Index Funds, Asset Prices, and the Welfare of Investors*

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June 22, 2023

Abstract

We study the impact of index funds on asset prices and on the choices and welfare of investors. To do so, we present a model in which investors choose among bonds, stocks, and an index Fund that holds the market portfolio. We define a notion of equilibrium in which asset prices are determined endogenously, and show that, under standard assumptions, an equilibrium exists. The availability of the index Fund induces investors to shift investment from individual stocks to the Fund and from bonds to the Fund. The former shift tends to reduces investor risk and hence to increase investor welfare; the latter shift tends to increase asset prices and decrease expected returns, and hence to reduce investor welfare. For a wide variety of parameters, we show that the welfare reductions from the latter dominate the welfare increases from the former; as a result, *the availability of the index Fund decreases welfare for all investors.*

JEL Classification: D14, D53, G11, G12, G23

Keywords: portfolio choice, asset pricing, ownership, indexing, inequality, household finance

*We thank Philip Bond (AFA discussant) Jonathan Parker, Glen Weyl, conference audiences at the AFA meetings, and seminar audiences at Berkeley, Yale University's Cowles Foundation, Carnegie Mellon University, Oslo Macro Group, University College London, and UCSD for helpful comments. Schmalz acknowledges support from Deutsche Forschungsgemeinschaft under Germany's Excellence Strategy EXC 2126/1390838866. Zame acknowledges the hospitality and support of the Geneva Finance Research Institute and the Einaudi Institute for Economics and Finance, and the hospitality of University College, London. Wentao Li and Wenchuan Zhao provided excellent research assistance.

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1 Introduction

Vanguard, and other index funds, are celebrated for allowing small investors to diversify their equity portfolios and enjoy market returns. (Indeed, this was the expressly stated objective of Vanguard when it was first introduced in 1975.) This view of index funds has dominated the academic literature and policy discourse for the past four decades. However, almost all of the existing analysis views index funds as having no effect on asset prices. This view might have been appropriate when index funds were small – but index funds are no longer small: among them, BlackRock, Vanguard, and State Street (the three largest funds) currently own 23% of the S&P 500 companies Amel-Zadeh et al. (2022), and the totality of index funds own almost half of the equity of all publicly traded firms Chinco and Sammon (2022). Indeed, index funds have grown so large that legal scholars and policy makers have begun to propose wide-ranging regulations on the size and behavior of index funds. Precisely because index funds are large, a proper analysis of effect of such regulations would seem to require abandoning the view that might have been appropriate when index funds were small and did not affect asset prices in favor of a more realistic view that acknowledges that index funds are large and might affect asset prices.

This paper offers a (stylized) formalization of this more realistic view. We offer a model in which investors choose among investments in individual firms, a single index Fund that holds the market portfolio, and a risk-free bond. We define a notion of equilibrium in which investors optimize, the market clears, and asset prices are determined endogenously, and show (in the presence of standard assumptions) that equilibrium exists. Investing in an individual firm exposes investors to both idiosyncratic firm-specific risk and market-wide aggregate risk; investing in the Fund (and paying the fee charged by the Fund) shields investors from idiosyncratic risk but not aggregate risk. Shifting invested wealth from individual stocks to the Fund tends to reduce investor risk and hence to increase investor welfare; shifting invested wealth from bonds to the Fund tends to increase asset prices and decrease expected returns, and hence to reduce investor welfare. We solve the model (numerically) for a variety of settings and parameters; in every case, we show that the welfare reductions from the latter shift dominates the welfare increases from the former shift; as a result, *the availability of the Fund decreases welfare for all investors*, and the decrease in welfare is greater when the fee charged by the Fund is smaller. When investors are heterogeneous (in terms of wealth and risk aversion), the welfare losses are greatest (in proportional terms) for upper-class investors (who are wealthier and less risk-averse) and for lower-class investors (who are poorer and more risk-averse), and smallest for middle-class investors.

Our work demonstrates that the introduction of index funds may have a significant effect on asset prices, and that any serious analysis of index funds – in particular, any analysis of the effects of regulation – must take this effect into account.[1] In particular, our work suggests a need to recast the existing policy debate about index funds, which has focused on the trade-off of *empirically observed* adverse effects on consumers against *assumed* benefits to investors. Our paper shows that it is not appropriate to simply *assume* that indexing benefit investors .

Because our model is stylized, we do not emphasize its quantitative predictions. However we note that its qualitative predictions about portfolio holdings as a function of wealth and risk aversion are consistent with empirical findings. In particular, our model predicts that the very rich don't diversify much, because they (correctly!) believe that their un-diversified investments will yield greater returns; this is consistent with survey evidence (Bender et al., 2022). Our model also predicts a non-monotonic relationship between wealth and the equity allocation in individuals' portfolios; this is consistent with the data as reported in Bach et al. (2020); Fagereng et al. (2020); Beutel and Weber (2022).

The present paper is sharply distinguished from extant theories of stock market equilibrium and asset pricing. The equilibrium model of Bond and Garcia (2022) examines the effect of a decrease in the cost of indexing on price efficiency and the welfare of heterogeneously informed investors, whereas we study the effect on the price level and the welfare of investors with different wealth and risk aversion. In contrast to the literature which assumes that firms are price takers (e.g. Hart (1979)), we allow firms to have arbitrary objectives. In contrast with the literature that argues that investor heterogeneity does not matter for asset prices (Panageas et al., 2020), we find that, in the presence of index funds, investor heterogeneity matters a great deal. In contrast to the literature on the effect of intermediaries, (e.g., He and Krishnamurthy (2013); Haddad and Muir (2021), we study the effect of an unlevered intermediary on asset prices. In contrast to the literature on cross-sectional differences (e.g., Jiang et al. (2022)) or many extant studies examining the effect of indexing on price efficiency (e.g. Baruch and Zhang (2022)), we focus on the general price level of the equity market. Indeed, our study examines the effect of textbook indexing, and

[1] One might view the introduction of index funds as similar to the introduction of new financial instruments. In that light, our finding that the introduction of index funds may decrease investor welfare echoes the example of Hart (1975) and the general results of Elul (1995).

does not engage with non-market variations of indexing that occur in practice.

In contrast to the seminal paper by Rotemberg (1984) and more recent theories proposed in the common-ownership literature (e.g., López and Vives (2019); Antón et al. (2022)) our model features endogeneous asset prices.[2] Piccolo and Schneemeier (2020) endogenize ownership by diversified investors in a model in which ownership affects firm behavior. By contrast, the present paper endogenizes ownership in a model in which ownership *does not* influence firm behavior. (Our companion paper Schmalz and Zame (2023) extends the present model to accommodate this possibility.) Azar and Vives (2021); Eeckhout and Barcelona (2020); Philippon et al. (2021); Azar et al. (2021) are complements to the present paper, as they debate equilibrium aspects of common ownership when the ownership of firms affects product and labor markets, but not asset prices.

2 Model

We build a model with a focus on aggregate qualitative predictions. To that end, we oversimplify in a number of dimensions. (We elaborate on some of these simplifications in the Conclusion.) In particular, we consider a setting in which investment decisions are made at date 0 and consumption (of a single good – wealth) takes place at date 1. (In the simulations, we think of the interval between date 0 and date 1 as 20 years.) We assume a large number of identical firms, operating in small industries; thus firms make positive profits, which are subject to both firm-specific idiosyncratic shocks and a market-wide random shock. We assume that the number of firms is sufficiently large that the firm-specific idiosyncratic shocks wash out in the aggregate. Because the index Fund is completely diversified across the whole market, investment in the Fund is immune to the firm-specific idiosyncratic shocks but remains subject to the market-wide random shock.

2.1 Industries and Firms

There are N_0 identical industries. Within each industry, there are $m \geq 1$ identical firms, so the total number of Firms in the market is $N = mN_0$. We think of m as small – so we allow for monopoly, duopoly, oligopoly – and N_0, and hence N, as large.[3] Within each industry, firms

[2] In theory, indexing creates some degree of common ownership across rivals in the same industry. In practice, much common ownership seems to be driven by active portfolio choice (Amel-Zadeh et al., 2022); it appears that no study has measured common ownership at the fund level. As a result, the extent to which indexing drives common ownership is not known.

[3] In the simulations, we take $N = 5,000$; this was roughly the number of publicly traded firms in 1980.

compete. Rather than specifying the objective of each firm, the mode of competition, and the (equilibrium) behavior of firms, we take the shortcut of specifying the *(random) profit* function Π of each firm. Thus, in keeping with the long tradition of Sharpe and Lintner, we view each firm simply as a risky asset.

The profit of each firm is subject to two kinds of random shocks: an idiosyncratic, firm-specific shock ϵ and a market-wide aggregate shock Δ; the total profit of a firm is the sum of a deterministic profit π and the two shocks;

$$\Pi = \pi + \epsilon + \Delta$$

We might view ϵ as arising from as a shock to the firm's cost of production and Δ as arising from a shock to the demand structure of the entire economy.[4] We assume that the idiosyncratic shocks ϵ are drawn independently from a distribution with mean 0, and that N is sufficiently large that they wash out in total.[5] We assume that firms have limited liability, so that $\pi + \epsilon + \Delta \geq 0$ for every realization of uncertainty. For simplicity, we assume that $\pi + \Delta$ is bounded away from 0.

2.2 Stocks

An investor who owns stock in an individual firm obtains fraction of the profits of that firm. We view the firms as having a single share of stock; investors own fractional shares, Because the firms are identical, the return on a share of stock in any firm is:

$$r_S = \pi + \epsilon + \Delta \tag{1}$$

(Recall that idiosyncratic shocks are drawn independently, so different firms – and investors who invest in them – may experience different shocks.) We assume below that each investor can hold only one stock; this is a proxy for the idea that the cost of creating and maintaining a diversified portfolio comprising many stocks is prohibitive. In reality, many investors held a small number of individual stocks, not entirely washing out idiosyncratic risk.

[4]In reality, each of π, ϵ, and Δ might depend on the firm's ownership, but for our present purpose we ignore this possibility. We will return to this point in the Conclusion.

[5]Because N is finite, this is an approximation, but if N is large it is a good approximation.

2.3 Index Fund

There is a single index Fund.[6] The Fund does not maximize profits; rather it charges a fixed fee $k \in [0, \infty]$, which simply covers its operating expenses. The Fund invests all of its Assets Under Management AUM to buy an equal share λ of all firms. If the price of firm n is p_n then the cost to the Fund to buy the share λ of firm n is λp_n. The Fund's total expenditure must equal its Assets Under Management so:

$$\text{AUM} = \sum_{n=1}^{N} \lambda p_n$$

By assumption, the Fund holds an equal fraction of all firms, so we can view a holding in the Fund as an indirect holding of an equal amount stock in each firm. Hence an investor who holds a total of x_F units of stock through the Fund holds x_F/N shares of stock in each firm. If the price of firm n is p_n, the cost of holding x_F/N units of stock in firm n is through the Fund is $(x_F/N)p_n$, so the investor's total expenditure is

$$\sum_n \left(\frac{x_F}{N}\right) p_n = \left(\frac{1}{N}\sum_n p_n\right) x_F$$

Note that $\bar{p} = (\sum_n p_n)/N$ is just the *average* price of all firms, so the cost of holding x_F units of stock through the Fund is just $\bar{p}x_F$.

An investor who holds stock through the Fund is perfectly diversified but must pay the fee k charged by the Fund; hence the the investor receives the return

$$r_F = \frac{\Pi + \Delta}{1 + k} \tag{2}$$

per share held through the Fund. Hence an investor who holds x_F shares of stock through the Fund receives the return $r_F x_F$.

This method of accounting is consistent with the idea that all investment takes place at date 0 and all consumption – hence all returns to investment – take place at date 1. In reality, index funds charge an annual fee that is a fraction of the current value of the investor's portfolio. Our method of accounting should be thought of as a convenient two-date proxy for this reality.

[6] Our examples are calibrated to the period of time in which Vanguard was the only operating index fund. At present, there are many such funds, of which Blackrock, Vanguard and State Street are the largest. To a first approximation, their fees are driven by the cost of indexing.

2.4 Bonds

An unlimited supply of riskless bonds is available at the fixed price of 1 with *return*

$$r_B = (1 + \rho) \tag{3}$$

where $\rho \geq 0$.

Here we make the simplifying assumption that the interest rate on bonds is fixed and that the supply is determined by demand. (E.g., the government offers an unlimited supply of bonds at a fixed interest rate.) An alternative would be to assume that the supply of bonds is fixed and that the interest rate is determined in equilibrium. We will discuss this alternative in the Conclusion.

2.5 Investors

There are a continuum of investors, indexed by $T = [0, \infty)$, and distributed according to some non-atomic measure ϕ with total mass M.[7]

Investor $t \in T$ is characterized by its invested wealth $w^t \in (0, \infty)$, its choice set X^t and a utility function U^t for *random* consumption. For simplicity, we assume that investors maximize expected utility with respect to some Bernoulli utility function for consumption; i.e. $U^t = E[u^t(c)]$, where u^t is continuous and strictly increasing.[8] We assume that the mapping

$$t \mapsto (w^t, X^t, U^t)$$

is measurable, and that total wealth

$$W = \int_T w^t d\phi(t)$$

is finite.

As noted above, we assume that investors can hold stock in only one firm. To formalize this, write

$$Y = \{x \in \mathbb{R}^N_+ : x(n) > 0 \text{ for at most one } n\}$$

[7] It is convenient to deviate from the usual convention and *not* normalize the mass of investors to one.

[8] It would be enough to require that the utility functions U^t depend only on the distribution of returns, a property that is guaranteed by the assumption of expected utility.

We assume that, for each investor t, $X^t \subset Y \times \mathbb{R}_+ \times \mathbb{R}_+$ is a closed cone containing 0.[9] Thus, a choice $x^t = (x^t_S, x^t_F, x^t_B) \in X^t$ for investor t specifies the number of shares $x^t_S(n)$ of each firm n that t holds directly, the total number of shares x^t_F of all firms that investor t holds through the Fund, and the number x^t_B of bonds that investor t holds. (In our formulation, there is a single share in each firm, so investors hold fractional shares.)

Now consider an investor t who holds the portfolio $x^t = (x^t_S, x^t_F, x^t_B)$ consisting of x^t_S shares of stock held directly, x^t_F shares of stock held through the Fund, and x^t_B bonds. Keeping in mind that we require $x^t_S(n) \neq 0$ for at most one n, our calculations of returns in equations (1) - (3) imply that the utility of this investor is

$$U^t(x) = U^t\Big(r_S\left[\sum_n x^t_S(n)\right] + r_F x^t_F + r_B x^t_B\Big)$$

(Recall that the return on Stock and the Fund are *random*, but that the return on bonds is not.)

3 Equilibrium

An *equilibrium* consists of

- *prices* $p_n > 0$ for each firm n
- a (measurable) investor choice function

$$t \to x^t = \left(x^t_S, x^t_F, x^t_B\right) \in X^t$$

such that

- each investor maximizes utility $U^t(x^t)$ subject to the feasibility constraint

$$x^t \in X^t$$

  and the budget constraint

$$\sum_n p_n x^t_S(n) + \bar{p} x^t_F + x^t_B \leq w^t$$

[9]We do not require that X^t be convex; this allows for the possibility that investors cannot hold mixed portfolios.

(Recall that $\bar{p}$ is the average price of all firms.)

- for each n, the market for stock in firm n clears

$$\int x_S^t(n) d\phi(t) + \int \left(\frac{x_F^t}{N}\right) d\phi(t) = 1$$

The first integral is the total of stock in firm n held directly by investors; the second is the total of stock in firm n held through the Fund.[10] (All shares are held by small investors.)

We are especially interested in equilibria in which all firms have the same price; i.e., $p_n = \bar{p}$ for all n; we refer to such an equilibrium as a *uniform price equilibrium*.

3.1 Discussion

Some discussion may be helpful. In our notion of equilibrium, there is no trade; more precisely, trade has already taken place at date 0 (and, because there is a single good, there is no trade at date 1). We fix the invested wealth of each investor. In effect, the stock in the various firms, the Fund, and the bonds represent *assets*. An equilibrium is defined by prices for stocks and holdings for investors that would be optimal at the specified price and clear the market for stocks. (As we have noted, bonds are in arbitrary supply.)

3.2 Uniform Price Equilibrium

Because all firms have the same (distribution of) profits, it might seem that, at equilibrium they would *necessarily* have the same price; i.e., all equilibria would be uniform price equilibria. Indeed, if $p_n > p_{n'}$ then no investor would hold stock in firm n; in this case it might seem that the market could not clear. However, this is not quite right: for some parameters, there will be equilibria in which no investor directly holds stock in *any* firm; rather, all stock is held through the Fund. In such a situation, *prices need not be uniform* . (We outline examples in Appendix A.)

[10] As we have noted, x_F^t is the total amount of stock held through the Fund and the Fund holds equal amounts of stock in each firm, so x_F^t/N is the amount of stock held in each individual firm.

3.3 Existence of Equilibrium

Theorem *There exists a uniform price equilibrium. Moreover, every equilibrium in which a positive mass of investors hold some stock directly is a uniform price equilibrium.*

We defer the proof to the Appendix.

4 Economic Forces

Aside from existence, the assumptions we have made so far do not imply any particular properties of the equilibrium. In particular, they do not tell us anything about how the equilibrium price and investor welfare depend on the presence of the Fund and on the fee it charges. For this, we will turn to simulations, but it might be useful to briefly consider the economic forces involved.

Consider first a world in which no Fund operates ($k = \infty$). In this world, investors apportion their entire wealth between stocks and bonds because those are the only choices available. Now suppose a Fund appears but charges no fee ($k = 0$). Stocks and the Fund have the same expected return, but the Fund is less risky; if investors are risk averse, they will use the Fund to diversify their holding – shifting their *entire* stock investment into the Fund; because investors are risk averse, this diversification will be welfare increasing. However, diversification is not the only force at work. Because the Fund is less risky than stock, it is more attractive than stock *in relation to bonds*. So investors will also shift some of their investment from bonds into the Fund. This shift increases the (indirect) demand for stock and hence its price; because the return on stock is fixed, this increase in price will be welfare decreasing.

Thus, the appearance of the Fund creates *two* forces, and these forces act in *opposite directions*. The magnitude of these forces depends on the parameters of the model; in the absence of assumptions about these parameters, it does not seem possible to evaluate the *net effect on welfare*. (Note however, that the effect on prices is unambiguously increasing, although the magnitude of this increase depends on the parameters of the model.)

We can also ask how prices and welfare depend on the fee k charged by the Fund.[11] In addition to the two forces identified above, there are other forces at work. Because investor choices of investments in stock and the Fund depend on prices, their exposure to risk also depends on prices.

[11] In fact, the fee charged by Vanguard has decreased over time, in part because Vanguard's cost of operation has decreased as computers have become faster and cheaper, etc.

In the following Sections, we explore these issues further by simulating the model for various settings and specifications of the parameters. In each setting and specification of parameters, we find that the presence of the Fund decreases welfare *for all investors* and this decrease in welfare is *greater* when the cost of investing in the Fund (the fee k) is less.

5 Numerical Simulations

It seems difficult – perhaps impossible – to solve the Model in closed form, even for simple specifications of the parameters.[12] Instead, we rely on numerical solutions for various choices of parameters and settings. Our choice of parameters is suggested by data from the US stock and bond markets circa 1980, at a time when Vanguard was just established but was the only significant index fund. Aside from this choice of parameters, we have made little effort to calibrate the model to real data. Our objective in these simulations is to provide intuition about the effect of index funds on asset prices, portfolio holdings, and investor welfare. We believe that some of the results are surprising – even counter-intuitive. We describe these parameters and settings below, and then describe our numerical procedure.

5.1 Timing

We view date 0 (the date at which investors make investments) and date 1 (the date at which investors realize the returns on investments) as approximately 20 years apart.

5.2 Bonds

Throughout, we assume that $\rho = 0.5$. This represents a real rate of return of roughly $0.02 = 2\%$ per year, compounded over the 20-year period.

5.3 Firms

Throughout, we assume the total number of (publicly traded) firms is $N = 5000$ and that the expected profit of each firm is $\pi = \$500$ Million. (Keep in mind that this is profit over a 20-year period.) We assume that idiosyncratic risk $\epsilon = \pm 0.5\pi$, each occurring with probability 0.5 and

[12] We elaborate on this difficulty in the Appendix.

that aggregate market risk is $\Delta = \pm 0.5\pi$, each occurring with probability 0.5. By definition, idiosyncratic risk and aggregrate risk are independent, so the distribution of realized profits for each firm is

$$\Pi = \begin{cases} \$1,000 \text{ Million} & \text{with probability} \quad 0.25 \\ \$500 \text{ Million} & \text{with probability} \quad 0.50 \\ \$0 & \text{with probability} \quad 0.25 \end{cases}$$

These parameters imply that, *ex-ante*, the probability that a given firm will go bankrupt during the 20-year period is 0.25. Lest this seem unreasonably large, note that the average bankruptcy rate of publicly traded firms is actually in the range of 1-2% per year. In our model, a firm will go bankrupt only if it experiences a negative idiosyncratic shock *and* the market experiences a negative aggregate shock; if the market shock is positive, no firm will go bankrupt. However, from the *ex ante* perspective of investors, what matters is that the return on a stock investment will be 0 with probability 0.25.

5.4 Investors

We consider three scenarios for investor characteristics:

- **Scenario 1** All investors have the same relative risk aversion and the same wealth.
- **Scenario 2** All investors have the same relative risk aversion, but wealth is exponentially distributed.
- **Scenario 3** Relative risk aversion is uniformly distributed, wealth is exponentially distributed; risk aversion and wealth are perfectly correlated.

(We will be more specific about details in the following Section.)

We offer this variety of scenarios in part as a robustness check. As we show, our qualitative findings are consistent across these scenarios: the presence of the Fund leads to a *loss* in the welfare of investors, and this loss in *greater* when the fee charged by the Fund is *lower*. As we show in Scenarios 2 and 3, the welfare losses are different for different investors.

In all settings, we measure wealth and consumption in units of \$10,000. We assume the total mass of investors (the number of investors) is $M = 100$ Million and the total invested wealth is

W = \$2 Trillion.[13] We assume that investors maximize expected utility with respect to CRRA utility for consumption.[14] Thus, an investor whose coefficient of relative risk aversion is s has Bernoulli utility function

$$u^s(c) = \begin{cases} \frac{c^{1-s}-1}{1-s} & \text{if} \quad s \neq 1 \\ \log s & \text{if} \quad s = 1 \end{cases}$$

5.5 The Fund

As in the Model, we assume there is a single Fund, so it is completely specified by k, the fee charged by the Fund. We consider various values of k in the interval $[0, 1]$ and $k = \infty$, which represents the setting in which no Fund is available to investors.

5.6 Numerical Procedure

We look for a uniform price equilibrium (the existence of which is guaranteed by Theorem 1). Our numerical procedure is as follows.

(1) We choose a candidate price for shares in a firm. (By assumption, at equilibrium, all firms have the same price so it is enough to consider that one price.)

(2) In the setting in which all investors have the same wealth and risk aversion, we calculate the investor's choices as a function of the candidate price; in the other settings, we calculate what each each investor in a fine grid of 1000 investors would choose. (Because firms are identical and have the same price, investors are indifferent across firms, so it is enough to calculate the investor's *total* direct purchase of stock, indirect purchase of stock through the Fund, and purchase of bonds.) In all settings, we then scale up to the total number of investors to determine the aggregate demands.

(3) We check whether the whether market for stock clears; i.e., whether the total demand for stock equals the total supply of stock.

[13]In 1980, total capitalization of all publicly traded US firms was approximately \$1 Trillion and the total capitalization of the US bond market is estimated to have been in the range of \$0.5-1.5 Trillion, so W = \$2 Trillion seems reasonable.

[14]We have also carried out the corresponding simulations for CARA utility; the qualitative findings are quite similar. Details of the CARA simulations are available from the authors on request.

We iterate this procedure until we find a price at which the market clears; this is an *equilibrium price*.[15] Once we have determined an equilibrium price, we then determine equilibrium choices and utilities.

6 Findings

In each Scenario, we solve for, and compare, equilibrium prices and investor utilities for various values of the fee k charged by the Fund. If k is large enough (e.g., $k = \infty$), no one will choose to invest in the Fund; this is equivalent to the *absence of the Fund*, and so provides a comparison of equilibrium prices and investor utilities in the absence of the Fund with equilibrium prices and investor utilities in the presence of the Fund. We also compare investor utilities at equilibrium with the counterfactual of what investor utilities would have been if the presence of the Fund did not affect prices. (We might also interpret this counterfactual as what investor utilities would be if the Fund were very small.) Finally, we compute the actual utility gain obtained by the marginal investor who faces actual equilibrium prices.

6.1 Scenario 1

In this simplest Scenario, we assume all investors are identical: they have the same coefficient of relative risk aversion and the same wealth. Because total wealth is \$2 Trillion and there are 100 Million investors, this means that each investor has wealth \$20,000 – but recall that we measure wealth in units of \$10,000. In Tables 1 and 2 below we show, for $k = 0.0, 0.01.0.02, 0.05, 0.10, 0.20.0.30, 0.40$:

- what the stock price p^{nF} (in Billions of \$) would be if there were no Fund; of course this does not depend on the fee k
- the equilibrium stock price p^* (in Billions of \$)

[15]Because the total demand for stock equals the total supply of stock, it will always be possible to arrange actual holdings so that, in each firm, the demand for stock equals the supply. Indeed, there will be many ways to do this, but they will all result in the same assignments of investor wealth to stock, to the Fund and to bonds, and to the same investor utility. In principle, our model might have multiple equilibria, but we do not find multiple equilibria in any of our simulations. We discuss the possibility of multiple equilibria in following the proof of the Theorem in the Appendix.

- what each investor's expect utility Eu^{nF} would be if there were no Fund; of course this does not depend on the fee k
- what each investor's expected utility Eu^* is in the presence of the Fund
- what an investor's expected utility Eu^Cwould be under the the counterfactual assumption that the price of stocks did not change because of the presence of the Fund
- the expected utility Eu^M of a marginal investor who faces actual equilibrium prices but does not invest in the Fund (perhaps because the investor did not know know of the existence of the Fund)
- the fractional change $\Delta^{nF} = (Eu^* - Eu^{nF})/Eu^{nF}$, expressed as a percentage; this is the actual loss incurred by an investor because the presence of the Fund drives up prices
- the fractional change $\Delta^C = (Eu^C - Eu^{nF})/Eu^{nF}$, expressed as a percentage; this is what the benefit derived by an investor would be if the presence of the Fund did not affect prices
- the fractional change $\Delta^M = (Eu^* - Eu^M)/Eu^M$, expressed as a percentage; this is the actual benefit derived by a marginal investor, given that the presence of the Fund does affect prices

Of course, the utilities and the percentage changes depend on our particular specification of utility functions and not only on the underlying preferences, so the actual magnitudes should not be overemphasized. What we do emphasize are the *signs* of the changes (which display welfare gains or losses) and the *relative magnitudes* of the changes for different values of the fee k charged by the Fund (which display how the gains and losses are affected by the fee k).

Table 1. Identical Investors; Relative Risk Aversion $s = 1$

k	p^{nF}	p^*	Eu^{nF}	Eu^*	Eu^C	Eu^M	Δ^{nF}	Δ^C	Δ^M
0	0.203	0.273	1.255	1.169	1.451	1.131	-6.89%	15.62%	3.31%
0.01	0.203	0.270	1.255	1.169	1.442	1.134	-6.88%	14.83%	3.05%
0.02	0.203	0.268	1.255	1.169	1.432	1.138	-6.85%	14.07%	2.80%
0.05	0.203	0.260	1.255	1.172	1.405	1.147	-6.64%	11.90%	2.15%
0.10	0.203	0.248	1.255	1.180	1.364	1.165	-5.97%	8.62%	1.31%
0.20	0.203	0.226	1.255	1.208	1.294	1.204	-3.74%	3.11%	0.34%
0.30	0.203	0.207	1.255	1.246	1.257	1.246	-0.72%	0.10%	0.01%
0.40	0.203	0.203	1.255	1.255	1.255	1.255	0.00%	0.00%	0.00%

Table 2. Identical Investors; Relative Risk Aversion $s = 2$

k	p^{nF}	p^*	Eu^{nF}	Eu^*	Eu^C	Eu^M	Δ^{nF}	Δ^C	Δ^M
0	0.128	0.233	0.732	0.700	0.829	0.682	-4.34%	13.32%	2.69%
0.01	0.128	0.231	0.732	0.700	0.828	0.682	-4.34%	13.09%	2.58%
0.02	0.128	0.229	0.732	0.700	0.826	0.683	-4.33%	12.86%	2.48%
0.05	0.128	0.222	0.732	0.700	0.821	0.685	-4.30%	12.17%	2.20%
0.10	0.128	0.212	0.732	0.701	0.813	0.689	-4.18%	11.06%	1.80%
0.20	0.128	0.194	0.732	0.704	0.797	0.696	-3.78%	8.91%	1.17%
0.30	0.128	0.179	0.732	0.708	0.782	0.703	-3.26%	6.85%	0.74%
0.40	0.128	0.166	0.732	0.712	0.767	0.709	-2.66%	4.86%	0.43%

In Table 1, we see that when $s = 1$ and $k = 0.4$, the fee charged by the Fund is so large that there is no investment in the Fund. In every other case, we see that:

- the presence of the Fund drives up the price of stock: $p^* > p^{nF}$
- the presence of the Fund decreases investor welfare: $\Delta^{nF} < 0$
- the presence of the Fund would increase investor welfare under the counterfactual that the presence of the Fund would not affect prices: $\Delta^C > 0$
- the opportunity to invest in the Fund benefits the marginal investor, even when we take into account that the presence of the Fund does affect prices $\Delta^M > 0$

Moreover, in every case, the effects (the increases or decreases) are *greater* when the fee charged by the Fund is *lower*.[16]

6.2 Scenario 2

In this scenario, we assume all investors have the same coefficient s of relative risk aversion (we will again consider the cases $s = 1, 2$) but different wealths. (This is evidently more realistic than Scenario 1.) We assume that investors are uniformly distributed on the interval $[0, 5]$ (this makes for easier comparison with the results of Section 3) and that wealth is exponentially distributed;

[16] We note that equilibrium prices p^* are strictly decreasing in k but that equilibrium utilities Eu^* appear to be constant for small values of k (and strictly increasing for larger values of k).In fact, equilibrium utilities are strictly increasing for all values of k (until the point where the fee is so large that there is no investment in the Fund); that the displayed values appear identical is a reflection only of rounding errors.

thus investor t's wealth is Ce^{-t}; the constant C is determined by the number of investors and the total invested wealth. We have assumed $M = 100$ Million investors so

$$M = \int_0^5 d\phi(t)$$

Because investors are uniformly distributed, this means that

$$d\phi(t) = \frac{M}{5} dt = 20 \text{ Million } dt$$

In order that total wealth $W = \$2$ Trillion we must have

$$W = \int_0^5 Ce^{-t} d\phi(t) = C(20 \text{ Million}) \int_0^5 e^{-t} dt$$

whence

$$C = \frac{\$2 \text{ Trillion}}{(20 \text{ Million}) \int_0^5 e^{-t} dt} = \frac{\$100,000}{1 - e^{-5}}$$

Because we measure wealth in units of \$10,000, this means $C = 10/(1 - e^{-5})$ units, so the wealth of investor t, in units of \$10,000, is

$$w^t = \left(\frac{10}{1 - e^{-5}} \right) e^{-t}$$

We find that the overall qualitative effects in Scenario 2 are the same as in Scenario 1; however, as might be expected, the quantitative effects for investor depend on the wealth of that investor. As in Scenario 1, we focus on risk aversion $s = 1$ and $s = 2$; for each, We display the results in Figures 1-8:

- Figures 1 and 2 show the equilibrium price p^* as a function of the fee k charged by the Fund
- Figures 3 and 4 show, for various values of k, the welfare loss in percentage terms Δ^{nF} suffered by investors because of the presence of the fund Figures 5 and 6 show, for the same values of k, the welfare gains in percentage terms Δ^C that investors would realize under the counterfactual assumption that the presence of the Fund did not affect prices

- Figures 7 and 8 show, for the same values of k, the welfare gains in percentage terms Δ^M that the marginal investor realizes, given that the presence of the Fund does affect prices

Figures 1-8 go here; to be added

As can be seen from these Figures, the qualitative results are the same as in Scenario 1:

- the presence of the Fund drives up the price of stock
- the presence of the Fund decreases investor welfare
- the presence of the Fund would increase investor welfare under the counterfactual that the presence of the Fund would not affect prices
- the opportunity to invest in the Fund benefits the marginal investor, even when we take into account that the presence of the Fund does affect prices

Moreover, as in Scenario 1, all of these effects are greater when the fee charged by the Fund is less.

6.3 Setting 3

In this Setting, we assume that investors are heterogeneous with respect to both wealth and risk aversion. In particular, we assume that investor t's coefficient of risk aversion is t, that the risk aversion of investors is uniformly distributed on the interval $[0, 5]$, and that the wealth of investors is exponentially distributed. As before, it follows that the wealth of investor t, in units of \$10,000, is

$$w^t = \left(\frac{10}{1 - e^{-5}}\right) e^{-t}$$

6.3.1 Equilibrium Prices

In Figure 9 we show the equilibrium price of equities as a function of the fee k charged by the Fund. The Figure plots prices for $0 \leq k \leq 1.0$, in increments of 0.01. Note that the equilibrium

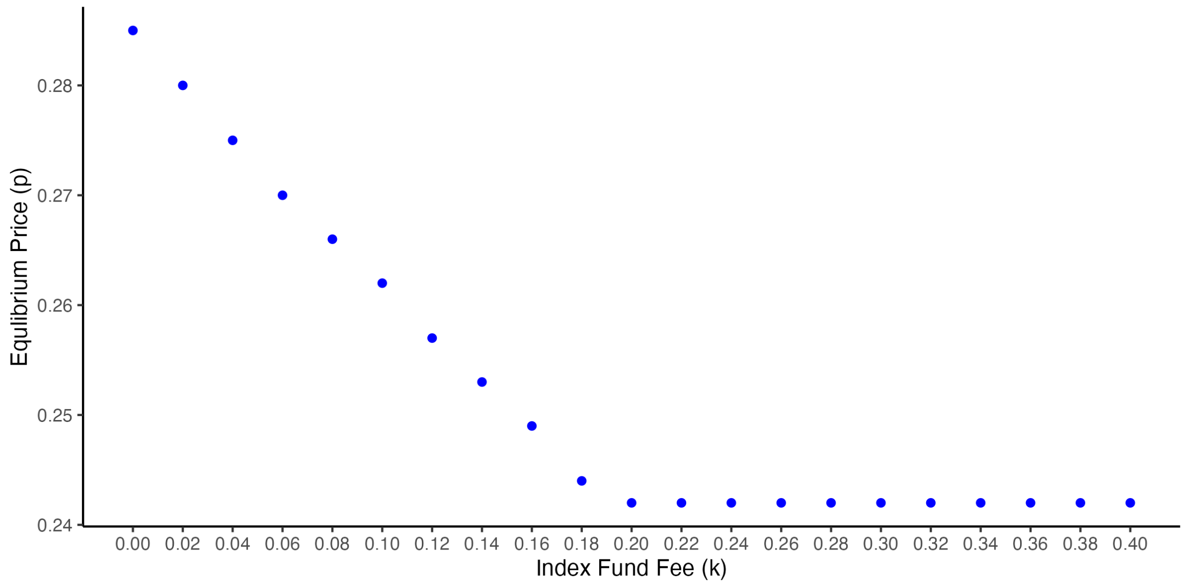

Figure 9. Equilibrium Price of Stock as a Function of the Fund Fee k

price is strictly monotonically decreasing for $k \in [0, 0.2)$ and flat for $k \geq 0.2$; this is because when $k \geq 0.2$ no investor buys shares in the Fund.[17]

6.3.2 Portfolio Choices

In Figures 10-14, we show the portfolio choices of investors as a function of their risk aversion, for values of $k = 0.2, 0.1, 0.05, 0.02, 0.0$. (When $k > 0.2$, no investor buys shares in the Fund.) As k decreases (so indexing becomes cheaper), money flows out of bonds and individual stocks and into the Fund. When $k = 0$ indexing is free so of course no investor chooses to hold individual stocks.

6.3.3 Welfare Comparisons

In Figure 15, we provide welfare comparisons (in percentage terms). The benchmark for comparison is welfare when $k = 0.2$, when no investor buys shares in the Fund – and which is equivalent to the scenario in which no fund is available. Remarkably, the availability of the fund *reduces welfare for all investors*, and *the reduction in welfare increases as the fund becomes cheaper*. Because asset prices rise when the Fund is available and continue to rise as investing

[17] In Appendix B, in which we assume that investors have constant absolute risk aversion, we find that investors hold shares in the fund – and the price continues to decrease – throughout the interval $[0, 1]$ and that the equilibrium price is even lower when $k = \infty$.

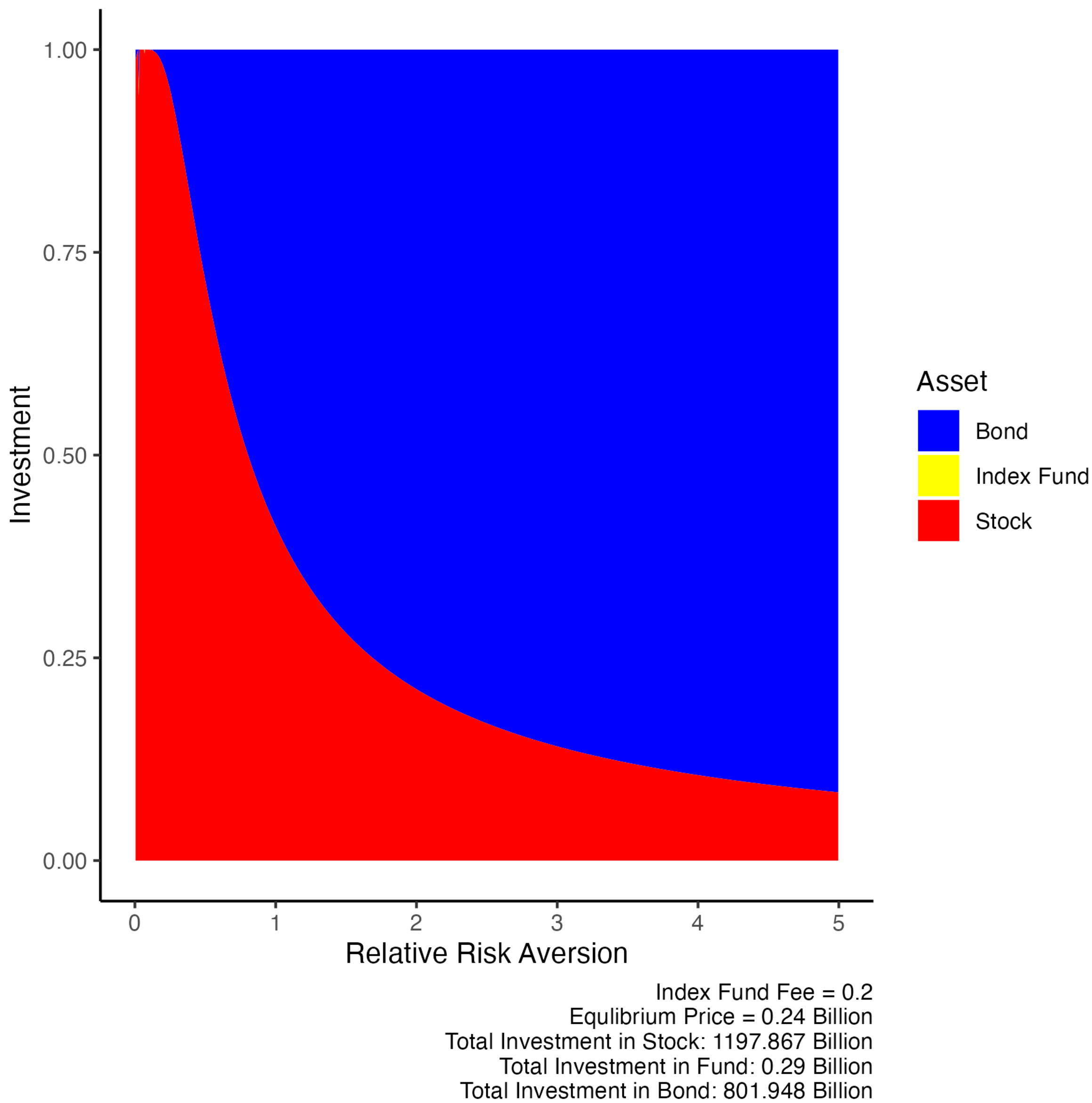

Figure 10. Portfolio Choices: CRRA utility; $k = 0.2$

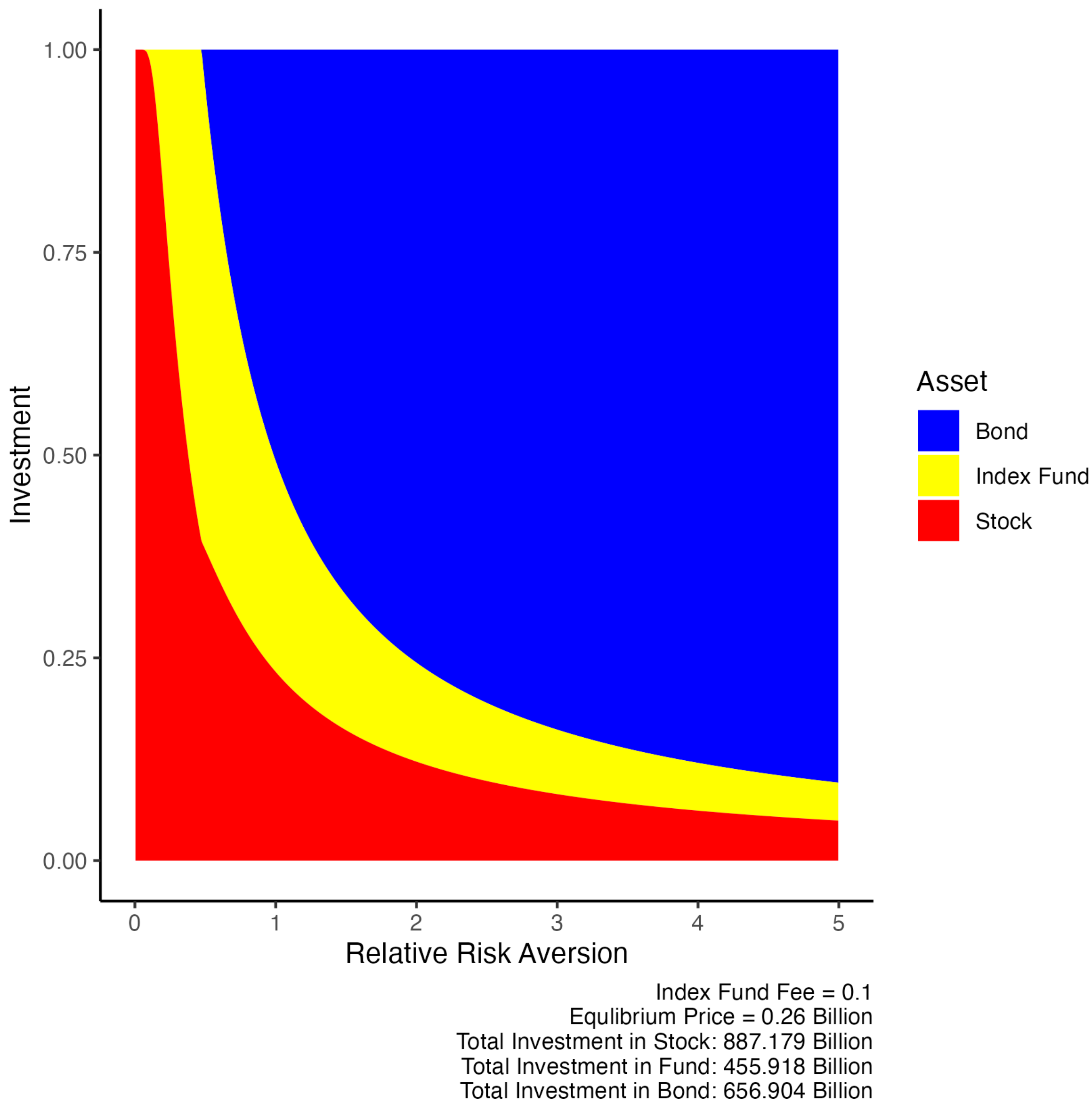

Figure 11. Portfolio Choices: CRRA utility; $k = 0.1$

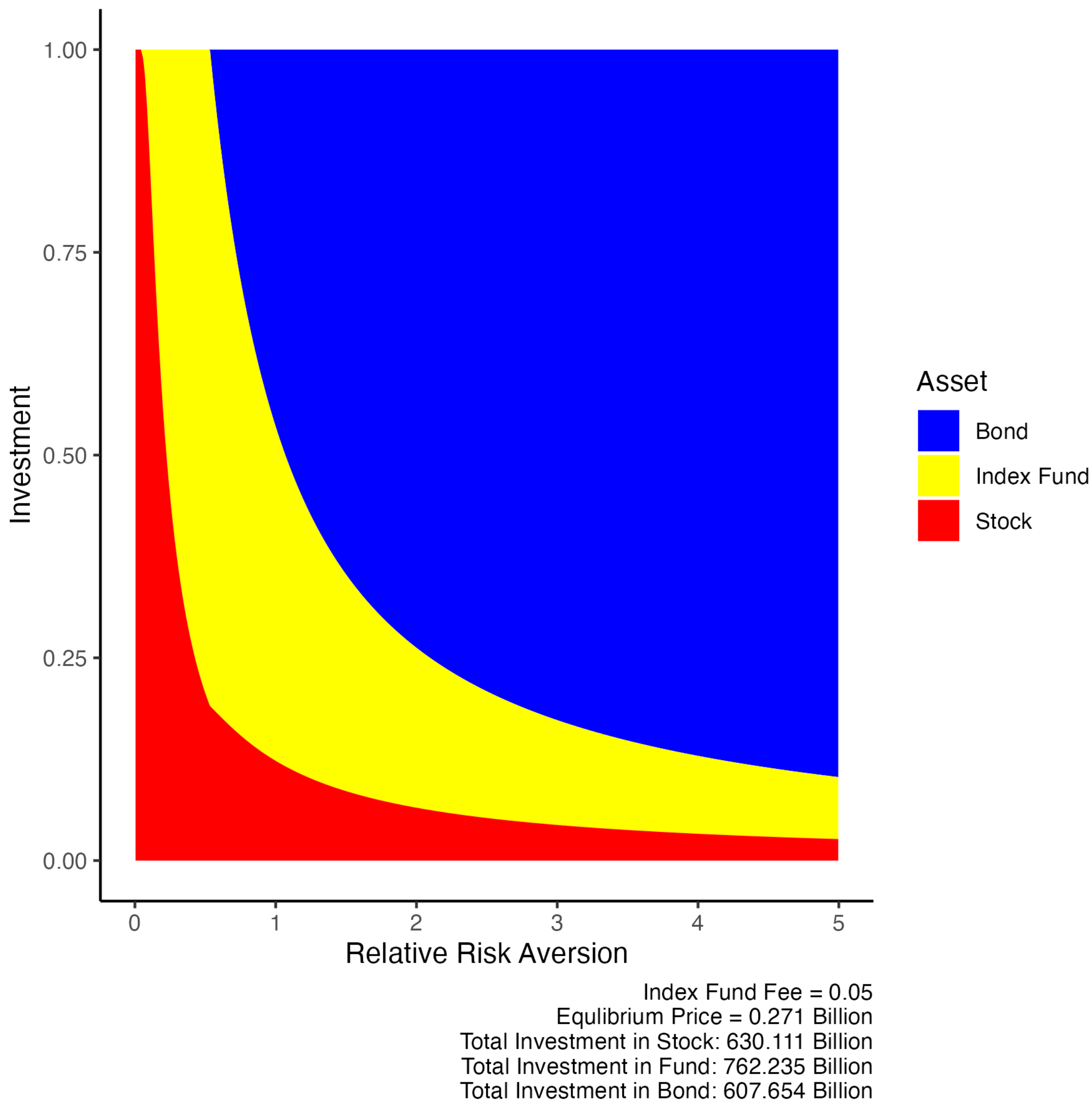

Figure 12. Portfolio Choices: CRRA utility; $k = 0.05$

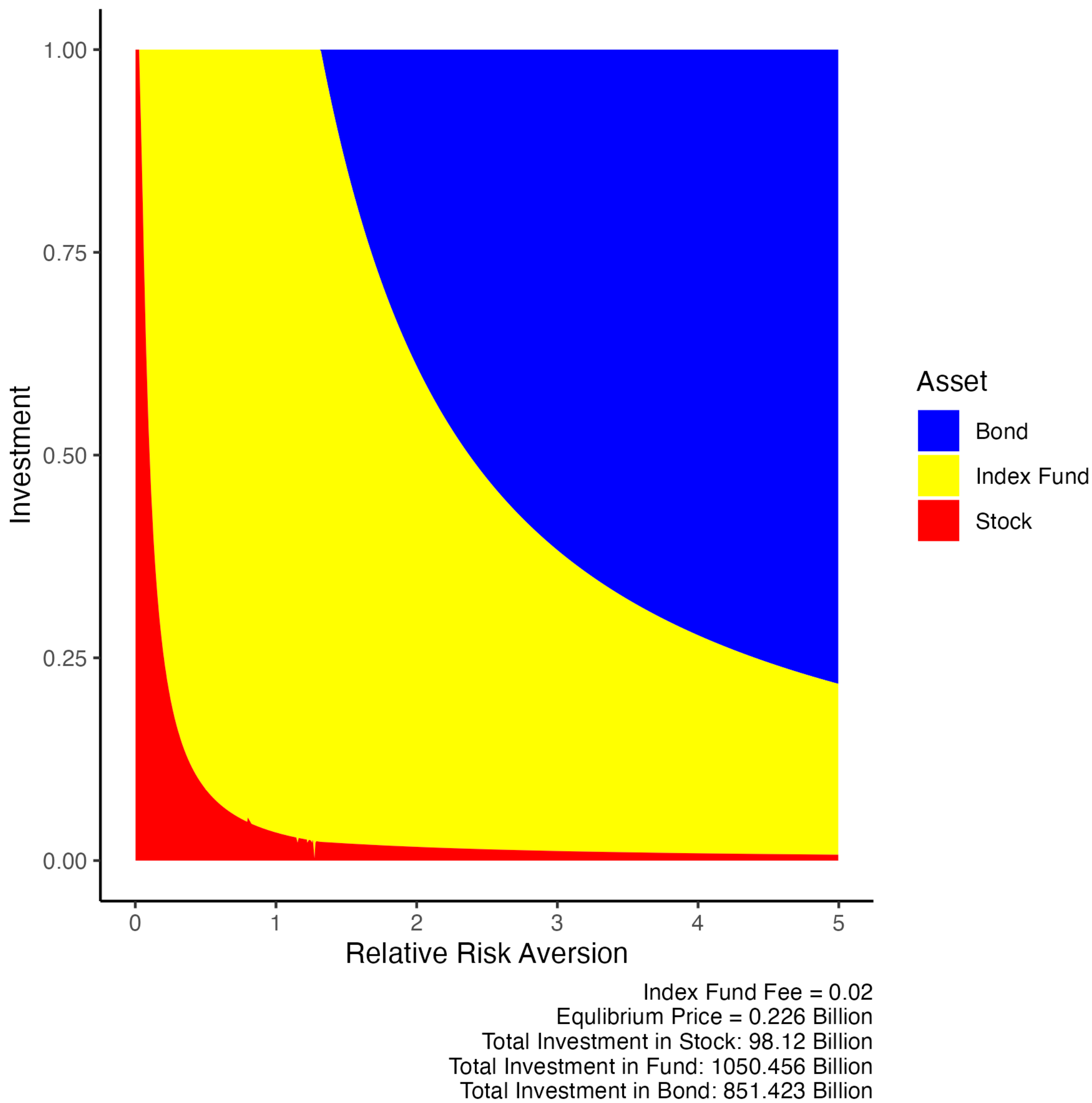

Figure 13. Portfolio Choices: CRRA utility; $k = 0.02$

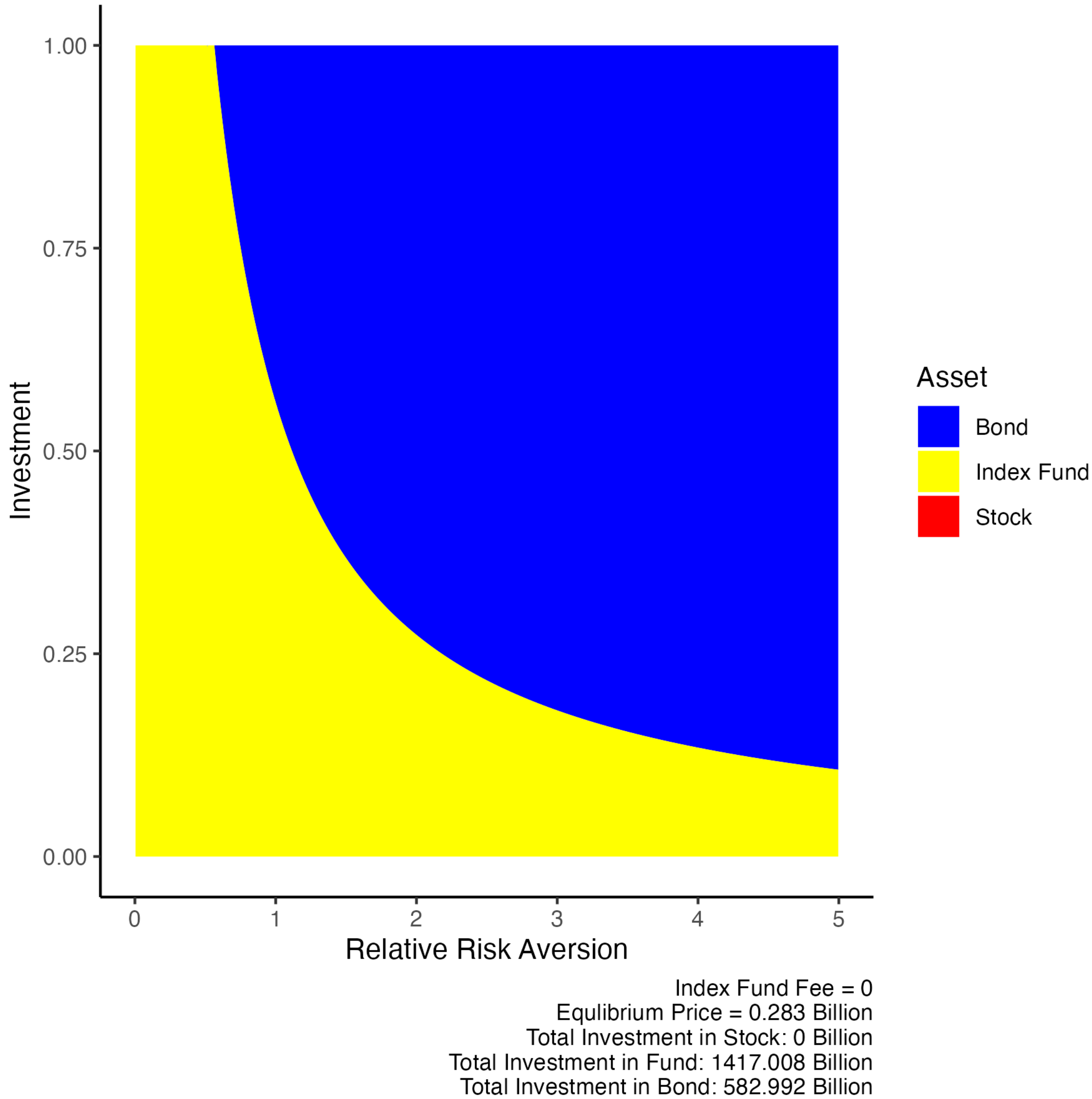

Figure 14. Portfolio Choices: CRRA utility; $k = 0$

in the Fund becomes cheaper, it is no surprise that the richest and least risk-averse investors – who invest most heavily in individual stocks – suffer large losses. It may be more surprising that middle class investors also suffer large losses – but it should be kept in mind that these investors are less wealthy and more risk-averse, to that their marginal utility of income is greater, and we are measuring welfare loss in percentage terms.

6.3.4 The Marginal Investor

As our Figure 15 shows, the welfare of investors *falls* when the Fund becomes available, and continues to fall as the Fund becomes cheaper. This is the equilibrium effect which we have discussed: the availability of the Fund drives up asset prices and the negative effect of this increase in asset prices is greater than the positive effect of the diversification that the Fund provides.

Of course, this does not mean that an individual investor – who does not influence prices – should not invest in the Fund. To make this point, we explore the portfolio choices and the welfare consequences for the *marginal investor*. We consider an investor who faces the *equilibrium asset prices* in a world in which the Fund operates and charges a given fee k. For such an investor, we show in Figures 16-20 the portfolios that investor would choose if s/he were unaware of the Fund. Comparing those Figures with Figures 10-14, we see how the choices of the marginal investor change when the investor becomes aware of the Fund. We then show, in Figure 21, the welfare gains realized by the marginal investor as s/he becomes aware of the Fund (but faces the actual equilibrium prices).

We note several important things about Figure 21. The first is that the welfare gain *increases* as the fee charged by the Fund decreases. Put differently: lower fees are good for the marginal investor. (Of course this confirms the obvious partial equilibrium analysis, but it is in stark contrast to the general equilibrium conclusions.) The second is that the percentage welfare gains are very different across the spectrum of investors. Roughly speaking we can say that

- Investors whose wealth is in the top 1% benefit little from the presence of the Fund because they are almost risk neutral – and invest little in the Fund
- Investors whose wealth is in the top 2-10% benefit substantially from the presence of the Fund because they shift a great deal of invested wealth into the Fund and are sufficiently risk-averse to enjoy the resulting reduction in risk.

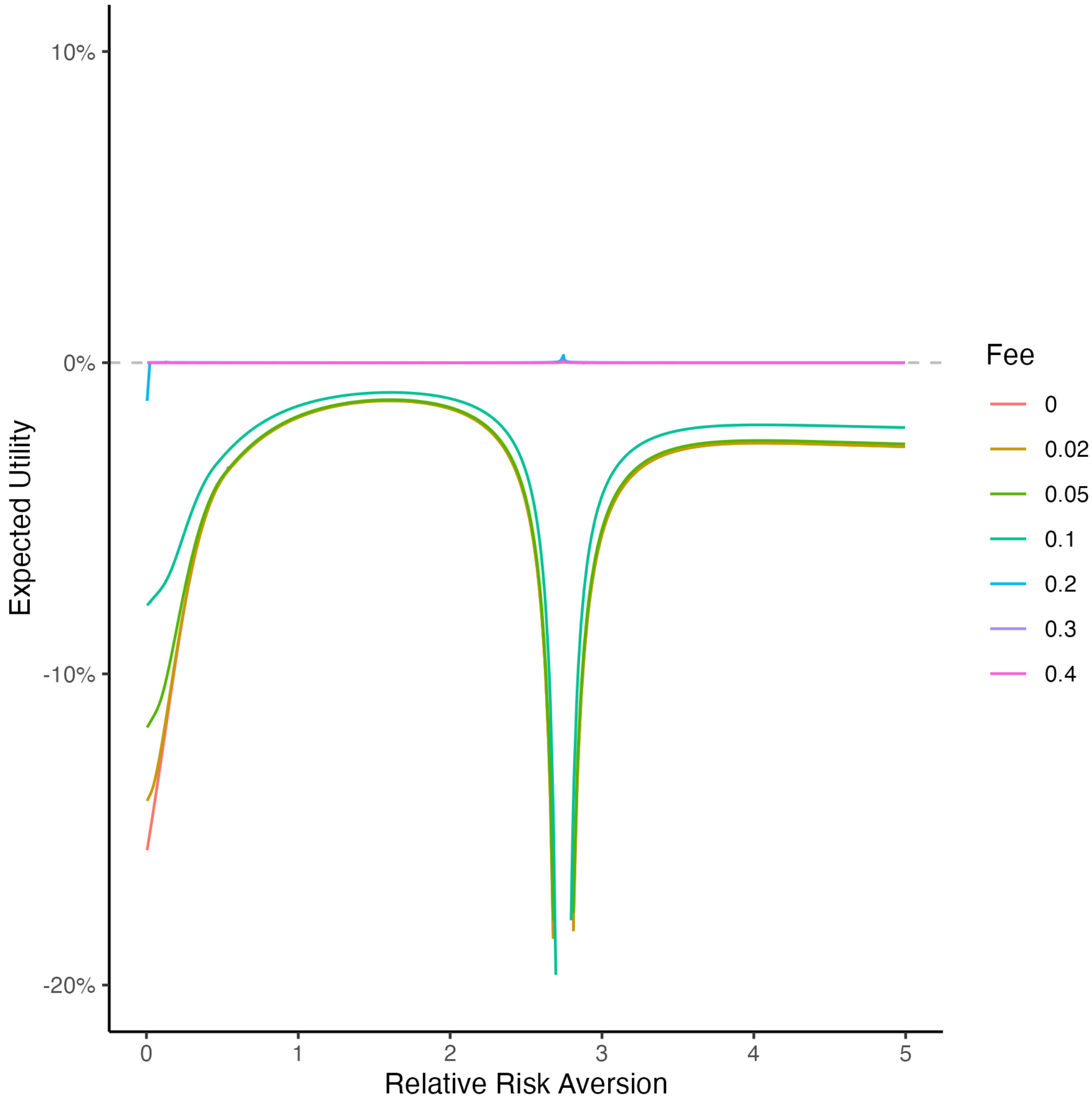

Figure 15. Welfare Losses; CRRA utility. Benchmark = No Fund

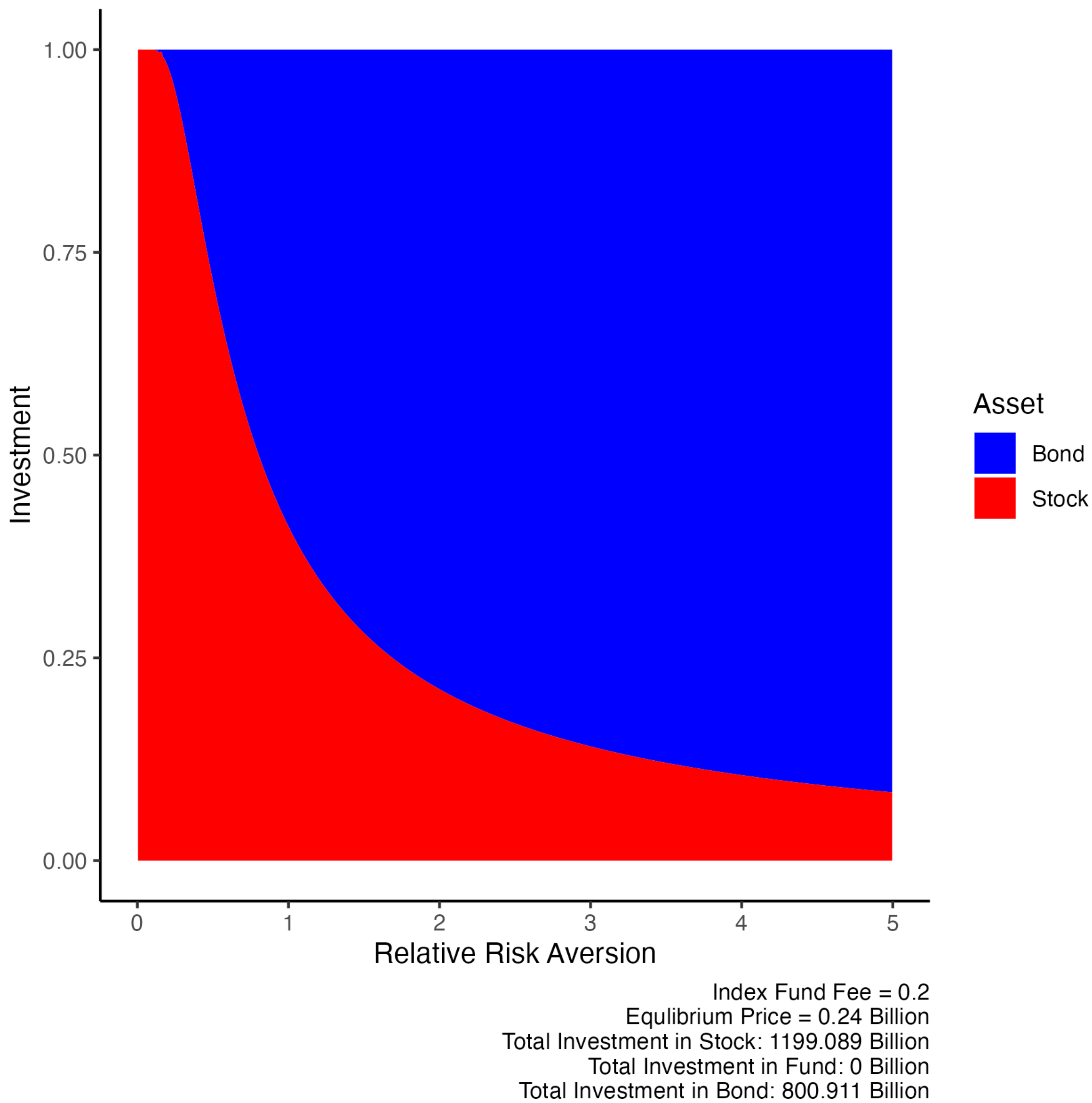

Figure 16. Portfolio Choices without the Fund: CRRA utility; $k = 0.2$

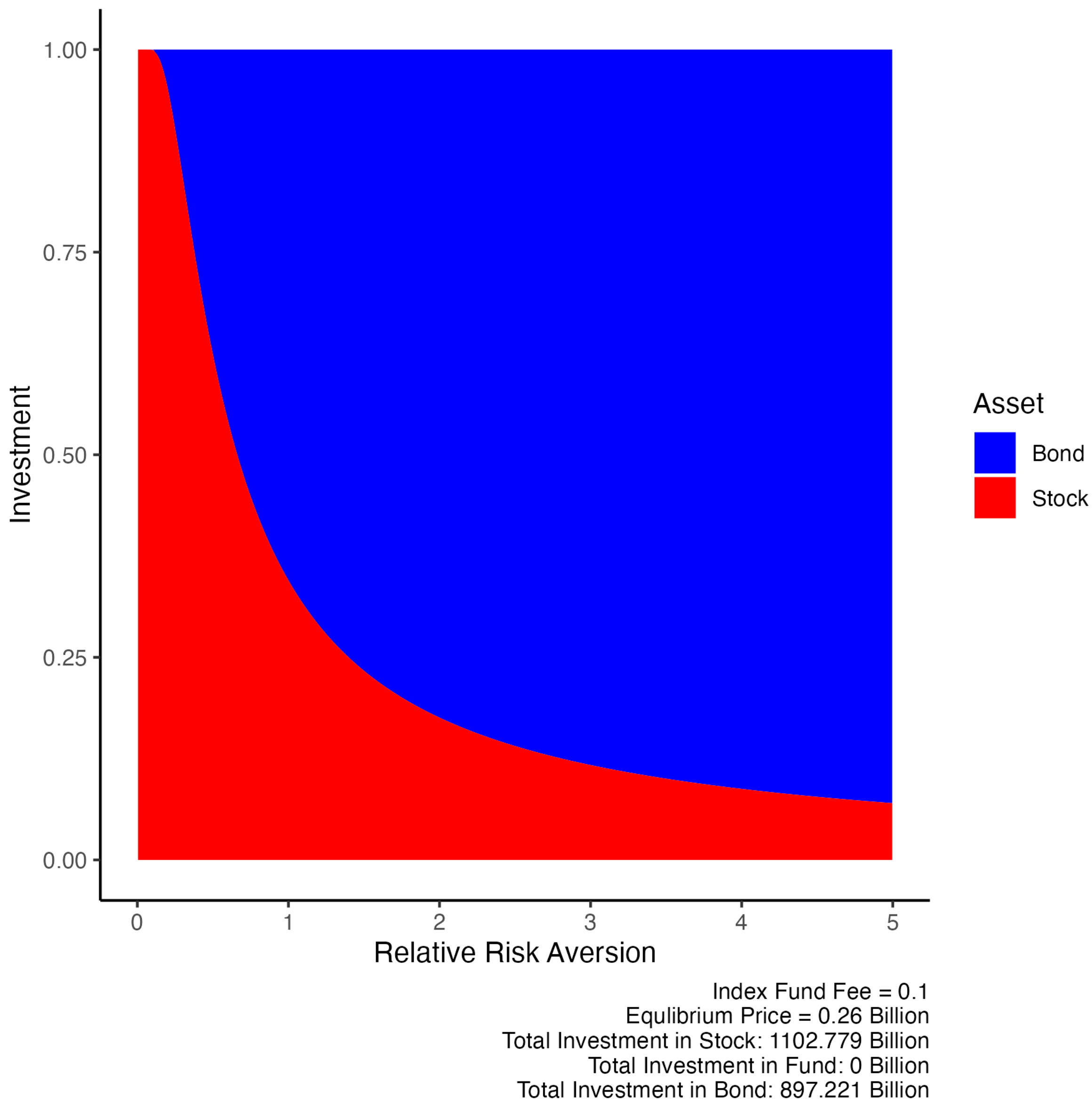

Figure 17. Portfolio Choices without the Fund: CRRA utility; $k = 0.1$

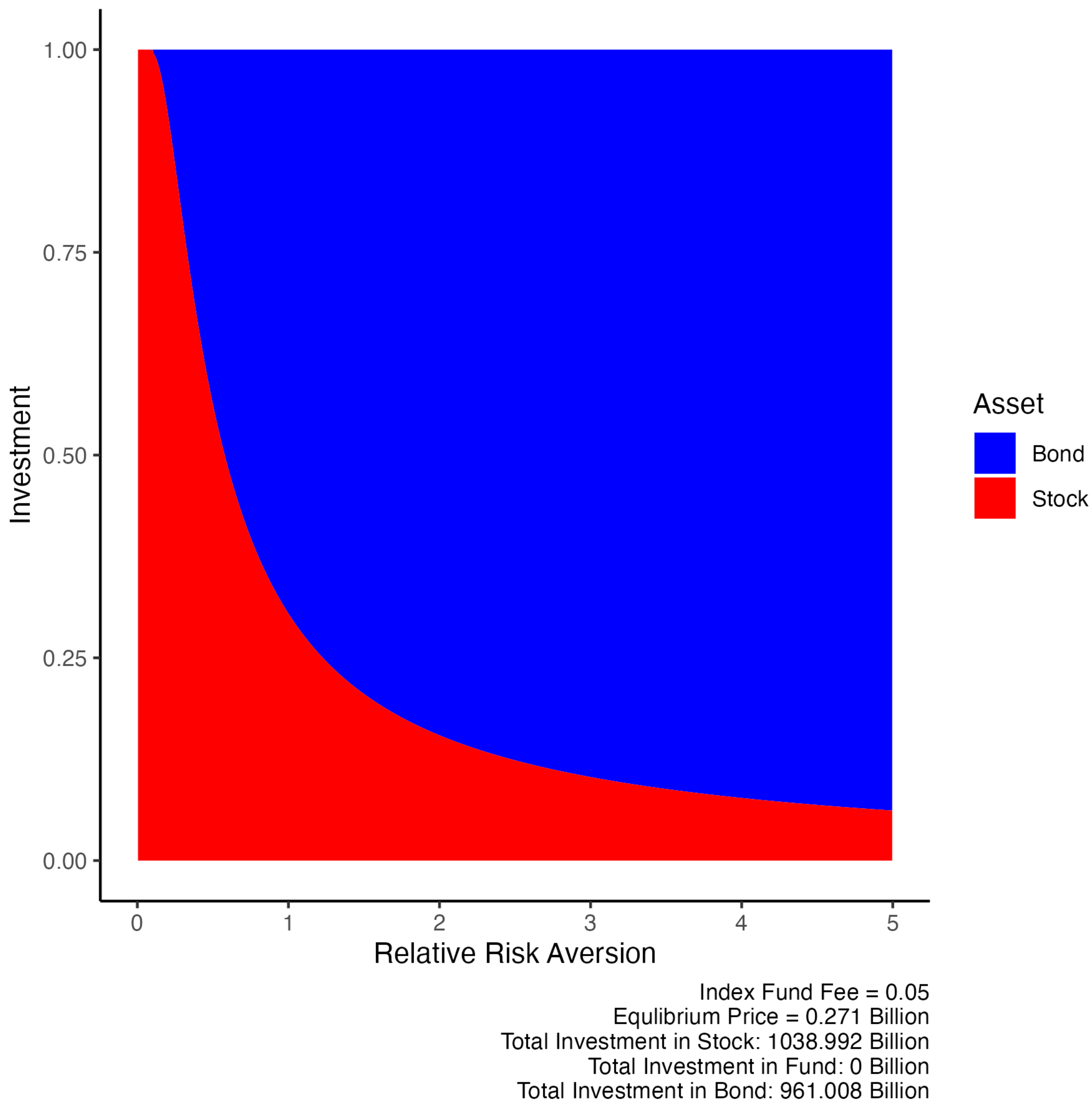

Figure 18. Portfolio Choices without the Fund: CRRA utility; $k = 0.05$

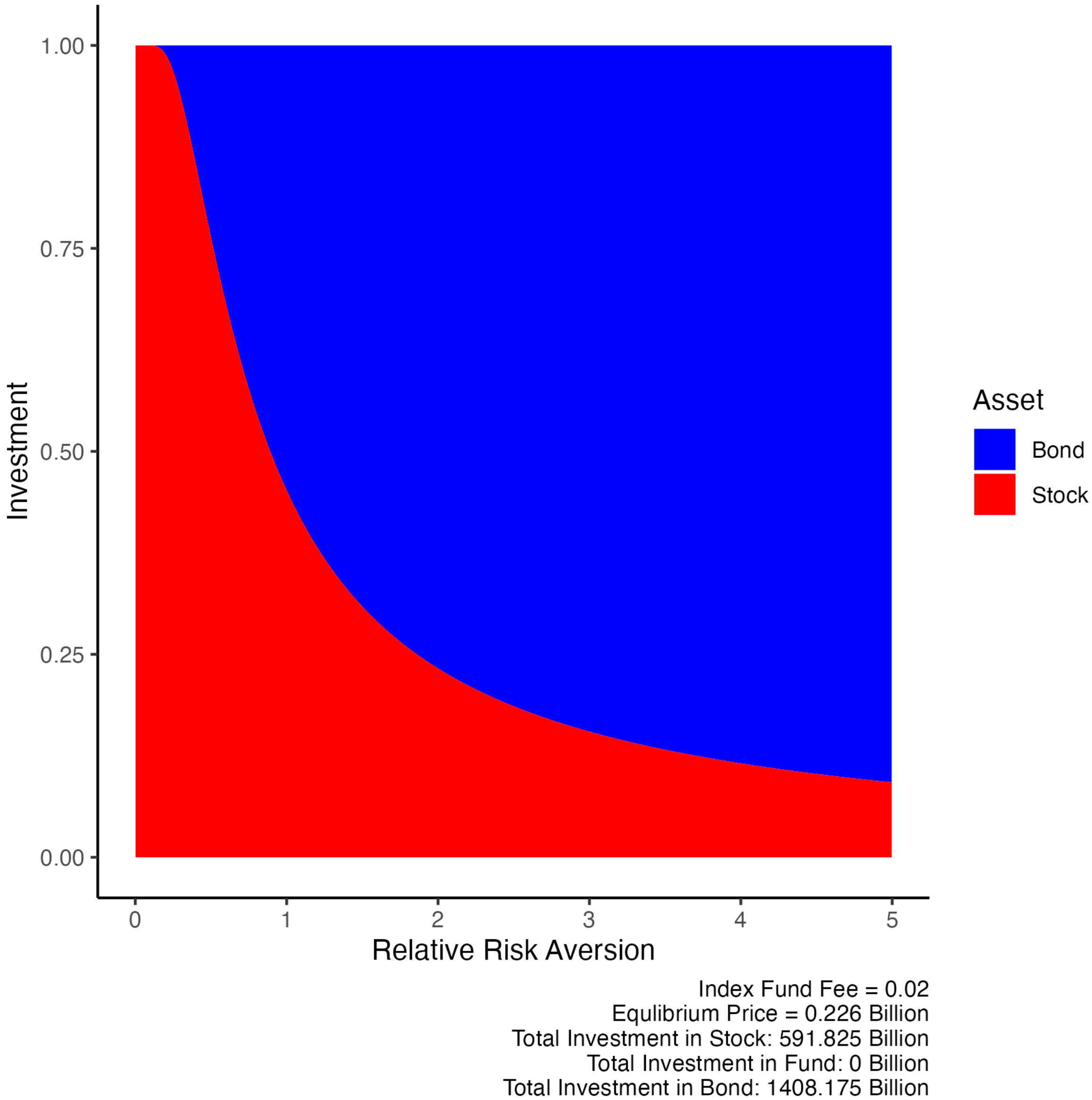

Figure 19. Portfolio Choices without the Fund: CRRA utility; $k = 0.02$

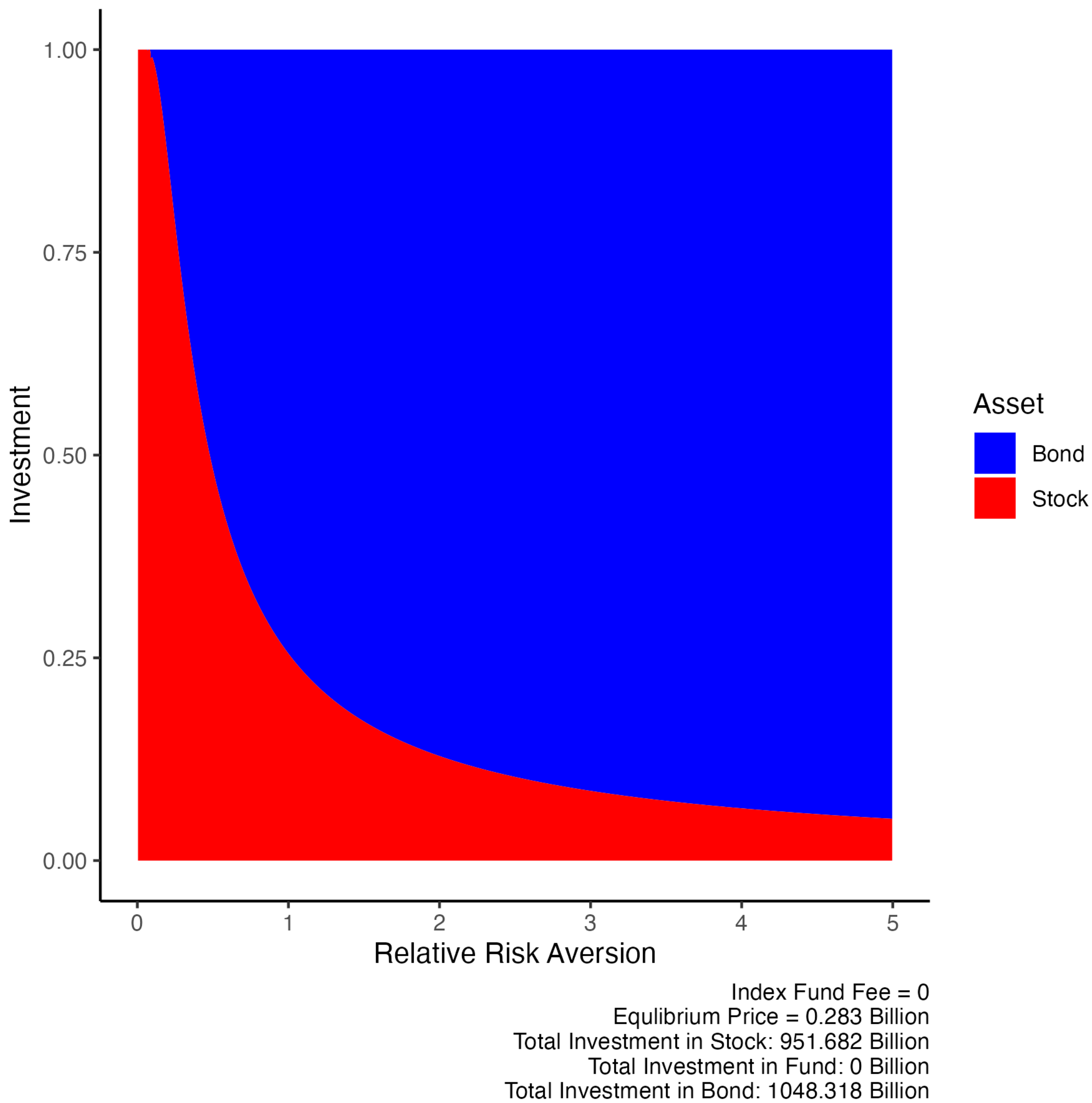

Figure 20. Portfolio Choices without the Fund: CRRA utility; $k = 0.0$

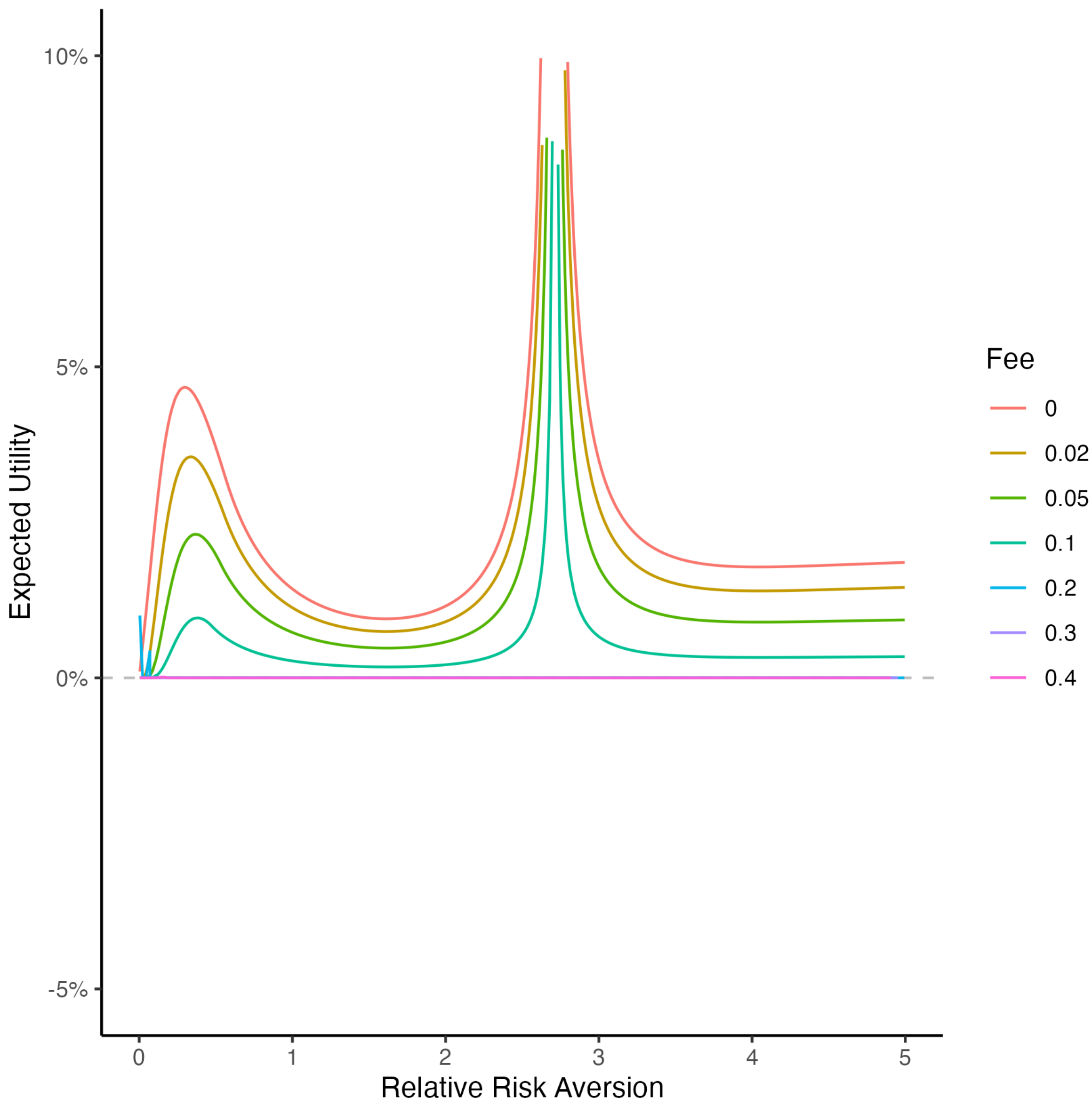

Figure 21. Percentage Utility Gain for the Marginal Investor; CRRA utility

- Investors in the middle class derive the largest benefit because they are quite risk-averse and have sufficient wealth to benefit from the reduction in risk that comes from investing in the Fund.
- The poorest investors benefit substantially in percentage terms from the presence of the Fund – but, because they have little invested wealth, their benefit in absolute terms is in fact small.

6.3.5 The Counterfactual

As we have emphasized, the presence of the Fund has a big effect on asset prices and hence on welfare; ignoring the effect on prices would lead to very wrong conclusions about welfare. To illustrate this point, we show in Figure 22 the counterfactual improvement in investor welfare if the Fund were to become available *but prices did not change*. (We might also view this as the improvement in investor welfare in that period when there was little investment in the Fund and so the influence of the Fund on prices was negligible.)

Figure 22 goes here: to be added

7 Conclusions

This paper studies the equilibrium consequences of the availability and cost of index funds when asset prices are determined endogenously. It shows that the availability of index funds tends to *increase* asset prices and *decrease* investor welfare, and that these effects are greater when the cost indexing is lower. While it is true that the availability of index funds allows small investors to enjoy market returns, at equilibrium, these market returns are *lower* than they would be in a world without index funds. Moreover, the welfare loss that investors experience as a result of these lower market returns may outweigh the welfare gain that they experience from reduced portfolio risk; on balance, the availability of index funds may not benefit any *investors*.

Our paper offers a very simple model and a variety of numerical simulations. In the simulations, the presence of the Fund has a big effect on prices and hence reduces the welfare of investors. We do not claim that our model is realistic; certainly it does not capture all the complexities of the world: active investors and funds, different kinds of firms, etc etc etc. Hence our model does not offer a realistic description of what really happens/happened. What our model does offer is very strong evidence that understanding the effect of index funds requires taking into account that they affect prices and hence welfare.

As noted in the Introduction, this paper does not offer – or intend to offer – any evaluation of policy. However, it does suggest that the policy debate on the impact of index funds (an corporate governance and on welfare) may need to be re-framed to take into account the effect of index funds on asset prices.

The model presented here is deliberately simplified to emphasize these issues. We highlight some of these simplifications – and the reasons for making them – below; Schmalz and Zame (2023) explores the consequences of relaxing (some of) these simplifications.

- We have assumed that the bond yield is fixed, for example by the policy of the central bank policy. An alternative would be to assume that the supply of the bond is fixed, and allow the yield (equivalently, the price or interest rate) to be determined in equilibrium. This alternative would present no challenges for the model or for the existence of equilibrium, but would present a challenge to the simulations, because it would require solving for *two* equilibrium prices, rather than one. Moreover, it is unclear what the supply of the bond should be.

- Perhaps the most significant simplifying assumption we have made is that Fund ownership does not affect the behavior of the firms, but there are a number of reasons that it might. Most obviously: if the Fund owns a significant fraction of the shares of a firm, and votes those shares, then this might lead to improvements in oversight and governance which would reduce the firm's costs. The firm's costs might also be reduced because the increase in the value of the firm (the asset price) might make it cheaper for the firm to raise capital. Reductions in cost would change the firm's production decisions, which would change both its expected profits and its exposure to risk. On the other hand, an increase in diversified ownership of firms may also decrease owners' incentives to hold management accountable, and thus increase production costs (e.g. Antón et al. (2022)) Moreover, because the Fund owns shares in *all* the firms in an industry, this might change firms' objectives (as in Rotemberg (1984)), and so might lead firms to compete less aggressively. All these changes in the behavior of firms will have consequences for profits – and hence for asset prices and the welfare of investors – *as well as for consumers.*

- In our model, there is no trade, hence no initial ownership of firms. A more elaborate model might specify the initial ownership; in such a model, higher asset prices would benefit the initial owners – but it would seem that, just as in the current model, new retail investors, who in the traditional partial-equilibrium narrative are those that benefit from index funds, would also be harmed in equilibrium by the availability of the index fund.

Appendix

We begin by giving a formal proof of the Theorem.

Proof of Theorem We first show that there is an equilibrium in which all firms have the same price p. Because all firms have the same return and we constrain investors to hold the stock in at most one firm, investors are indifferent among all firms. We will therefore adopt a reduced form in which we consider each investor's *total demand* for direct stock holdings and look for an equilibrium price p^*; once we have found such a p^* we can then apportion demands to clear the market for the stock of each firm.

In principle, the price of stock could be arbitrarily large or arbitrarily small. However, if the price p of stock is too small then (because we have assumed $\pi + \Delta$ is bounded away from 00, the rate of return on the Fund will exceed the rate of return on bonds – so no investor will wish to buy bonds. (Investors might prefer to buy stocks rather than the Fund.) Hence the total wealth W of all investors will be spent on stocks – either directly or through the Fund. In either case, the demand for stocks will be W/p. However, if p is small enough then W/p will exceed N; i.e., demand for stock will exceed supply. Similarly, if the price of stock is too large, then the expenditure on stock, which cannot exceed W/p will be strictly less than N i.e., the supply of stock will exceed demands. Hence we can restrict our attention to stock prices in some interval $[\underline{p}, \overline{p}]$.

Thus, or each $p \in [\underline{p}, \overline{p}]$ and each investor $t \in T$, let

$$F^t(p) = \{(x^t_S, x^t_F)\} \subset R^2_+$$

be the set of investor t's optimal choices of total shares of individual stocks and shares of the Fund, assuming that the price of stocks is p. (The holding of Bonds is determined by the budget constraint and the assumption that utility is strictly increasing in certain consumption.) Because investor's utility functions are continuous and choice sets are closed, $F^t(p)$ is a non-empty compact set. Moreover, for each investor t, the correspondence

$$p \mapsto F^t(p)$$

is upper-hemi-continuous. By assumption, the distribution ϕ of consumers is non-atomic. The integral of a correspondence with respect to a non-atomic measure is convex so the correspondence

$$p \mapsto F(p) = \int_T F^t(p)\, d\phi(t)$$

is upper hemi-continuous and convex-valued.

In principle, the price of stock could be arbitrarily large or arbitrarily small. However, if the price of stock is too high (greater than W/N) then the total demand for stock will be less than then supply. Similarly, if the price of stock is too low then, because we have assumed that $\pi + \Delta$ is bounded away from 0, the rate of return on the Fund will be so large that the demand for stock will exceed the supply. Hence we may restrict our attention to stock prices in some interval $[\underline{p}, \overline{p}]$.

By definition, if $x = (x_S, x_F) \in F(p)$ then x_S is the number of shares of stock purchased directly and x_F is the number of shares of stock purchased through the Fund, so $x_S + x_F$ is the total number of shares purchased. Define the correspondence $G : [\underline{p}, \overline{p}] \to \mathbb{R}$ by

$$G(p) = \{x_S + x_F - N : x \in F(p)\}$$

Note that $y \in G$is the market excess demand for stock. It is easily checked that G is upper-semi-continuous and has compact, convex values.

For $y \in (-\infty, +\infty)$ set

$$h(y) = \arg\max_q \{qy : q \in [\underline{p}, \overline{p}]\}$$

Note that

$$\begin{aligned} y < 0 &\Rightarrow h(a) = \underline{p} \\ y > 0 &\Rightarrow h(a) = \overline{p} \\ y = 0 &\Rightarrow h(a) = [\underline{p}, \overline{p}] \end{aligned}$$

and define a correspondence $H : \mathbb{R} \to [\underline{p}, \overline{p}]$ by

$$H(y, \eta) = h(y)$$

It is evident that H is an upper-hemi-continuous correspondence with compact, convex values. Finally, consider the composite correspondence $H \circ G : [\underline{p}, \overline{p}] \to \overline{p}]$. This is an upper-hemi-continuous correspondence with compact, convex values, and so (by Kakutani's fixed point theorem), $H \circ G$ has a fixed point. That is, there is some p^* such that $p^* \in H \circ G(p^*)$. In view of the definition of h and our choices of $\underline{p}, \overline{p}$, market excess demand for stock at price p^* must be 0. We can now apportion demand for the stock in individual firms so each of those markets clears. Hence we have found an equilibrium in which the stock in all firms has the same price, as asserted.

To see the second assertion, consider an equilibrium in which some investors hold stocks directly, and suppose that there are firms n, n' for which $p^*_n > p^*_{n'}$. Because some investors hold stocks directly, the share of stock held by the Fund is strictly less than 1; in particular, some investors must hold stock in firm n. These investors could obtain a better return by holding the same amount stock in firm n' instead and investing the amount saved in bonds, so these investors cannot be optimizing. This is a contradiction, so we conclude that in any equilibrium in which some investors hold stock directly, the prices of all firms must be the same, as asserted. This completes the proof. **QED**

Some comments may be useful. It is evident from the proof that if the demand for stock were decreasing in the price, then the equilibrium price would be unique (although choices might not be). However, it is not obvious that the demand for stocks is necessarily decreasing in the price. To see why, fix an investor t and a price p, assume that investor t optimally chooses to hold shares of stock and of the fund and bonds, and think about the investor's optimal choice at some (slightly) lower price $p' < p$. At this lower price, the investor might find it optimal to shift some investment from the Fund to individual stocks and to hedge the additional risk by also shifting some investment from the Fund to bonds. In that case, the investor's overall demand for stock would *fall*, despite the price being lower. Of course, the same might be true for many investors, so total demand for stocks might also fall; i.e., the demand for stock need not be decreasing in the price. Given this possibility, there seems to be no reason to suppose that the equilibrium price would necessarily be unique. Finding conditions that guarantee uniqueness seems challenging.

We have asserted previously that there can be equilibria in which no investors hold stock directly and stock prices are not uniform. To see this, we sketch two (admittedly extreme) examples (leaving the omitted details to the interested reader).

- Assume $k = 0$ and all investors are strictly risk-averse. In view of the Theorem, there is an equilibrium in which all prices are the same, say p^*. Because $k = 0$ and investors are strictly risk averse, at the price p^*, every investor will *strictly prefer* its equilibrium holding of the Fund and bonds to any alternative that includes a direct holding of stock. Now consider an alternative configuration of stock prices $\{p_n\}$ with the property that $\sum p_n = p^*$. At this alternative configuration of prices, the same holdings of the Fund and bonds will be budget feasible. Moreover, if for each n, the deviation $|p_n - p^*|$ is sufficiently small, then every investor will continue to prefer its equilibrium holding of the Fund and bonds to any alternative that includes a direct holding of stock. Thus, there will be an equilibrium in which stock prices are not uniform.

- Assume $k > 0$, that there is a strictly positive probability that the realization of $\pi + \epsilon + \Delta$ is 0 and that all investors are infinitely risk averse; e.g., for every holding $X = (x_S^t, x_F^t, x_B^t)$ of investor t we have

$$U^t(X) = \inf c$$

where the infimum extends over all realizations c of the holding X. In view of the Theorem, there is an equilibrium in which all prices are the same, say p^*. Because the realization of stock purchased directly might be 0, investors obtain *no utility* from stock held directly, so at equilibrium, no investors hold stock directly. Now consider an alternative configuration of stock prices $\{p_n\}$ with the property that $\sum p_n = p^*$ and assign the same holdings as in the equilibrium with prices all equal to p^*. Because investors obtain no utility from stock held directly, these alternative prices, together with the same holdings, constitute an equilibrium in which stock prices are not uniform.

Finally, we have asserted in the text that it is difficult or impossible to obtain solutions in closed form. A simple example may illustrate this point.

Consider a very simple case:

- all investors have the same wealth: 2 units = \$20,000 and the same coefficient of constant relative risk aversion $s > 0$

- the fee charged by the Fund is $k = 0$

- the expected profit, idiosyncratic risk, aggregate risk, and interest rate on the bond are as in the Numerical Simulations.

Under these assumptions, investors choose not to invest in stock directly, so are not subject to idiosyncratic shocks, but only to the aggregate shock, which is positive half the time and negative half the time. Hence, for a given candidate price p each investor chooses to purchase x_F shares of stock through the Fund and x_B bonds, to maximize

$$(0.5)u_s\Big(\pi + (0.5)\pi x_S + (1.5)x_B\Big) + (0.5)u_s\Big(\pi - (0.5)\pi x_S + (1.5)x_B\Big)$$

subject to the budget constraint

$$px_F + x_B = 2$$

where u_s is CRRA utility with coefficient s. Substituting $x_B = 2 - px_F$ and differentiating with respect to x_F the first-order constraint becomes

$$0 = (0.5)\Big(1.5\pi x_F + (3 - 1.5px_F)\Big)^{-s}[1.5\pi - (1.5)p] + (0.5)\Big(0.5\pi x_F + (3 - 1.5px_F)\Big)^{-s}[0.5\pi - (1.5)p]$$

If we clear denominators we obtain

$$0 = (0.5)\Big[0.5\pi x_F + (3 - 1.5px_F)\Big]^{s}[1.5\pi - (1.5)p] + (0.5)\Big[1.5\pi x_F + (3 - 1.5px_F)\Big]^{s}[0.5\pi - (1.5)p]$$

If s is a positive integer then this is a polynomial equation in p of degree $s+1$. In particular, if $s = 1, 2, 3$, it will be solvable (for the price p) in closed form (because polynomial equations of degree ≤ 4 admit solutions in terms of radicals), but if $s \geq 4$ it will (probably) not be solvable in closed form (because polynomial equations of degree > 4 do not (in general) admit solutions in terms of radicals). But if the coefficient of risk aversion s is not an integer, solving in closed form seems entirely impossible.

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