highly sophisticated bidder will shave her bid even more assuming that, in a risk-neutral Nash equilibrium (RNNE), every one else bids like her (see Equation 1c).<sup>6</sup> The break-even and the RNNE bid do not differ by much; the analyses

<sup>&</sup>lt;sup>6</sup>We constrain our attention to the signal domain  $(\gamma_l + \varepsilon < s_i < \gamma_h - \varepsilon)$  for which the

will therefore mainly focus on the naive and the RNNE benchmark as these represent the highest and the lowest bidding benchmark, respectively.

Because predictions vary with parameters and signals, aggregate data will be mainly described with the measure of bid factors. Bid factors correspond to deviations from the naive bidding function and allow us to focus on statistics that are independent from the private signals. The bid factors with respect to RNNE (Naive bid - RNNE bid) and Break-Even bid (Naive bid - BE bid) are thus:

Break-Even bid factor: 
$$k \cdot \varepsilon \cdot \left(\frac{n-1}{n+1}\right)$$
 (2)

RNNE bid factor: 
$$k \cdot \left[\varepsilon - \frac{2\varepsilon}{n+1}e^{-(\frac{n}{2\varepsilon})[s_i - (\gamma_l + \varepsilon)]}\right]$$
 (3)

Thus, the computation of theoretical bid factors (with respect to both the break-even and the RNNE bid) is identical across the two auction formats, with the only difference being that in the CV auction k = p (in percentage points) and in the CP auction k = v. Like in standard common-value auctions, bid factors depend mainly on the signal's precision (that is inversely related to  $\varepsilon$ ) and the market size n.<sup>7</sup>

Despite the strategic equivalence of our two auction formats, we might very well suspect that behaviorally subjects treat them differently. Subjects may find it more difficult to process uncertainty about probabilities than uncertainty over values – a situation they face more frequently in their every day lives. Whether this difference leads to a difference in bidding behavior or a different incidence in the winner's curse across auction formats is something we will let our data determine. We test two null behavioral hypotheses.

Hypothesis 1 CV and CP auctions do not differ with respect to bids.

**Hypothesis 2** There is no difference in the incidence of the winners' curse across our auction formats.

Hypothesis 2, which focuses on the subsample of winning bids, can be valid even if Hypothesis 1 is rejected. In contrast to Hypothesis 1, however,

above risk-neutral Nash equilibrium (RNNE) bid function is defined.

<sup>&</sup>lt;sup>7</sup>We chose n = 4 because in the experimental literature the winner's curse has been extensively studied in auctions with four bidders (see Kagel and Levin, 2002).

Hypothesis 2 is not deduced from theoretical predictions but derives from the observation that none of the existing behavioral explanations for the winner's curse effect hinge on a particular definition of uncertainty.

# 4 Results

In this section we investigate the two hypotheses stated above.<sup>8</sup> We first test whether the bidding behavior of subjects differ across our two auction formats (which it does) and then investigate whether this difference has a consequence for the incidence of the winner's curse. We follow this up by regression analyses to shed light on the source of these differences.

**Result 1** Overall, bids significantly differ between the two auction formats: Subjects generally overbid in common-value but bid according to Nash equilibrium in common-probability auctions.

Hypothesis 1 of identical bidding behavior across auction formats is clearly rejected. To study all auctions jointly, we consider bid factors that, for a better visualization, we define here as the difference between the subject's bid and the Nash equilibrium bid.<sup>9</sup> Bid factors are then zero when subjects bid according to Nash equilibrium but are positive (negative) when they bid above (below) the Nash equilibrium bidding function. Figure (2) shows the distribution of bid factors. Bid factors are significantly different between the two treatments: They are predominantly positive in CVA (indicating a fair amount of overbidding) but slightly negative albeit consistent with Nash equilibrium in CPA. Appendix Table (A1) gives a more detailed picture with the mean and median bid factors when the bid factor is computed not only with respect to the RNNE bid but also with respect to the break-even bid and the naive bid. In CVA, subjects bid more than the expected value of the lottery given their private signal. As mean and median

<sup>&</sup>lt;sup>8</sup>Despite passing the comprehension test, some subjects chose dominated bids that were above the highest possible value. We exclude 3 and 14 subjects in treatment CV and CP on this basis, respectively. It is important to note that, first, removing those subjects does not affect our main conclusion as we continue to observe a substantial difference in bidding behavior with the entire sample. Second, this reduced sample is balanced in the sense that across treatments the remaining subjects do not differ with respect to personal characteristics measured at the end of the experiment (see Appendix Table A11.)

<sup>&</sup>lt;sup>9</sup>Bid factors are usually defined as the difference between the naive bid and the subject's bid. We opted for a varying definition of bid factors that in our opinion offers an easier interpretation and better visualization of the data.

bids are above the naive bid, all three computations lead to, on average, positive bid factors. In CPA, however, all three computations lead to, on average, negative bid factors because subjects bid even slightly below the RNNE bid. In sum, subjects significantly overbid for CV lotteries, but bid according to Nash equilibrium for CP lotteries.



Note: Solid line corresponds to the median bid factor.

**Figure 2:** Bid Factors (=bid - RNNE bid) in Treatments CVA (left) and CPA (right)

The question remains whether this difference in bid factors affects the incidence of the winner's curse. Figure (3) shows the distribution of bid factors in winning bids separately for treatments CVA and CPA. Winners in both auctions fell prey to the winner's curse as average winning bids were significantly above the naive bid (see Appendix Table A2). Yet, the data reject Hypothesis 2: The difference in bid factors between the two auction formats remains substantial. Winners bid higher in CVA and lost, on average, more than in CPA (mean loss of  $\oint -24.47$  in CVA vs.  $\oint -9.34$  in CPA, p-value< 0.001 in t-test of differences with cluster-robust standard errors). As shown in Figure (4), the cumulative distribution function (CDF)

of winning payoffs in CV auctions first-order stochastically dominates the CDF in CP auctions.



**Figure 3:** Bid Factors (=bid - RNNE bid) in Winning Bids in Treatments CVA (left) and CPA (right)

**Result 2** The winner's curse effect is attenuated in the common-probability compared to the common-value auction.

This difference between CVA and CPA occurs for all parameter combinations, i.e., for all eight lottery types. Appendix Figure (A 1) shows the estimated median bid as a function of signals. In all eight auction types, the median bidding curves for CV lotteries lie substantially above while those for CP lotteries are slightly below the RNNE curve.

# 5 Individual Non-Strategic Pricing

The lower bids in CP auctions raise the question whether uncertain probabilities affect strategic reasoning, reducing hereby the winner's curse effect or, whether differences in bids simply emanate from different valuations of the lotteries across the two types of uncertainty. Understanding at what level subjects' behavior diverge across auction formats will help us assess



Figure 4: Cumulative Distribution Function of Winners' Payoff

the environments in which different types of uncertainties may trigger different decisions.

We consider two main origins for these differences in bids: First, subjects could possibly evaluate lotteries with uncertain outcomes differently from those with uncertain probabilities. If so, differences could be captured by differences in individual valuations, and would naturally extend to individual choice problems. In Section 6, we discuss how risk aversion or non-expected utility models like rank-dependent utility or salience introduce differences in valuations of these lotteries.

Second, differences might be triggered by the strategic context. That is, even if subjects value the two types of lotteries equally, the auction game requires translating these valuations into strategic bids. The extent of strategic sophistication may differ between the two auction formats in that, for instance, it might be easier to reason through the adverse selection problem with one versus the other type of uncertainty.

A straightforward way of assessing the importance of strategic uncertainty is to strip the auction game off of its strategic elements.<sup>10</sup> To do

<sup>&</sup>lt;sup>10</sup>Similar approaches can be found in (Charness et al., 2014, i.a.)

this we devise an additional experiment that resembles the auction game but contains decision problems only. Analogous to the first experiment, the second experiment also consists of two treatments, CVL and CPL, with the main difference being that subjects do not compete against other bidders. In treatment CVL we elicit subjects' willingness-to-pay (henceforth WTP) for a series of CV lotteries whereas in treatment CPL we do the same for CP lotteries. The WTP, rather than the certainty equivalent or the willingness to accept, serves here as the counterpart to a bid when a subject does not engage in any strategic reasoning. In a nutshell, treatments CVL and CPL provide an empirical benchmark for subjects' non-strategic naive bidding function, i.e., their bid as a function of a private signal when the latter is the only relevant information. At this point, it is worth mentioning that we view the naive bidding curve as the theoretic non-strategic benchmark. In reality, it could well be that other non-strategic considerations like competitiveness, thrill of winning, etc. induce naive bidders do bid differently than their individual valuations, leading to a discrepancy between bids and valuations that is not due to "strategic" reasoning in its strict sense. In the following we use the term "non-strategic" to refer to individual considerations, abstracting from any strategic and *non-strategic* effects arising from social interaction.

In the first case of different valuations, we further distinguish between three possible reasons for why subjects might value CV and CP lotteries differently. First, when presented with both lotteries, subjects might value uncertainty in values differently than uncertainty in probabilities. The type of uncertainty would then affect subjects' valuations even before receiving any signal. Second, differences could arise at the information processing stage. More specifically, supposing subjects evaluate lotteries ex-ante similarly, they still may process signals about values in CV lotteries differently than signals about probabilities in CP lotteries. The processing of equivalent value and probability signals would then lead to different interim valuations of CV and CP lotteries. Third, a more fundamental skill that is required in this context is subjects' ability to reduce compound lotteries. One of the features of our auctions is that the objects for which subjects bid are lotteries and, in fact, compound lotteries. They differ, however, in that in the CV lottery the compounding is first over whether the good for sale has a positive value or not and then over its exact value while, in the probability lottery, the compounding is first over the exact probability of receiving the big prize and then, given this success probability, whether the lottery realizes to the big prize or zero. If subjects have different approaches to reducing these compound risks, this may affect the way they value the lotteries they are bidding for and hence be responsible for our auction results. Such differences, however, would stem from cognitive difficulties rather than preferences. We design the second experiment in a way that allows us to further disentangle these three possible sources.

The two treatments, CVL and CPL, share the same structure: each consists of three parts. In the part "Compound Lotteries" (CL) we investigate how subjects value lotteries in their ex ante and interim form (before and after receiving a signal about the lottery's worth) while in another part "Reduced Lotteries" (RL) we examine the impact of compounding risk by eliciting subjects' valuations for the reduced form of lotteries. In the last part of the experiment we measure our subjects' attitudes toward risk, compound risk and ambiguity. This last part is identical to the last part in the auction treatment (for a more detailed description see Appendix B). This last stage is also followed by a similar, unincentivized questionnaire.

#### 5.1 Experimental Design

Part CL: Valuation of Compound Lotteries Before and After Private Information. In our CVL and CPL treatments we elicited subjects' WTP for the lotteries with a Becker-DeGroot-Marschak mechanism (1964, henceforth BDM). Different subjects were recruited for the CVL and CPL treatments, but within each treatment subjects performed a variety of tasks. Hence we have a between-subjects treatment with respect to whether the lottery had CV or CP features, but a within-subjects treatment with respect to the tasks each subject is asked to perform.

In Part CL of the treatment, subjects engage in the same eight environments presented in the auction game. To isolate the effect of signal processing from ex ante evaluation, we separate the subject's valuation of a lottery type before and after receiving a signal. For example, in the treatment CPL a subject would be presented with a lottery with a fixed prize, say  $\notin 60$ , and a probability p of getting that prize that could be any integer between 10% and 70% with equal probability. With the complementary probability she receives zero. That means that instead of knowing the probability for sure, only a range of possible probabilities (i.e., the probability could be 10%, 11%, 12%, ..., 69%, or 70%) was shown to the subject and she has to specify a WTP based on this description. We call this the *ex ante* lottery since the subject is asked to state a WTP based only on this description of the lottery and without any signal as to which probability might be the actual probability used to determine the subject's chance of receiving the big prize of  $\mbox{\/}60$ .

To determine whether or not a lottery is bought we endow the subjects with  $\textcircled$  100 with which to bid, with any unspent credits paid to the subject. We then use the BDM mechanism so that after she submits her WTP, a random number between 0 and 100 sets the lottery price. The subject buys the lottery if its price is weakly less than her WTP. In that case, any gains or losses are added to or subtracted from her endowment of  $\oiint$  100. Otherwise, she does not engage in the lottery and ends the round with her initial endowment. The experimental interface remains essentially the same as in the auctions, with the only difference being that the subject submits her WTP for lotteries in a non-strategic setting rather than a bid in an auction.

After stating her ex-ante WTP, she submits 10 different WTP in 10 subsequent rounds after observing a random signal in each round. Since these rounds rely on signal processing, we refer to these decisions as interim WTP. Hence, in Part CL, the subject submits 11 decisions per environment: a first one without signal and 10 after receiving a random signal.

After every round with signal, the subject sees the actual lottery ticket, its price and its outcome (irrespective of whether or not she buys). To keep learning dynamics as similar as possible to the auction treatments, there is no feedback after submission of the ex-ante WTP.

#### Part RL: Valuation of Reduced Lotteries.

Part RL of the experiment focuses on subjects' ability to reduce compound lotteries. To identify whether cognitive difficulties are at work in our auction, we present subjects with the same lottery types but in reduced form. More precisely, we reduce the lotteries our subjects face by compounding the probabilities for them and present them with lotteries defined over



Figure 5: Example for Screen Interface in Part RL

final payoffs. In each round, a wheel is used to display the (up to 62) possible outcomes of a reduced lottery in a simple and condensed graph (see Figure 5 for an example of a CV lottery). Similarly to Part CL, the subject sees her endowment of  $\notin$  100, the lottery wheel and then states her WTP in a BDM mechanism. In Part RL, we abstract from signal processing and present the subjects only with lotteries without signals to elicit their ex-ante WTP. Like in Part CL, the subject does not receive any feedback after submitting her ex-ante WTP.

A total of 104 subjects participated in the lottery treatments, of which 54 (50) were assigned to treatment CVL (CPL). We collected data across a total of nine sessions, where every session lasted approximately 90 minutes. We reversed the order of the first two parts CL and RL for one third of the subjects.

## 5.2 Results

In Absence of Strategic Incentives. To present our results we compute the analog to the bid factor in a non-strategic setting by measuring the difference between  $w_i$ , subject's willingness to pay for a lottery, and  $E[L|s_i]$ , the lottery's objective expected value given the subject's signal. To emphasize its correspondence to the bid factor, we call it the *non-strategic price* factor and denote it with  $PF = w_i - E[L|s_i]$ . In other words, the price factor is equivalent to the negative of a risk premium.

For a better comparison with the auction data, we use the fact that the  $E[L|s_i]$  represents the naive bidding curve in the auction and compute the same measure with the bids, leading to a non-strategic bid factor  $(BF^{ns})$ measure with respect to the naive benchmark  $(BF^{ns} = bid - E[L|s_i])$ . Figure 6 shows the distribution of non-strategic bid factors by treatment CVA and CPA. We juxtapose Figure 7 that shows the distribution of nonstrategic price factors by treatment CVL and CPL. The treatment effect that we found in the auction experiment, while still present in the non-strategic context, is largely attenuated. There is a small significant difference as subjects priced CV lotteries above, but CP lotteries below their expected value, willing to pay, on average,  $\oplus 4$  more when the uncertainty was defined over values rather than probabilities (p-value < 0.001 in median test). This average difference of 04 (in the median, 06 in the means) is, however, substantially smaller than the difference of (17.6) observed in the auctions. In this sense, strategic uncertainty seems to be one major amplifier of the main treatment effect.<sup>11</sup> 12

To understand how these valuations come about we next contrast valuations for lotteries in ex ante valuations, stated before getting a signal, to interim valuations, stated after observing a signal.

**Ex-ante Valuation of Lotteries.** One interesting result is that we find no differences in median WTP for ex ante CV and CP lotteries (see Figure 8).

<sup>&</sup>lt;sup>11</sup>This is consistent with the observation that the winner's curse is more prevalent with increasing number of bidders (Charness et al., 2014).

<sup>&</sup>lt;sup>12</sup>Note that incentives differ between the auction and the lottery treatments. In firstprice auctions, a Nash equilibrium bidder pays his bid and in expectations makes small profits. In the lottery treatment, subjects pay the random price, which is, in expectation and conditional on buying, half the subject's WTP. Monetary incentives are therefore, on average, higher in the lottery treatment and could partly explain the smaller differences in the lottery treatments. However, monetary incentives should have a similar effect across CV and CP treatments, but the asymmetry in findings between CV and CP treatments casts some doubts on monetary incentives being the main reason for the observed differences between auction and lottery treatments.



**Figure 6:** Distribution of Strategic Bid Factors  $(=bid-E[L|s_i])$  in the Auction Treatments CVA (left) and CPA (right)



**Figure 7:** Distribution of Non-strategic Price Factors  $(=w_i - E[L|s_i])$  in the Lottery Treatments CVL (left) and CPL (right)

More specifically, without any further information in the form of a signal, subjects chose an average uncertainty premium of  $\clubsuit$  4 regardless of whether uncertainty was represented by a range of values or a range of probabilities. In that sense, the general perception of lotteries with uncertain values versus uncertain probabilities does not explain why subjects were willing to pay more or less than the expected value after receiving a value or a probability signal.



**Figure 8:** Distribution of *Ex ante* Price Factors for Compound Lotteries  $(=w^A - E[L])$  in Treatments CVL (left) and CPL (right)

*Reduced lotteries.* Remember, however, that the description of lotteries involves some understanding of compound risk. Since we find no difference in the values of lotteries ex ante, it is of interest to investigate whether their perception of lotteries is accurate in either case. To do this, we compare subject's valuations for the same lotteries in compound and reduced form.

This within-subject comparison of WTP for otherwise identical compound and reduced lotteries reveals that, in the aggregate, subjects priced compound and reduced lotteries equally. In CVL, subjects made no such distinction when valuing reduced and compound CV lotteries. The median premium for compound risk in values is zero, suggesting that compound risk in values may not necessarily be perceived as such.<sup>13</sup> In CPL, they chose a small average compound risk premium of  $\notin 2$  for CP lotteries. That is, subjects priced the reduced CP lotteries slightly higher than their compound analog.



**Figure 9:** Differences Between Valuations for Compound and Reduced Lotteries  $(=w_i^{CL} - w_i^{RL})$  in Treatments CVL (left) and CPL (right)

Note that, while subjects priced CV lotteries above expected values upon receiving value signals, in valuation of ex-ante lotteries we observe the opposite pattern. Subjects priced CV lotteries below their expected value. In that sense, ex-ante valuations for both CV and CP lotteries are equally below expected values and do not explain why with private signals subjects

<sup>&</sup>lt;sup>13</sup>There are some order effects in the comparison of reduced and compound lotteries. Whether subjects first saw reduced or compound lotteries turns out to matter, albeit only in the CV treatments. In the aggregate subjects chose similar WTP with and without compound risk when they valued the compound lottery before its reduced form version (median compound risk premium of 0 in CV lotteries). Seeing the reduced lottery first, on the other hand, *increases* (rather than decreases) their WTP for the compound version of CV lotteries by C3.5. In other words, the median premium for compound risk defined over values is even negative, implying that the average subject was more averse to the reduced than to the compound version of the CV lottery.

are willing to pay more than the expected value of CV lotteries but less than the expected value for CP lotteries.

Interim Valuation of Lotteries. We next study the importance of information processing in this decision problem. The empirical value of a signal is obtained by comparing subjects' willingness to pay before and after receiving signal  $s_i$ . To this end, we regress subjects' willingness to pay  $w_i$  on objective measures like the prior expected value E[L] and the information content of the signal given by  $(E[L|s_i] - E[L])$ . We also include a dummy  $D_{signal}$  that equals one when the willingness to pay was submitted after observing a signal.

WTP	EV	CV	CP	Diff
E[L]	1	$0.898^{***\dagger}$	$0.830^{***\dagger\dagger\dagger}$	0.068
		(0.053)	(0.039)	(0.068)
E[L s] - E[L]	1	$1.051^{***}$	$0.905^{***\dagger\dagger\dagger}$	0.146
		(0.084)	(0.030)	(0.103)
$D_{signal}$	0	$5.292^{***}$	$2.074^{**}$	3.218
		(2.027)	(0.947)	(2.419)
Cons	0	-0.335	0.074	-0.409
		(2.349)	(1.497)	(2.783)
N		4256	4000	8256
Subjects		54	50	104
$R^2$		0.234	0.390	
F-Test		0.0434	0.0000	0.0000

 Table 2: MEDIAN REGRESSION COEFFICIENTS

*Note*: Median regression with cluster robust standard errors (CRSE) at subject-level in parentheses. Significant difference from 0: \*: p-value<.1,\*\*: p-value<.05, \*\*\*: p-value<.01. Significant difference from pricing at expected value (EV):  $^{\dagger}$ : p-value<.1, $^{\dagger\dagger}$ : p-value<.01.

As shown in Table 2, we do not find substantial differences in the way subjects processed these value and probability signals. Under risk-neutral expected utility, pricing occurs at the expected value. That is, an increase of  $\notin 1$  in prior and interim beliefs is reflected in an equivalent increase of  $\notin 1$  in prices, while uncertainty premia (captured by the constant and the dummy variable) should be zero (cf. first column of Table 2). In treatment

CVL, subjects reacted reasonably to variations in both, prior parameters and signals as the corresponding coefficients do not substantially differ from the RNEU benchmark. In treatment CPL, subjects slightly underreacted to variations in the parameters, but more importantly coefficients do not differ from the ones in CVL. Hence, subjects processed value and probability signals similarly.

Another striking observation is that in both CVL and CPL, the mere fact of observing a signal significantly increased WTP by  $\oplus 5$  and  $\oplus 2$ , respectively. In other words, even when objective prior and interim expectations coincided, subjects were willing to pay more after observing a signal. This could be rationalized to some extent with a reduced uncertainty premium in interim beliefs, as seen in the treatment CPL where after getting a signal subjects bid closer to expected value. Rather surprising is that in CVL subjects bid, on average, even above expected values after seeing a signal, implying that the mere fact of getting a signal led subjects to move from an average positive to a negative uncertainty premium.

We next investigate the discrepancy between the auction and the pricing data. A comparison between strategic bids and non-strategic WTP for the same lotteries reveals that strategic considerations have a different impact on bids depending on whether subjects evaluated CV or CP lotteries. Valuations for CP lotteries with and without strategic incentives are rather stable. On average, subjects priced CP lotteries almost  $\oplus$  1.8 below their expected value, and in auctions, bid (2.21) less for the same CP lotteries (p < 0.001 in median comparison of bids and prices for CP lotteries). In contrast, valuations for CV lotteries substantially differed across settings. In CV auctions subjects bid, on average,  $\product{CV}$  13 above the expected value of the lottery, but priced the same lotteries close to the expected value in the non-strategic environment ( $\mathbb{C} 2$  above expected value, i.e., on average  $\mathbb{C} 11$ less than in auctions, p < 0.001 in median test). Comparing information processing across auction and lottery settings (cf. Table 2 and Appendix Table A4), we note that behavior in CP treatments is rather consistent: subjects underreacted to variation in signals when both bidding for and pricing CP lotteries. In contrast, subjects in CV treatments reacted more to value signals in the auction than in the lottery treatment. In other words, the observed difference across strategic and non-strategic settings is mainly

driven by a different bidding behavior for CV lotteries in contrast to their non-strategic valuations. In general, both in the auctions and lottery treatments, uncertainty over outcomes generates more variance than uncertainty over probabilities.

Experiment II provided us with new insights. The lack of differences in ex-ante valuations teaches us that, in fact, differences in our auctions are not a direct consequence of the different types of uncertainty (i.e., uncertainty over values vs. over probabilities). They also cannot be attributed to difficulties of reducing compound risk in values vs. probabilities. While information processing introduces small differences, it is also not the main driver. Signals generally led subjects to price both CV and CP lotteries higher, but in particular subjects then priced CV lotteries *above* their expected values. This difference is small in simple lottery pricing but substantially exacerbated by strategic uncertainty when subjects have to translate prices into bids. In fact, strategic uncertainty matters particularly in the CV environment where we observe more differences between pricing and bidding behavior.

### 6 Discussion

In this section we will discuss a number of possible explanations for our observed difference in behavior between our CV and CP auctions. As was discussed above, submitting a bid in an auction whose underlying prize is a lottery is a three stage process. First one has to value the lottery ex ante or before one receives a signal as to its worth. Then, in the interim stage after receiving a signal, one has to update that value. Finally, in the bidding stage, one has to transform one's updated value into a bid under some assumption on the behavior of one's opponent. Differences in the bids submitted across our auction formats may occur at any of these stages. Subjects may value CVL's differently from CPL's and hence bid differently for them, or update the value of the lottery differently once receiving a signal. However, if in the first two stages we cannot find any difference in the way our subjects value the objects they are bidding for, then the only explanation remaining is that the strategic uncertainty existing across these auctions appears to be different and leads our subjects to bid differently. In our discussion below we explore a number of theories that one might use to explain the difference in the way our subjects value the lotteries in their common values and common probability formats. In all the theories discussed, however, we do not find any that can explain the differences in the observed bidding behavior of our subjects and hence we come to the conclusion that if behavior differs across our two auction formats, that difference must derive from the different ways our subjects treat the strategic uncertainty existing in these two auctions rather than from the way they value the underlying objects they are bidding for. This conjecture is further supported by the lack of substantial differences in valuations in the lottery treatments.

This suggests, however, that what we observe are persistent differences in non-equilibrium behavior supported by a process of thinking that reacts differently to the two environments we place our subjects in. Since game theory only explains equilibrium behavior, there are no obvious strategic theories that come to mind that could help unravel our puzzle. Given the need for more elaborate research in this direction, we are content to leave our common probability puzzle unsolved for the moment.

# 6.1 Differences in Underlying Lotteries Under Expected Utility

One obvious way in which bidding behavior can differ across our auctions is for subjects to value the lotteries they are bidding for differently across the CV and CP environments. For example, while we present subjects with equivalent lotteries in terms of their expected value, they are not equivalent with respect to their variance. Theoretically, the effect of risk-aversion in first-price common-value auctions is ambiguous (see Kagel and Levin, 2002), but we find a general negative correlation between the risk premia elicited in the last part of our experiment and the bids submitted by our subjects (see Appendix Tables A5 and A6). Hence, if our subjects were risk-averse then we would expect lower bids in the CV as opposed to the CP auction given that CV lotteries are generally riskier. The fact that we discovered no difference in our subjects' willingness to pay for the underlying lotteries (despite the bigger variance in the CV lottery) and also found no difference in the average degree of risk aversion across our treatments, suggests that risk aversion cannot be relied on to explain behavioral differences in our experiments. Similarly, if our subjects processed these signals about the common probability or common value differently before converting them into bids, then we might observe the type of difference we observed. Again, as we have shown in our lottery treatments, such differences are negligible. Hence we cannot look to simple explanations like risk aversion or information processing to explain why subjects may value probability auctions differently than value auctions.

# 6.2 Differences in Underlying Lotteries Under Non-Expected Utility

Another avenue to help explain our results would be to posit non-expected utility for our subjects which might lead them to value the lotteries they face differently depending upon whether they are CV or CP lotteries. Here it is important to note that, because of the similarity of CV and CP lotteries, preferences that differ from linear expected utility introduce systematic deviations from expected values to different extents but in the same direction for both auctions. Therefore, unless one is willing to allow for decisions weighting or utility functions that differ across sources of uncertainty (Abdellaoui et al., 2011; Klibanoff et al., 2005; Nau, 2006; Seo, 2009), none of these non-expected utility models can rationalize biases in opposite direction from the expected value and hence none can serve as the basis to explain our results. Let us explore some of these models.

#### 6.2.1 Rank-Dependent Utility

Rank-dependent utility, that allows for different attitudes toward variation in utilities and probabilities, also generates different valuation of the lotteries. Here decision-makers evaluate lotteries via a rank-dependent expected utility of the general form:

$$\begin{split} RDU(L) &= \sum_j \pi_j u(v_j) \\ \texttt{with} \pi_j &= w(\sum_{j=1}^n p_j) - w(\sum_{j=1+1}^n p_j) \text{ and } v_1 < \ldots < v_j < \ldots < v_n \end{split}$$

where  $v_j, j = 1, ..., n$  represent all possible outcomes of a lottery ranked from low to high values and  $w(\cdot)$  is the weighting function applied to cumulative probabilities.

Suppose, for instance, that subjects have pessimistic decision weights (i.e., the weighting function of cumulative probabilities,  $w(\cdot)$ , is convex). For a simple illustration consider only the high outcome of the lottery and linear utility. In that case, pessimistic decision weights induce the decision-maker to put more weight on lower outcomes and lead then to a lower expected high prize in CV than the known high prize in CP lotteries  $(E[\tilde{u}]^{CV} < k^{CP})$ , with  $\tilde{u} \sim U[v_l, v_h]$  and  $v_l < k = \frac{v_l + v_h}{2} < v_h$ ). With our considered set of parameters, a convex weighting function leads to an undervaluation of both lotteries. However, because possible values in CP lotteries are a subset of the possible values in CV lotteries, the latter are valued lower than CP lotteries. This would lead bidders to bid higher for CP lotteries than for CV lotteries, which is the opposite of what we observe. A concave (i.e., optimistic) weighting function obtains the reversed pattern, i.e.,  $E[L] < EU[L^{CP}]_{RDU} < EU[L^{CV}]_{RDU}$ .

Assuming that weighting and utility functions do not, on average, differ between subjects in the two treatments, rank-dependent utility would create a deviation from equilibrium bids in the same direction for both lotteries. The bias would be attenuated for CP lotteries, like it is in our experiment. However, while decision weights would have to be optimistic to match our observed pattern in CV auctions, the slight underbidding for CP lotteries would not be consistent with optimistic decision weights.

#### 6.2.2 Non-Neutrality Toward Compound Risk

Another avenue through which subjects may value our CVL's and CPL's differently is through their perception of the compound risk in each lottery. More precisely, compound risk is present in both CP and CV lotteries, which both consist of a combination of a binary lottery and a uniform distribution. Even though the perceived complexity of reducing these compound lotteries might differ, any failure to reduce compound lotteries would impact decisions in both environments. As our data in Section 5 and introspection suggest, there is no obvious erroneous pattern of reducing compound lotteries that introduces systematic biases in opposite direction of the expected value.

#### 6.2.3 Salience Theory

Different dimensions of uncertainty might emphasize different features of a lottery and might, therefore, generate different valuations for them. In their salience theory, Bordalo et al. (2012) postulate that the salience of payoffs but not of probabilities determines decision weights. In our context, the uncertainty in values renders extreme high values (e.g.,  $\oplus$  90 in lottery type 1) more salient. Uncertainty in probabilities, on the other hand, should have no effect on the subjective perception of the lottery.

When subjects evaluate a lottery in isolation, the predictions of salience theory depend on the value to which subjects compare the lottery. They may compare the lottery to a status quo of nothing (i.e., the value zero) or, alternatively, to the expected value of the lottery.<sup>14</sup> For instance, when the status quo is zero, lottery outcomes that differ from zero become more salient. A subject will then assign higher decision weights to these higher outcomes, and will consequently overvalue the lottery. Because the CV lottery has a higher maximum than the CP lottery  $(\gamma_h > v)$ , she will value a CV lottery more than a corresponding CP lottery (see Appendix Figure A 4). In contrast, if she compares the lottery to its expected value, the location of the expected value will determine whether the minimum or the maximum outcome become more salient. If the expected value is sufficiently high, she will assign more decision weight to the minimum payoff of 0 and will undervalue the lottery (see Appendix Figure A 5). In our experiment, because the value domain of the CP lottery is a subset of the value domain of CV lottery, the salience bias is always more pronounced for lottery CV regardless of the comparison benchmark. The comparison benchmark determines only whether the decision maker over- or undervalues the lottery compared to a decision maker with standard linear utility.

In a nutshell, although the salience bias generates different valuation of the lotteries, the bias always go in the same direction for both lotteries. As a result, based on their evaluations of the underlying lotteries, salience theory does not explain why subjects overbid for CV but slightly underbid for CP lotteries.

<sup>&</sup>lt;sup>14</sup>Bordalo et al. (2012) distinguish between the valuation and the revealed preference approach. In the valuation approach the comparison benchmark is zero while in the revealed preference approach any nonzero sure payoff could be used to compute certainty equivalents.

#### 6.2.4 Differences Due to the Source of Uncertainty

Note that one might rationalize differences across auction formats if we allow decision weights to differ across these two types of uncertainty. Abdellaoui et al. (2011) study decisions under uncertainty by allowing weighting functions in rank-dependent utility to differ across sources of uncertainty. While they distinguish, in particular, between events with known and unknown probability distributions, one could similarly perceive uncertainty over outcomes and probabilities as different sources of uncertainty. An alternative form of modeling variations are source-dependent utilities (instead of source-dependent decision weights) that vary across the type of uncertainty. For instance, in models with recursive expected utilities (Abdellaoui et al., 2011; Klibanoff et al., 2005; Nau, 2006; Seo, 2009), one could model different utility functions not only for risk and ambiguity, but also for different types of uncertainty. While such differences in decision weights might help explain our observed difference across CV and CP auctions, positing such difference is ad hoc and would not be based on any logical foundation.

# 6.2.5 Different Ways of Processing Signals: Scale Compatibility and Anchoring.

Ever since Tversky and Kahneman (1974) experiment anchoring has been found to be a persistent phenomenon in the experimental literature. Subjects' responses are affected by irrelevant or relevant information. In our design, the signal provides a starting point for subjects' reflections. Chapman and Johnson (1994) show that anchoring matters only to the extent that anchors and responses have the same scale. In our treatment comparison, subjects might be more prone to anchor to value signals in CV auctions that are on the same scale as bids as opposed to probability signals in CP auctions that do not represent possible outcomes. In that case, bids in CV auctions would be biased towards signals whereas bids in CP auctions would remain unaffected. In that sense, our data is to some extent consistent with anchoring on compatible anchor/response scales.

An alternative model of compatibility is Tversky et al. (1988)'s contingent weighting model, according to which decision attributes that are compatible with the response mode receive more weight in the judgment process. This interpretation is slightly different from the Chapman and Johnson (1994) analysis of scale compatibility because, even without additional signal to anchor on, having uncertainty over values rather than probabilities might induce subjects to focus more on the value than on the probability component of a lottery. Our data is only partly consistent with this interpretation. Indeed, our subjects in CV auctions put four times more weight on their value signal than on the probability (cf  $(\beta/\alpha)$  in regression Table A9 ), but those in CP auctions put nevertheless 1.25 times more weight on probability signals than on values despite the response scale being in values. Finally, there are also no apparent reasons why anchoring would matter more in bidding than in pricing, where we do not observe important discrepancies.

#### 6.3 Strategic Components

Our analysis above tends to rule out differences in bids across our CV and CP auctions as resulting from the different ways subjects value the lotteries they are bidding on. From what we have seen, several theories propose differences in lottery valuations pointing in a direction opposite to what we observe. Hence, if bidding behavior differs it must be the result of the strategic uncertainty that exists in our auctions and the different ways in which subjects view this uncertainty rather than the way they value lotteries. This is curious since the process of submitting an equilibrium bid in each auction is identical.

# 7 Conclusion

We consider an alternative modeling of common-value auctions, one in which uncertainty is defined over probabilities and bidders receive private information about probability distribution rather than outcome values. These common-probability auctions give rise to a different bidding behavior that conforms with the risk-neutral Nash equilibrium and stands in stark contrast to the robust observation of overbidding in common-value auctions. We investigate possible explanations for this unconventional bidding behavior, and find that the type of uncertainty per se does not trigger differences. Without private information, subjects valued the auctioned item similarly. Instead the type of uncertainty appears to interact with information processing (to some extent), but primarily with strategic considerations. While subjects priced lotteries slightly higher after updating on signals in the CV auctions, this slightly suboptimal updating behavior was substantially amplified in a strategic setting when valuations were translated into bids. In other words, bids and prices were much more erratic when uncertainty was defined over values rather than probabilities. Our experimental findings expose the need for more thorough research on the link between the type of uncertainty, information processing and strategic considerations. Not only will it allow us to further understand to what frameworks robust empirical anomalies may or may not extend to, it might also shed more light on the precise sources of the winner's curse effect.

## References

- M. Abdellaoui, A. Baillon, L. Placido, and P. P. Wakker. The Rich Domain of Uncertainty: Source Functions and Their Experimental Implementation. *The American Economic Review*, 101(2):695–723, 2011.
- P. J. Astor, M. T. P. Adam, C. Jähnig, and S. Seifert. The joy of winning and the frustration of losing: A psychophysiological analysis of emotions in first-price sealed-bid auctions. *Journal of Neuroscience, Psychology,* and Economics, 6(1):14–30, 2013.
- A. Baillon, Y. Halevy, and C. Li. Experimental Elicitation of Ambiguity Attitude using the Random Incentive System. 2015.
- M. H. Bazerman and W. F. Samuelson. I Won the Auction But Don't Want the Prize. *Journal of Conflict Resolution*, 27(4):618–634, 1983.
- G. M. Becker, M. H. DeGroot, and J. Marschak. Measuring utility by a single-response sequential method. *Behavioral Science*, 9(3):226–232, 1964.
- O. Bock, A. Nicklisch, and I. Baetge. hroot: Hamburg registration and organisation online tool. WiSo-HH Working Paper Series, 1(13):1–14, 2012.
- P. Bordalo, N. Gennaioli, and A. Shleifer. Salience Theory of Choice under Risk. *The Quarterly Journal of Economics*, pages 1243–1285, 2012.
- J. D. Carrillo and T. R. Palfrey. No trade. Games and Economic Behavior, 71(1):66–87, jan 2011.
- B. M. Casari, J. C. Ham, and J. H. Kagel. Selection Bias , Demographic Effects , and Ability Effects in Common Value Auction Experiments. *American Economic Review*, 97(4):1278–1304, 2007.
- G. B. Chapman and E. J. Johnson. The limits of anchoring. Journal of Behavioral Decision Making, 7(4):223–242, 1994.
- B. G. Charness and D. Levin. The Origin of the Winner's Curse: A Laboratory Study. American Economic Journal: Microeconomics, 1(1):207–236, 2009.

- G. Charness, D. Levin, and D. Schmeidler. A Generalized Winner 's Curse: An Experimental Investigation of Complexity and Adverse Selection. 2014.
- D. L. Chen, M. Schonger, and C. Wickens. oTreeâAn open-source platform for laboratory, online, and field experiments. *Journal of Behavioral and Experimental Finance*, 9:88–97, 2016.
- J. C. Cox, V. L. Smith, and J. M. Walker. Theory and Misbehavior of First-Price Auctions : Comment. American Economic Review, 82(5):1392–1412, 1992.
- V. P. Crawford and N. Iriberri. Level-K Auctions : Can a Nonequilibrium Model of Strategic Thinking Explain the Winner 's Curse and Overbidding in Private-Value Auctions ? *Econometrica*, 75(6):1721–1770, 2007.
- M. R. Delgado, A. Schotter, E. Y. Ozbay, and E. A. Phelps. Understanding Overbidding :. Science, 575(September):1849–1852, 2008.
- R. Dorsey and L. Razzolini. Explaining overbidding in first price auctions using controlled lotteries. *Experimental Economics*, 6(2):123–140, 2003. ISSN 13864157.
- N. Du and D. V. Budescu. The effects of imprecise probabilities and outcomes in evaluating investment options. *Management Science*, 51(12): 1791–1803, 2005.
- D. Dyer, J. H. Kagel, and D. Levin. A Comparison of Naive and Experienced Bidders in Common Value Offer Auctions: A Laboratory Analysis. *The Economic Journal*, 99(394):108–115, 1989.
- K. Eliaz and P. Ortoleva. Multidimensional ellsberg. Management Science, 62(8):2179–2197, 2016.
- P. Eso and L. White. Precautionary bidding in auctions. *Econometrica*, 72 (1):77–92, 2004. ISSN 00129682.
- I. Esponda and E. Vespa. Hypothetical Thinking and Information Extraction in the Laboratory. American Economic Journal: Microeconomics, 6 (4):180–202, 2014.

- E. Eyster. Errors in strategic reasoning. In Handbook of Behavioral Economics, volume 2, chapter 3, pages 187–259. Elsevier B.V., elsevier edition, 2019.
- E. Eyster and M. Rabin. Cursed equilibrium. *Econometrica*, 73(5):1623– 1672, 2005.
- S. Frederick. Cognitive reflection and decision making. Journal of Economic Perspectives, 19(4):25–42, 2005.
- B. Gillen, E. Snowberg, and L. Yariv. Experimenting with measurement error: Techniques with applications to the caltech cohort study. *Journal* of *Political Economy*, 127(4), 2019.
- J. K. Goeree, C. A. Holt, and T. R. Palfrey. Quantal response equilibrium and overbidding in private-value auctions. *Journal of Economic Theory*, 104(1):247–272, 2002.
- C. González-Vallejo, A. Bonazzi, and A. J. Shapiro. Effects of vague probabilities and of vague payoffs on preference: A model comparison analysis. *Journal of Mathematical Psychology*, 40(2):130–140, 1996.
- B. Grosskopf, L. Rentschler, and R. Sarin. An experiment on first-price common-value auctions with asymmetric information structures: The blessed winner. *Games and Economic Behavior*, 109:40–64, 2018.
- Y. Halevy. Ellsberg revisited: An experimental study. *Econometrica*, 75(2): 503–536, 2007.
- G. W. Harrison and J. A. List. Naturally Occurring Markets and Exogenous Laboratory Experiments: A Case Study of the Winner's Curse. The Economic Journal, 118(April):822–843, 2008.
- B. C. A. Holt and R. Sherman. The Loser 's Curse. American Economic Review, 84(3):642–652, 1994.
- J. H. Kagel and D. Levin. The winner's curse and public information. American Economic Review, 76:894–920, 1986.
- J. H. Kagel and D. Levin. Common value auctions and the winner's curse. Princeton University Press, Princeton, princeton edition, 2002. ISBN 0691016674.

- J. H. Kagel and J.-f. Richard. Super-Experienced Bidders in First-Price Common-Value Auctions : Rules of Thumb , Nash Equilibrium Bidding, and the Winner 's Curse. *Review of Economics and Statistics*, 83(August): 408–419, 2001.
- J. H. Kagel, D. Levin, R. C. Battalio, and D. J. Meyer. First-price common value auctions: bidder behavior and the "winner's curse". *Economic Inquiry*, 27(2):241–258, 1989.
- P. Klibanoff, M. Marinacci, and S. Mukerji. A Smooth Model of Decision Making under Ambiguity. *Econometrica*, 73(6):1849–1892, 2005.
- C. Koch and S. P. Penczynski. The winner's curse: Conditional reasoning and belief formation. *Journal of Economic Theory*, (174):57–102, 2018.
- M. G. Kocher, J. Pahlke, and S. T. Trautmann. An experimental study of precautionary bidding. *European Economic Review*, 78:27–38, 2015.
- K. M. Kuhn and D. V. Budescu. The relative importance of probabilities, outcomes, and vagueness in hazard risk decisions. Organizational Behavior and Human Decision Processes, 68(3):301–317, 1996.
- A. Martínez-Marquina, M. Niederle, and E. Vespa. Failures in Contingent Reasoning : The Role of Uncertainty. 2019.
- J. Moser. Hypothetical thinking and the winner's curse: an experimental investigation. *Theory and Decision*, 87(1):17–56, 2019.
- R. F. Nau. Uncertainty Aversion with Second-Order Utilities and Probabilities. *Management Science*, 52(1):136–145, 2006.
- T. Neugebauer and R. Selten. Individual behavior of first-price auctions: The importance of information feedback in computerized experimental markets. *Games and Economic Behavior*, 54(1):183–204, 2006.
- K. Ngangoue and G. Weizsäcker. Learning from unrealized versus realized prices. 2019.
- P. J. Schoemaker. Choices involving uncertain probabilities. Tests of generalized utility models. *Journal of Economic Behavior and Organization*, 16(3):295–317, 1991.

- K. Seo. Ambiguity and Second-Order Belief. *Econometrica*, 77(5):1575– 1605, 2009.
- A. Tversky and D. Kahneman. Judgment under uncertainty: Heuristics and biases. *Science*, 185:1–92, 1974.
- A. Tversky, S. Sattath, and P. Slovic. Contingent Weighting in Judgment and Choice. *Psychological Review*, 95(3):371–384, 1988.
- W. Van Den Bos, J. Li, T. Lau, E. Maskin, J. D. Cohen, P. R. Montague, and S. M. Mcclure. The value of victory: Social origins of the winner's curse in common value auctions. *Judgment and Decision Making*, 3(7): 483–492, 2008.

# A Descriptive statistics

# A.1 Bid and Price Factors

		CVA	CPA	Diff.
Naive bid (bid- <i>bid<sup>Naive</sup></i> )	mean	$ \begin{array}{c} 10.59^{***} \\ (1.877) \end{array} $	$-4.79^{***}$ (1.446)	$15.38^{***}$ (2.364)
	median	13.6***	-4***	17.6***
Break-Even bid $(\text{bid-}bid^{BE})$	mean	$ \begin{array}{c c} 12.32^{***} \\ (1.877) \end{array} $	$-3.04^{**}$ (1.446)	$ \begin{array}{c} 15.36^{***} \\ (2.364) \end{array} $
	median	15.48***	-2.08***	17.56***
Nash-Eq. bid $(\text{bid-}bid^{RNNE})$	mean	$13.39^{***}$ (1.876)	-1.96 (1.447)	$15.35^{***}$ (2.363)
	median	16.60***	-1.00	17.60***

Table A1: Bid Factor (BF)

Note: Cluster robust standard errors (CRSE) in parentheses. \*: p-value<.1,\*\*: p-value<.05, \*\*\*: p-value<.01.

		CVA	CPA	Diff
Naive bid (bid- <i>bid<sup>Naive</sup></i> )	mean	$24.83^{***} \\ (0.778)$	$9.06^{***}$ (1.061)	$ \begin{array}{c} 15.77^{***} \\ (1.314) \end{array} $
	median	23.6***	8.20***	5.40***
Break-Even bid $(\text{bid-}bid^{BE})$	mean	$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	$10.80^{**}$ (1.086)	$ \begin{array}{c c} 15.76^{***} \\ (1.333) \end{array} $
	median	25.44***	10.16***	15.28***
Nash-Eq. bid (bid- $bid^{RNNE}$ )	mean	$27.63^{***} \\ (0.775)$	$11.89^{***}$ (1.099)	$ \begin{array}{c c} 15.73^{***} \\ (1.343) \end{array} $
	median	26.80***	$11.26^{***}$	15.54***

Table A2: BID FACTOR (BF) - WINNNING BIDS

*Note:* Cluster robust standard errors (CRSE) in parentheses.\*: p-value<.1,\*\*: p-value<.05, \*\*\*: p-value<.01.

Table A3:PRICE FACTOR

		CVL	CPL	Diff.
Part CL with signal				
Price Factor	mean	$5.12^{***}$	-1.03	$6.16^{***}$
(price - E[L s])		(1.676)	(1.006)	(1.945)
	median	$2.4^{***}$	-1.8***	4.2***
Part CL without signal				
Price Factor	mean	0.75	-3.00**	$3.75^{*}$
(price - E[L s])		(1.679)	(1.417)	(2.188)
	median	-4	-4	0
Part RL				
Price Factor	mean	-2.93*	-1.37	-1.56
(price - E[L s])		(1.684)	(1.132)	(2.022)
	median	-5	-1	-4***

Note: Cluster robust standard errors (CRSE) in parentheses. \*: p-value<.1,\*\*: p-value<.05, \*\*\*: p-value<.01.



**Figure A 1:** Estimated Median Bids in CV and CP Auctions by Lottery Types

#### A.2 Regression analyses

We use the similarity of the auction and pricing treatments to contrast here the regression estimates from Table (2) with a similar regression on the auction data (in Appendix Table A4 ). In auctions, our parameter variations allow us to estimate subjects' sensitivity to three potentially relevant components of bidding. The first row of Table A4 renders the reaction to changes in the ex ante expected value of the lottery, E[L]. The average response to signals is measured by the coefficient of the difference between the naive bid and the ex ante expectation, (E[L|s] - E[L]). Here, instead of the dummy variable  $D_{signal}$  that compares decisions with and without signal processing, we include a possible indicator for strategic sophistication. The extent of bid shaving with less precise signals is given by the coefficient of the difference between the RNNE and the naive bid,  $(b^{RNNE} - E[L|s])$ , which in turn depends on the signal's precision. Table A4 also shows the predicted values of the coefficients in the risk-neutral Nash equilibrium, to which estimated coefficients can be compared.

In contrast to the treatments CVL and CPL, differences between the two auction formats appear at all three levels. In CPA, individual coefficients do not differ from the Nash equilibrium: subjects reacted appropriately to variation in both the ex-ante expected value of the lottery and the signal. However, a joint test depicts a deviation from the RNNE bid as they exante slightly undervalued the lotteries, slightly overreacted to the signals and shaved their bid slightly less than they should (p=0.0322 in F-test), leading overall to an estimated bidding curve that lies below the RNNE bid (see Figure A 1).

Subjects in CV auctions, on the other hand, significantly deviated from the RNNE prediction (in both joint and individual coefficient tests). They overvalued lotteries ex-ante and overreacted to signals about values. Interestingly, the signal's precision had a bigger impact in CV than in CP auctions. Subjects in CV auctions shaved their bids disproportionately with less precise signals but the amount of shaving was not sufficient to offset the general overbidding. Notice, however, that it is not clear whether subjects' reaction to the signal's precision reflects strategic sophistication or varying confidence in the signals.

Comparing the regression estimates from the strategic auction and the non-strategic pricing data (Tables 2 and A4) shows that, overall, estimates in

Bid	RNNE	CVA	CPA	Diff.
$\overline{E[L]}$	1	1.386***†††	0.841***	0.545***
		(0.059)	(0.118)	(0.138)
E[L S] - E[L]	1	$1.694^{***\dagger\dagger\dagger}$	$1.126^{***}$	$0.568^{***}$
		(0.045)	(0.084)	(0.097)
$(E[L S] - E[L])^2$	0	$-0.017^{***}$	0.005	-0.022**
		(0.005)	(0.008)	(0.009)
$Bid^{RNNE} - E[L S]$	1	$2.820^{***\dagger\dagger\dagger}$	0.474	$2.346^{***}$
		(0.442)	(0.460)	(0.629)
Cons	0	13.949***	0.873	$13.075^{***}$
		(3.017)	(1.730)	(3.629)
N	5762	3260	2502	5762
Subjects	90	52	38	90
$R^2$	1.00	0.277	0.282	0.371
F-Test on NE	0	0.000	0.032	

 Table A4:
 MEDIAN REGRESSION COEFFICIENTS

*Note*: Median regression with cluster robust standard errors (CRSE) at subject-level in parentheses. Significant difference from 0: \*: p-value<.1,\*\*: p-value<.05, \*\*\*: p-value<.01. Significant difference from RNNE coefficient: <sup>†</sup>: p-value<.1,<sup>††</sup>: p-value<.05, <sup>†††</sup>: p-value<.01. CP treatments vary little. The only small difference is that in CPL subjects underreacted but in CPA overreacted to variations in signals. Estimates differ more in CV treatments, where in CVA we observe stronger reactions to both prior parameters and signals relative to CVL.

To better understand differences in bidding and pricing behavior, we insert additional covariates in the regression model (see models (1)-(4) in Appendix Tables (A5) to (A8). The effect of covariates is not necessarily the same across strategic and non-strategic environments, and across CV and CP treatments.

For instance, experience effects differ. Subjects generally chose lower bids and prices after losing money in the previous period  $(Loss_{t-1})$ , except in CV auctions where losses induced them to bid even more aggressively. Incurring a loss appears to be the only experience effect in auctions where a reciprocal time trend (1/t) does not detect any other relevant dynamics; in the lottery treatments, on the other hand, subjects adjusted their chosen prices upward over time.

In all treatments, subjects who scored higher on the cognitive reflection test chose, on average, lower bids or prices. Apart from the cognitive reflection test, individual covariates are non-significant in most treatments. The exception are CP auctions where lower bids are associated with female and more risk averse students, and CV treatments where bids are negatively related to aversion to compound risk.

Bid	(1)	(2)	(3)	(4)
$\overline{E[L]}$	1.398***†††	1.380*** <sup>†††</sup>	1.336*** <sup>†††</sup>	1.350*** <sup>†††</sup>
	(0.058)	(0.050)	(0.052)	(0.050)
E[L s] - E[L]	$1.696^{***\dagger\dagger\dagger}$	$1.671^{***\dagger\dagger\dagger}$	$1.624^{***\dagger\dagger\dagger}$	1.632***†††
	(0.051)	(0.054)	(0.087)	(0.076)
$[E[L s] - E[L]]^2$	-0.016***	-0.018***	-0.015**	-0.017***
	(0.004)	(0.004)	(0.006)	(0.005)
$Bid^{RNNE}$	2.935***†††	$2.814^{***\dagger\dagger\dagger}$	$2.590^{***\dagger\dagger\dagger}$	$2.374^{***\dagger\dagger}$
	(0.470)	(0.492)	(0.584)	(0.576)
$Payoff_{t-1}$	-0.134***			
	(0.028)			
$Loss_{t-1}$		$4.665^{***}$	$4.960^{***}$	4.896***
		(1.678)	(1.713)	(1.789)
$(Loss \times Payoff)_{t-1}$		-0.072***	-0.051**	-0.061**
		(0.022)	(0.022)	(0.026)
$\frac{1}{t}$	-0.383	-0.702	-0.397	0.093
-	(1.106)	(1.138)	(0.914)	(1.002)
RP			-3.074	-1.168
			(4.140)	(5.502)
AP				-0.078
				(5.502)
CRP			$-10.770^{**}$	
			(6.778)	
Male			-1.088	-0.430
			(2.806)	(2.644)
CRT			-2.609**	-2.229*
			(1.236)	(1.253)
Cons	13.412***	$13.077^{***}$	$18.433^{***}$	$16.397^{***}$
	(2.937)	(2.903)	(3.538)	(3.116)
N	3213	3213	3213	3213
Subjects	52	52	52	52

 ${\bf Table \ A5: \ Median \ Regression \ Estimates \ in \ CVA}$ 

*Note:* Median regression with cluster robust standard errors (CRSE) in parentheses.\*: p-value<.1,\*\*: p-value<.05, \*\*\*: p-value<.01.

Bid	(1)	(2)	(3)	(4)
$\overline{E[L]}$	0.834***	0.814***	0.871***	0.846***
	(0.120)	(0.123)	(0.086)	(0.108)
E[L s] - E[L]	$1.114^{***}$	$1.050^{***}$	$1.068^{***}$	$1.037^{***}$
	(0.087)	(0.096)	(0.075)	(0.087)
$[E[L s] - E[L]]^2$	0.004	0.002	0.005	$0.004^{***}$
	(0.007)	(0.008)	(0.008)	(0.008)
$Bid^{RNNE}$	0.422	0.411	0.261	$0.176^{***\dagger}$
	(0.426)	(0.443)	(0.470)	(0.474)
$Payoff_{t-1}$	-0.061			
	(0.096)			
$Loss_{t-1}$		-2.233	-5.934**	-6.181***
		(2.514)	(2.540)	(2.359)
$(Loss \times Payoff)_{t-1}$		-0.230***	-0.223***	-0.233***
		(0.126)	(0.110)	(0.123)
$\frac{1}{t}$	0.250	0.233	-0.257	1.120
L	(1.158)	(1.246)	(1.137)	(1.074)
RP		· · ·	-7.204**	-7.712**
			(3.948)	(4.243)
AP			· · · ·	5.731
				(4.947)
CRP			1.810	( )
			(7.356)	
Male			5.973*	$5.942^{**}$
			(3.076)	(2.924)
CRT			-3.116**	-3.004***
			(1.301)	(1.134)
Cons	0.865	1.088	-0.190	0.267
	(1.839)	(1.874)	(4.008)	(3.628)
N	2467	2467	2467	2467
Subjects		38	38	
	00	00	00	00

 Table A6:
 MEDIAN REGRESSION ESTIMATES IN CPA

*Note:* Median regression with cluster robust standard errors (CRSE) in parentheses.\*: p-value<.05, \*\*\*: p-value<.01.

WTP	(1)	(2)	(3)	(4)
$\overline{E[L]}$	0.913***	0.897***	0.903	0.898***
	(0.064)	(0.060)	(0.073)	(0.073)
E[L s] - E[L]	1.032***	1.032***	1.036***	1.049***
	(0.081)	(0.077)	(0.108)	(0.092)
$D_{signal}$	0.068	1.924	0.406	1.378
-	(2.582)	(2.386)	(3.126)	(2.993)
$Payoff_{t-1}$	-0.265***			
	(0.070)			
$Loss_{t-1}$		0.580	-1.309	-1.227
		(2.208)	(2.065)	(1.644)
$(Loss \times Payoff)_{t-1}$		-0.265***	-0.266***	-0.296***
		(0.070)	(0.093)	(0.079)
$\frac{1}{4}$	-5.013**	-3.200*	-4.629*	-3.383
L	(2.214)	(1.920)	(2.524)	(2.094)
RP	· · /	· · · ·	-9.099	-9.565
			(5.702)	(5.935)
AP			· · · ·	-7.085
				(5.855)
CRP			-11.641*	· /
			(5.571)	
Male			2.912	-0.044
			(4.034)	(3.172)
CRT			-2.211	-2.202*
			(1.451)	(1.245)
Cons	5.635	2.914	9.643	9.005*
	(3.469)	(3.156)	(5.941)	(5.104)
 N	4634	4634	4381	4381
Subjects	54	54	51	51
Dubjects	94	94	01	51

 $\it Note:$  Median regression with cluster robust standard errors (CRSE) in parentheses.\*: p-value<.1,\*\*: p-value<.05, \*\*\*: p-value<.01.

WTP	(1)	(2)	(3)	(4)
$\overline{E[L]}$	$0.817^{***\dagger\dagger\dagger}$	$0.798^{***\dagger\dagger\dagger}$	$0.794^{***\dagger\dagger\dagger}$	$0.799^{***\dagger\dagger\dagger}$
	(0.402)	(0.040)	(0.042)	(0.043)
E[L s] - E[L]	$0.906^{***\dagger\dagger\dagger}$	$0.904^{***\dagger\dagger\dagger}$	$0.905^{***\dagger\dagger\dagger}$	$0.901^{***\dagger\dagger\dagger}$
	(0.030)	(0.029)	(0.032)	(0.032)
$Payof f_{t-1}$	-1.114e-15			
	(0.010)			
$D_{signal}$	0.391	-0.003	0.831	1.011
	(1.277)	(1.193)	(1.367)	(1.310)
$Loss_{t-1}$		-4.435***	-4.528***	-4.614***
		(1.006)	(0.995)	(1.030)
$(Loss \times Payoff)_{t-1}$		-0.319***	-0.305***	-0.316***
		(0.085)	(0.082)	(0.078)
$\frac{1}{t}$	-1.820*	-2.047**	-1.491	-1.416
	(0.955)	(0.959)	(1.018)	(1.049)
RP			-1.662	-1.711
			(3.230)	(3.201)
AP				-0.538
				(2.434)
CRP			-0.190	
			(3.728)	
Male			1.015	1.037
			(1.733)	(1.681)
CRT			-1.312	-1.322
			(0.904)	(0.812)
Cons	2.423	3.326	$4.718^{*}$	4.444*
	(2.036)	(2.055)	(2.631)	(2.415)
N	4000	4000	3920	3920
Subjects	50	50	49	49

Note: Median regression with cluster robust standard errors (CRSE) in parentheses.\*: p-value<.1,\*\*: p-value<.05, \*\*\*: p-value<.01.

We next estimate the elasticity of the bid with respect to the known and the unknown (i.e., signal) component of the lottery. To this end, we use a simple Cobb-Douglas bidding function in the form of  $b(s_i) = k^{\alpha} \cdot s^{\beta}$ . A naive agent, for instance, would bid  $E[L|s] = k^{\alpha} \cdot s^{\beta}$  with  $\alpha = \beta = 1$ .

ln(Bid)	CVA	CPA	Diff.
ln(k)	$0.254^{*\dagger\dagger\dagger}$	$0.745^{***\dagger}$	-0.491**
	(0.137)	(0.133)	(0.204)
$ln(s_i)$	$1.081^{***\dagger\dagger\dagger}$	$1.295^{***\dagger}$	-0.214
	(0.028)	(0.175)	(0.191)
Cons	-0.298**	$0.994^{**}$	$-1.293^{**}$
	(0.137)	(0.496)	(0.627)
N	3260	2502	5762
Subjects	52	38	90
$R^2$	0.015	0.075	
F-Test	0.000	0.032	
MRS	$\approx 0.23 \frac{s}{k}$	$\approx 0.57 \frac{s}{k}$	

Table A9: MEDIAN REGRESSION COEFFICIENTS IN BIDDING

Note: Median regression with cluster robust standard errors (CRSE) at subject-level in parentheses. Significant difference from 0: \*: p-value<.1,\*\*: p-value<.05, \*\*\*: p-value<.01. Significant difference from 1: †: p-value<.1,††: p-value<.05, †††: p-value<.01. F-test refers to a test on equal weighting of known parameter and signal ( $\alpha = \beta$ ).

We use the marginal rate of substitution (MRS) to compare the estimated bidding functions. The MRS represents here how much units of the signal subjects are willing to trade against a unit of the known parameter to maintain the same bid. For a naive bidder, the MRS equals  $\frac{\alpha s}{\beta k} = \frac{s}{k}$ . For our parameter variation, MRS under Nash equilibrium should be close to  $\frac{s}{k}$ . In both auction formats, the estimated MRS is smaller than  $\frac{s}{k}$  ( $\approx 0.23\frac{s}{k}$  in CVA vs.  $\approx 0.57\frac{s}{k}$  in CPA in Appendix Table A9), indicating that subjects overweighted their private signal but underweighted the known component. Subjects in CV auctions put relatively more weight on the signal compared to those in CP auctions. Similar results are obtained with the pricing data, where MRS are closer to the naive benchmark  $\frac{s}{k}$  (see Appendix Table A10). While subjects put more attention on signals in both CV and CP formats it is important to keep in mind that these signals are about different com-

ln(bid)	CVL	CPL	Diff.
$\overline{ln(k)}$	$0.546^{***\dagger\dagger\dagger}$	0.810***†††	-0.264
	(0.157)	(0.056)	(0.067)
$ln(s_i)$	$0.947^{***}$	$0.974^{***}$	-0.027
	(0.069)	(0.044)	(0.066)
Cons	-2.500***	-3.847***	1.347
	(0.721)	(0.346)	
N	4256	4000	8256
Subjects	54	50	104
$R^2$	0.141	0.3919	
F-Test	0.0209	0.0017	
MRS	$\approx 0.57 \frac{s}{k}$	$\approx 0.83 \frac{s}{k}$	

Table A10: MEDIAN REGRESSION COEFFICIENTS IN PRICING

Note: Median regression with cluster robust standard errors (CRSE) at subject-level in parentheses. Significant difference from 0: \*: p-value<.1,\*\*: p-value<.05, \*\*\*: p-value<.01. Significant difference from 1:  $^{\dagger}$ : p-value<.1, $^{\dagger\dagger}$ : p-value<.05,  $^{\dagger\dagger\dagger}$ : p-value<.01.

ponents of the lotteries. In CV treatments, subjects paid more attention to values in the lottery whereas in CP treatments they rather focused on the probabilities. In a nutshell, it appears that the uncertain component determines how subjects allocate their attention to different features of the auctioned item.

# **B** Individual Covariates

#### B.1 Attitudes toward risk, compound risk and ambiguity

In the last part of the experiment, we elicited subjects attitudes toward risk, compound risk and ambiguity. Subjects started this part by first selecting the payoff relevant task. To this end they threw a dice, knowing that the number on top of the dice would define the selected task. The correspondence between the dice numbers and the tasks were, however, revealed only at the end of the experiment (Baillon et al., 2015). The exchange rate remained the same (\$1 for 6 credits), but payoffs from the main part of the experiment were weighted more heavily than those in this last part (3:1).

This part consisted of only six decision problems. The six decision screens corresponded to three types of decision problems with two replicate measurements each.

#### B.2 Elicitation

We elicited risk attitudes with a multiple price list akin to Abdellaoui et al. (2011) and Gillen et al. (2019). Subjects faced virtual bags with red and blue chips. First subjects chose the color to bet on and then gave their certainty equivalent (henceforth CE) for their chosen bet. Risky bets were implemented with the following lottery (100:0.5;0) and (150:0.5;0) (i.e., a 50% chance of winning (0, 0, 0) (0, 0, 0) (0, 0, 0).

To implement bets with compound risk, subjects were told that the computer would first select with equal probability one virtual bag from a set of virtual bags containing each a different mixture of red and blue balls (Figure A 2 shows an example of the screen for a bag with 20 chips), and would then randomly draw a chip from the selected bag. Subjects received  $\notin 100$ ( $\notin 150$  in the replicate measurement) if the color of the drawn chip matched the color they bet on.

The implementation of ambiguous bets was similar, except that the mixture of red and blue chips was determined ex ante by a research affiliate and was not known to subjects. The virtual bag contains 20 chips, but its composition is randomly determined. That is, the computer will first randomly choose one of the 21 possible and equally likely mixtures displayed below. The letters R and B in the table denote the number of red and blue chips, respectively. Note, it must be the case that R+B=20 in every possible bag.

One chip will then be randomly drawn from the bag with the selected mixture. You will receive 100 credits if its color matches your bet, otherwise nothing. Choose first the color you would like to bet on and state then your minimum compensation for your chosen bet.

	19	2 18	3 17 4 16		5 15	
6 14 7	13	8 12	9 11 10 10		11 9	
12 🛯 13	7	14 6	15 5 16 4		17 3	
18 2 19		20 0				
Bet: 100 on Red	d, 0 on	Blue	Bet: 100 on Blu	ie, 0 oi	n Red	Your Bet
	Red	Blue		Red	Blue	100 on Red, 0 on Blue
Value	100	0	Value	0	100	Your minimum compensation for this be
Number of Balls	R	в	Number of Balls	R	В	
				_		50

Figure A 2: Example for a decision screen to elicit attitudes toward compound risk (after selecting to bet on red and a certainty equivalent of 50 credits.)

#### **B.3** Descriptive statistics

**Methods.** We classify attitudes as averse toward a type of uncertainty if subjects' prices display a premium for the corresponding lottery. In other words, we classify subjects as averse if they chose a CE that is smaller than the expected value. The subject's premium for a lottery is defined as the difference between its expected value and the subject's CE. A positive (negative) premium reflects aversion (proclivity).

We mitigate possible measurement error by taking the mean of the two replicate measurements: To this end we first normalize the CE by the lottery's expected value and average the normalized CE across the two replicate measurements.<sup>15</sup> <sup>16</sup> Note that all decisions under uncertainty should be affected by a risk premium, if a subject is not risk-neutral. In a crude

<sup>&</sup>lt;sup>15</sup>For the ambiguous bets, we assume uniform beliefs over possible probabilities to compute the lotteries' expected value.

<sup>&</sup>lt;sup>16</sup>Most subjects were also consistent in their attitudes, especially in their attitudes toward ambiguity. The redundant measures yield the same classification for 71.15%, 75.96% and 79.81% for attitudes toward risk, compound risk and ambiguity, respectively (in the full sample).

attempt to control for risk attitudes in decisions with compound risk and ambiguity, we subtract the subject's average risk premium from the chosen premium for lotteries with compound risk and ambiguity (cf. Gillen et al., 2019). This yields a conservative measure for premia under compound risk and ambiguity since risk premia for binary lotteries should be highest when the success probability equals 50% (as in the risky lotteries). Thus, premia for compound risk and ambiguity that are corrected for individual risk premia become also negative in the cases where subjects were less averse toward compound risk/ ambiguity than toward risk ( applies to 59 (60) out of 194 subjects for the compound risk (ambiguity) premium).



**Figure A 3:** Distribution of premia – by treatments CV (left) and CP (right).

**Results.** Figure A 3 shows the distribution of risk, compound risk and ambiguity premia, averaged across the two duplicate measures. Most subjects were averse.

Distributions of premia are not significantly different from each other across treatments (the Kolmogorov-Smirnov statistics yields p-values of p = 0.21, p = 0.45, p = 0.89 for risk, compound risk and ambiguity premia, respectively). Most subjects chose a premium close to zero, and attitudes toward compound risk and ambiguity are positively correlated (consistent with Halevy (2007)'s finding). The pairwise correlation coefficients are  $\rho_{RC} = -0.24, \rho_{RA} = -0.10, \rho_{CA} = 0.54.$ 

#### **B.4** Individual Characteristics

In general, individual characteristics do not significantly differ between CVA and CPA, or between CVL and CPL. The treatments are also balanced with respect to gender and the cognitive reflection test. In contrast, the elicitation of attitudes toward different types of uncertainty seems to be affected by the main experiment. Elicited attitudes differ (albeit non-significantly) depending on whether prior to the elicitation subjects participated in the strategic or in the non-strategic decision game. We observe less aversion in the individual decision treatments CVL and CPL. This warrants a cautious interpretation of the regression findings presented in model (4) and (5) of Appendix Tables A7 to A6.

	Auction				Lottery	Auction - Lottery		
	CVA	CPA	Diff.	CVL	CPL	Diff.	CV	CP
RP	-0.0090	-0.0671	0.0581	0.1512	0.0499	0.1013	0.1602	0.1170
	(0.0508)	(0.0659)	(0.0832)	(0.0586)	(0.0483)	(0.0776)	(0.0852)	(0.0816)
$\operatorname{CRP}$	0.1118	0.1560	-0.0442	0.0568	0.0938	-0.0370	-0.0550	-0.0622
	(0.0353)	(0.0338)	(0.0488)	(0.0346)	(0.0370)	(0.0507)	(0.0494)	(0.0484)
AP	0.1348	0.1221	0.0127	0.0427	0.0899	-0.0473	-0.0921*	-0.0321
	(0.0343)	(0.0511)	(0.0615)	(0.0426)	(0.0422)	(0.0600)	(0.0547)	(0.0663)
CRT	1.5385	1.3421	0.1964	1.3889	1.5	-0.1111	-0.1496	0.1579
	(0.1516)	(0.1612)	(0.2212)	(0.1638)	(0.1545)	(0.2252)	(0.2232)	(0.2233)
Male	0.5385	0.6053	-0.0668	0.4340	0.4800	-0.0460	-0.1045	-0.1253
	(0.0698)	(0.0804)	(0.1064)	(0.0688)	(0.0714)	(0.0991)	(0.0980)	(0.1074)

 Table A11: MEANS OF INDIVIDUAL VARIABLES BY TREATMENT

*Note:* \*: p-value<0.1, \*\*: p-value<0.05, \*\*\*: p-value<0.01. Robust error clustered by subject in parentheses.

# C Non-Expected Utility Models

## C.1 Salience

Figures A 4 and A 5 show the valuation of a (reduced) lottery given a specific salience bias  $\delta \in (0, 1]$ . A standard expected utility maximizer has  $\delta = 1$ , whereas a salience bias is more accentuated with decreasing  $\delta$ . For a given parameter  $\delta$  the salience bias is always less accentuated in lottery CP.



**Figure A 4:** Valuation of the lottery type 1 as a function of salience bias  $\delta$  (compared with a payoff of zero)



Figure A 5: Valuation of the lottery type 1 as a function of salience bias  $\delta$  (compared with E[V])

In a nutshell, although the salience bias generates different valuation of

the lotteries, the bias always go in the same direction. Salience theory alone would not explain why subjects overbid for CV but slightly underbid for CP lotteries.