Too Many Meetings?
Scheduling Rules for Team Coordination

(Authors' names blinded for peer review)

Workers in knowledge-intensive industries often complain of having too many meetings, but organizations still give little thought to deciding when or how often to meet. We investigate the efficiency and robustness of various coordination scheduling rules. We consider workers who are engaged in a common activity (e.g., software programming) that can be divided into largely independent, parallel production tasks, but that necessitates periodic coordination. Coordination enables workers to address issues they have encountered in their independent work, but takes time away from production. Using a stylized game-theoretic model, we show that small teams allow a more fluid, i.e., worker-driven, approach to scheduling coordination, such as preemptive coordination (or production), under which any worker can impose coordination (or production). In larger teams this becomes inefficient. Several approaches can mitigate this effect. One option is to allocate the decision rights to produce or coordinate to the most productive worker. A more general version is to implement a voting-based scheme, where a minimum number of workers from a predetermined subset choose to coordinate. A third approach is to modify the preemptive coordination and production rules by adding time-based controls, to reserve some minimal amount of productive time or to enforce coordination after some point. Finally, a fixed-interval meeting schedule works well for very large teams. Our research helps formalize the tension between meeting (coordinating) and producing, and indicates how to adapt team coordination scheduling rules to the degree of worker heterogeneity and team size.

Key words: Collaboration and coordination models; Game theory; Teams and group decisions; New product design and development

---

1. Introduction

Alice and Bob are responsible for managing a software product. They each have their own domain (e.g., graphical user interface vs. database handling), but they occasionally need to coordinate to ensure the product will work. This might involve negotiating which of several possible approaches to take or which new functions to prioritize—issues that require in-person discussions that cannot be settled with a short email exchange or chat.

Initially they interrupt each other whenever such an issue arises, but they both feel they get nothing done. They agree to meet every Monday morning to iron out any such issues and promise not to interrupt each other the rest of the week. Often they find out on Monday that much of what they had done last week is incompatible with the other's work, so at least one of them wasted most of the previous week. Reluctant to rigidly increase the frequency of their meetings, they agree instead to meet every Monday, then to work independently for at least two days, and to allow interruptions on Thursday and Friday. This works reasonably well; some weeks they end up meeting on Thursday and/or Friday, other times they work independently all week and meet again on Monday. Senior management notices their approach and adopts it for the entire company. Alice and Bob join a much larger team working on a more complex product, but are surprised to see their approach break down: the team spends every Monday, Thursday, and Friday meeting and does not get as much done as it should.

This vignette points to a fundamental challenge in many organizations: agents need some coordination to maintain productivity, but not too much. The complaint about too many meetings is widely heard and not new. Drucker (1967) observed that "one either meets or one works. One cannot do both at the same time." Cross et al. (2016) warn that the benefits of collaboration are clear but the costs are often ignored. Even organizations that value time management tend to view it as an individual rather than institutional issue (Bevins and De Smet 2013), though some have adopted rules like Tommy Hilfiger's "no-meeting Friday" (Staats 2018, p. 92); see also Saunders (2017). Perhaps the most in-depth investigation is the ethnographic work by Perlow (1999), describing experiments with various forms of "quiet time" in a high-tech software firm; despite their success, these rules did not endure after the experiments concluded.

Various practitioner-oriented blogs and books address meeting frequency. One focuses on the optimal cadence,[1] not taking into account the workers' needs. Another[2] argues that stochastic environments call for a more fluid approach to meeting scheduling. Nelson Repenning[3] suggests that meeting frequency should be linked to workers' needs. Computer scientist Cal Newport emphasizes the importance of uninterrupted time (Newport 2016) and of having protocols for how to schedule meetings (Newport 2021). Schwarz (2017) recognizes that coordination can range from standardized to dynamic and should depend on the interdependence between workers. A mismatch might reveal itself through complaints about too many meetings or about lack of communication. Perlow et al. (2017) observe that even well-run meetings become too frequent as an organization grows, often forcing workers to sacrifice their personal time to get work done, endangering their well-being and organizational success.

A large and growing literature in operations management (OM) and organizational behavior (OB), originating from Simon (1947), examines the need for and the costs of coordination in teams. One might be able to reduce coordination demands by spending more time upfront on planning and specifications. However, once the need to coordinate and the costs of doing so are given, this formal and informal literature says relatively little about several fundamental questions related to coordination scheduling:

[1] https://blog.lucidmeetings.com/blog/how-often-should-you-meet-selecting-the-right-meeting-cadence-for-your-team

[2] https://robinpowered.com/blog/how-meeting-coordination-is-replacing-reservation-systems-in-the-workplace/

[3] https://mitsloan.mit.edu/ideas-made-to-matter/5-ways-to-avoid-ineffective-meetings

- How often should agents coordinate? How should they decide when to coordinate?
- How well do simple worker-driven coordination scheduling rules work if agents have different levels of productivity and coordination demands? How does it depend on team size?
- How do such rules compare with simple fixed-interval rules, such as weekly meetings?

We develop a simple game-theoretic framework to address these questions. By focusing on coordination scheduling, we address the symptom rather than the root causes of the need for coordination. Our stylized approach aims to uncover some key dynamics of these settings, to stimulate further research on prescriptive insights, perhaps using numerical or simulation-based methods.[4]

We consider workers who are engaged in a common activity (e.g., software programming) that can be divided into largely independent, parallel production tasks, but that necessitates periodic coordination. As a base case, we consider in §4 two workers operating in a discrete-time, infinite-horizon, stationary environment with a binary productivity function, meaning that as soon as a worker encounters an issue requiring coordination, her productivity drops to zero. (We explore more general productivity functions in Appendix B.) In each period, workers produce or coordinate to maximize their individual future discounted sum of value from production across time periods. When workers coordinate, all pending issues are resolved, restoring productivity to its highest level. The workers thus face a trade-off between generating output, perhaps not both at full productivity if they have accumulated issues, or spending this period coordinating to both generate output at full productivity later. We explore several worker-driven coordination scheduling rules:

- Preemptive Coordination (PC), where any worker can interrupt the other at any time—this is similar to an open-door policy;
- Preemptive Production (PP), where coordination occurs only when all workers want it—this is closer to a closed-door policy;
- Hierarchical Structure (HS), where one designated worker decides when coordination occurs.

When the workers' productivity function is binary, each of these rules can achieve the first-best (FB) outcome for certain parameter values. The productivity loss from choosing the wrong rule can be quite large, especially when the workers have heterogeneous coordination demands and productivity levels. For instance, under PC, a worker who adds little value but frequently needs coordination will severely limit the more valuable worker's output. All coordination scheduling rules lead to outcomes with which some workers might disagree. For instance, coordination may occur even when one worker would like to keep producing. That worker may perceive the meeting

[4] Meetings may have different purposes (e.g., information sharing, decision making), requiring different scheduling rules. We abstract away from this finer-grained classification of meetings by framing them generically as coordination meetings whose purpose is to resolve pending issues that prevent a worker from being productive. We recognize other types of coordinating meeting in the conclusions.

Table 1 Categorization of scheduling rules for team coordination

	Time-Independent	Time-Dependent
Not Worker-Driven	always (every period) or never	Fixed Interval (FI)
Worker-Driven	Preemptive Coordination (PC) Preemptive Production (PP) Hierarchical Structure (HS)	PC with Minimum Cycle Duration (PC-C^{min}) PP with Maximum Cycle Duration (PP-C^{max}) HS with Min/Max Cycle Duration (HS-C^{min}, HS-C^{max})

as a waste of time but that ignores the benefits for the team (Appendix A). Of course, when workers are rewarded for the team's production, not just their own, their incentives are aligned and such disagreement disappears (Appendix C).

To mitigate that productivity loss, we consider worker-driven coordination scheduling rules enhanced with time-based controls or "safeguards," such as a minimum production cycle duration (under PC or HS) before coordination is allowed (PC-C^{min} or HS-C^{min}), or a maximum production cycle duration (under PP or HS) after which coordination is required (PP-C^{max} or HS-C^{max}). These enhanced rules substantially reduce the suboptimality gap experienced by their basic counterparts.

Larger teams pose further challenges, explored in §5: PC, PP, and HS can again lead to substantial productivity loss relative to FB. In a large team under PC, some worker will almost always want to coordinate, so the team rarely produces. This is consistent with the coordination neglect hypothesized by Heath and Staudenmayer (2000) and the related team scaling fallacy observed by Brooks (1975) and Staats et al. (2012). As with small teams, adding safeguards helps, especially for PC and PP. A voting-based scheme, where a minimum of workers from a predetermined subset need to agree to coordinate, can also mitigate the effect of larger team size.

We then analyze in §6 the purely time-based fixed-interval (FI) rule, such as standing weekly meetings. Its suboptimality loss is limited to 28% across all team sizes; whether this is a large or modest gap will depend on the context. For larger teams, the FI rule outperforms PC, PP, and HS. The enhanced versions of PC and PP (PC-C^{min} and PP-C^{max}) outperform FI, but as team size increases they converge towards FI. In light of our stylized analysis, these are not meant as immediate practical prescriptions, but initial insights into the trade-offs involved.

We classify the rules according to whether they are based on the workers' stated preferences and/or the time since the last coordination, in the 2-by-2 matrix in Table 1. PC, PP, and HS are worker-driven but time-independent; FI is not worker-driven but time-based. Rules that account for both workers' stated preferences and time, by adding a time-based coordination constraint (C^{min} or C^{max}) to a worker-driven rule (PC, PP, or HS), can do better, but not always. Rules that are neither worker-driven nor time-dependent (always or never coordinate) are obviously suboptimal.

2. Literature Review

Our work is related to research in OM and OB, as well as to the large popular literature on time management. Research on time allocation in economics and OB focuses mostly on choosing between work vs. leisure. A smaller literature in OB, often descriptive or experimental, focuses specifically on time at work. A key study, which helped to inspire our research, is the ethnographic work by Perlow (1999), describing the frustration of engineers in a high-tech software firm who were constantly interrupted by requests for help from others. Her analysis of their logs showed they rarely got the blocks of uninterrupted time they needed to do their engineering work, due to constant interactive activities. Those interactions were almost always helpful, but 95% occurred spontaneously while 86% could have been planned for a later time with no negative consequences. The firm first tried "quiet time," during which engineers did not interrupt each other, three days a week until noon, and then switched to designating every day between 11am and 3pm as "interaction time," before switching back to "quiet time" in response to engineers' feedback. This appeared effective, but did not persist after the experiment concluded, which suggests that allowing unlimited interruptions is a dominant default coordination mode. Our introductory vignette is motivated by this experiment.

Several studies look at the effect of interruptions. Tucker and Spear (2006) examine how processes can be redesigned to reduce the impact of the frequent interruptions experienced by nurses. Coordination is less frequently needed in more stable teams (Narayanan et al. 2011), with important nuances on the team members' degree of specialization, diversity, and familiarity (Huckman and Staats 2011), or between teams that work on decoupled parts of a product (Sosa et al. 2004). We focus here on the symptom and not the root cause of interruptions. Not all interruptions are detrimental (Jett and George 2003), including informal interactions (Metiu and Rothbard 2013). (Our model does not preclude those but we assume extensive coordination only takes place as specified by the prevailing rule.) Among their benefits, meetings coordinate expertise (Faraj and Sproull 2000), engender relational coordination (Gittell 2002), and enhance knowledge (Kotlarsky et al. 2008). Similarly, we assume that coordination boosts future productivity.

Coordination needs are often unpredictable, especially in agile environments (Wiesche 2018), making time management planning less effective than contingency planning (Parke et al. 2018). The daily scrum meetings associated with agile development are a mechanism to flag coordination needs; they are too short to hash out issues in depth. The daily 15-minute huddles at Intermountain Healthcare (Harrison 2018) are similarly aimed more at conveying relevant information up and down the hierarchy quickly, rather than tackling a specific issue in depth. This motivates us to study worker-driven coordination scheduling rules (Table 1). In lean software development, Staats et al. (2011) suggest that identifying problems early is important; this again raises the question of when one worker should be allowed to interrupt another to resolve a newly-emerged question. Gurvich et al. (2020) report that interruptions may result in significant changeover times and propose to batch them—an integral part of our model.

There is an extensive OB literature on team performance, but very little on the operational issue of coordination within teams. Heath and Staudenmayer (2000) argue that aligning goals (the agency problem) has received far more attention than organizing individuals (the coordination problem). The books by McGrath and Kelly (1986) and McGrath and Tschan (2004) include brief discussions of "synchronization" and "entrainment" in groups, but do not translate them into guidelines for coordination frequency. Mohammed and Nadkarni (2011) find that how team leaders schedule and synchronize tasks among members affects performance, which suggests that coordination scheduling rules do matter. We contribute to this work by developing a formal framework for analyzing coordination scheduling rules. Our simple model does not capture the richness of these practical settings, but our approach can inform further numerical studies that do.

Most OM work on team coordination relates to New Product Development (NPD) or project management. The NPD literature has traditionally adopted a macroscopic perspective on teamwork. Ha and Porteus (1995) consider the trade-offs in concurrent design: more frequent reviews allow the process and product design teams to detect potential design flaws earlier, but at the cost of more time spent on the reviews. Others have examined how to manage information exchange in a concurrent process (Krishnan et al. 1997, Özkan-Seely et al. 2015) and the optimal degree of concurrency (Loch and Terwiesch 1998, Roemer and Ahmadi 2004). This literature investigates the trade-off between sequential and reciprocal dependency, following the typology by Thompson (1967). In contrast, we consider pooled dependency, in which agents do not directly depend on one another. Adopting an organization-wide perspective, Mihm et al. (2003), Mihm et al. (2010), and Sting et al. (2020) use simulations to study the challenges of coordination in distributed search and potential remedies such as setting up hierarchies and distributing knowledge. We consider a more operational aspect of coordination, scheduling, similar to Thomke and Bell (2001) who consider testing as a mechanism to reduce outstanding design problems in a single development process (in contrast to our ongoing parallel activities). We adopt a more microscopic perspective, viewing a team as a collection of autonomous agents working in parallel on distinct tasks, but requiring periodic coordination due to the stochastic nature of the environment.

Taking a similar microscopic view, recent literature on project management studies the dynamics of collaboration between agents. To maximize a team's collaboration potential, different levers are considered such as deadlines (Bonatti and Hörner 2011), milestones (Rahmani et al. 2017), monitoring (Georgiadis 2015), and leadership style (Rahmani et al. 2018). This literature is mostly concerned with the co-productive nature of the project, in which a single output results from the agents' joint input. We focus instead on the coordination dynamics between agents. Gurvich and Van Mieghem (2015, 2018) study how the need for collaboration can cause a network to have lower capacity than its bottleneck resource, and show that in certain settings policies that prioritize collaboration perform better. Their modeling framework is a continuous-time network with discrete tasks, while we assume a continuous stream of work in discrete time periods. Siemsen et al. (2007) and Crama et al. (2019) distinguish between help, knowledge sharing, and co-productive linkages. Crama et al. (2019) analytically model the "red card" system observed by Sting et al. (2015), who highlight the importance of psychological safety to encourage engineers to ask for help (using red cards) when their tasks are becoming critically delayed. Although our focus is not on accelerating project completion, the same concern about reluctance to interrupt others applies; explicit coordination scheduling rules can help mitigate that concern. This literature typically models investments in coordination as unconstrained (but costly) efforts. We contribute to that literature by explicitly modeling coordination as an activity that takes time at the expense of production.

The vast practical literature on time management for individuals, including Drucker (1967), MacKenzie (1972), and Griessman (1994), typically does not examine the organizational ramifications of protecting one's time. Several analytical papers on time allocation, including Radner and Rothschild (1975), Seshadri and Shapira (2001), and Yoo et al. (2016), also ignore interactions between individuals. Recent collaboration software includes mechanisms for avoiding interruptions, e.g., the Slack's Do-Not-Disturb hours or Basecamp's Focus Mode, but the academic and practitioner literatures to date provide minimal guidance on how organizations should use them.

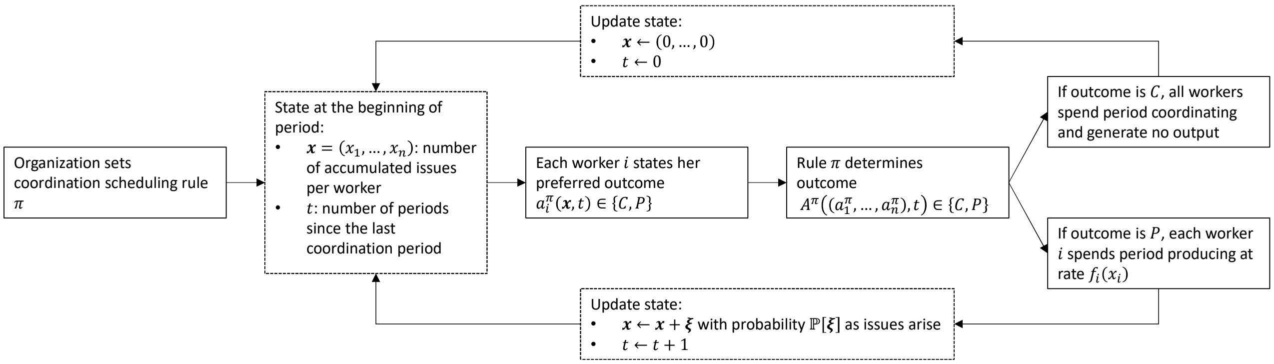
3. Model

We consider a team of n workers (e.g., software developers) involved in a common activity, which is divided into largely independent, parallel tasks (e.g., user interface vs. database, or iOS vs. Android versions) but necessitates periodic coordination (e.g., to link objects to each other). While producing independently, workers may encounter issues which require dedicated coordination time to resolve. (We do not consider minor issues that can be resolved in a quick conversation or email exchange.) Coordination improves future productivity by resolving the accumulated issues, but takes time away from production.

We model this complex situation in the stylized way shown in Figure 1. Time is discrete; a period corresponds to a few hours of work. In each period, workers engage in either Production (P) or Coordination (C). During a production period, the value of worker i's output, or productivity, $f_i(x_i)$, depends on the number x_i of accumulated issues, with $f_i(0) > 0$ and $f_i'(x_i) \leq 0$; there is a number of issues $\overline{x}_i$ beyond which the value of the worker's production is zero, i.e., $f_i(x_i) = 0$ for all $x_i \geq \overline{x}_i > 0$. We refer to x_i as the worker's (coordination) "needs". We assume workers care only about their own productivity. In Appendix C we briefly consider team incentives.

The likelihood of encountering additional issues is stationary and independent of the current stock of issues. Let $\mathbb{P}[\boldsymbol{\xi}]$ be the probability distribution of $\boldsymbol{\xi} \doteq (\xi_1, \ldots, \xi_n) \geq \mathbf{0}$ issues arising during

Figure 1 **Timeline**



any production period. By coordinating, workers get all their issues resolved, akin to the full-fidelity test in Thomke and Bell (2001), so worker i's productivity after coordination is reinitialized to $f_i(0)$, for all $i = 1, \ldots, n$. Coordination takes one period, irrespective of the accumulated issues, has no setup time beyond the period spent coordinating, and requires the participation of all workers. There is no monetary cost associated with production or coordination; only time matters. Time is discounted by $\delta \in (0,1)$, which introduces a trade-off between generating output now at rate $f_i(x_i)$ and coordinating to generate output in the next period at the maximum productivity $f_i(0)$. Relative to the rate at which issues emerge, the activity has a long-term horizon, which we approximate as infinite (as in, e.g., Bonatti and Hörner 2011). These assumptions are made for simplicity to generate preliminary insights into the coordination dynamics.

At the start of a period, the state is $(\mathbf{x}, t)$, where $\mathbf{x} = (x_1, \ldots, x_n)$ is the workers' needs and t is the time since the last coordination period. The period immediately following a coordination period has $t = 0$. At the beginning of each period, every worker states their preferred outcome, i.e., whether they prefer to coordinate or produce, yielding a vector $\mathbf{a} = (a_1, \ldots, a_n) \in \{C, P\}^n$.

A *coordination scheduling rule* π is a mapping from any set of workers' preferred outcomes $\mathbf{a} \in \{C, P\}^n$ and any elapsed time since coordination $t \geq 0$ to an outcome (C or P). The outcome implemented under rule π when given $(\mathbf{a}, t)$ will be denoted as $A^\pi(\mathbf{a}, t) \in \{C, P\}$. We omit the argument $\mathbf{a}$ whenever π does not depend on workers' preferences and omit t whenever it is time-independent. Of particular interest are the following coordination scheduling rules:

- Preemptive Coordination (PC): coordinate if and only if any worker chooses to, i.e.,

$$A^{\mathrm{PC}}(\mathbf{a}) = C \Leftrightarrow \sum_{i=1}^{n} \mathbb{1}[a_i = C] \geq 1, \tag{1}$$

  in which $\mathbb{1}[X]$ is the indicator function, equal to 1 if X is true and zero otherwise.

- Preemptive Production (PP): produce if and only if any worker chooses to, i.e.,

$$A^{\mathrm{PP}}(\mathbf{a}) = P \Leftrightarrow \sum_{i=1}^{n} \mathbb{1}[a_i = P] \geq 1. \tag{2}$$

- Hierarchical Structure (HS_j): decision delegated to worker j, i.e.,

$$A^{HS_j}(\mathbf{a}) = C \Leftrightarrow a_j = C. \tag{3}$$

  Whenever the choice of the worker who is allocated all decision rights is arbitrary, we will identify her as worker 1 and will simply refer to this coordination scheduling rule as HS.

- Fixed Interval (FI): coordinate every $T \geq 2$ periods, i.e.,

$$A^{FI}(t) = C \Leftrightarrow t \geq T - 1. \tag{4}$$

  Coordinating every two periods means each coordination period is followed by production and vice versa.

Under PC, any worker is free to disrupt the production of the others and call for a coordination meeting. This is similar to an open-door policy or one under which workers are expected to answer phone calls from co-workers regardless of what they are doing. Under PP, coordination happens only if all workers want it, similar to a closed-door policy or one that lets the "Do Not Disturb" status on Microsoft Teams signal that one is strictly unavailable. HS assigns the decision rights to coordinate or produce to one worker. And FI specifies a predetermined meeting frequency.

While the first three rules are worker-driven, the fourth is time-based. Under the worker-driven rules, coordination need never happen, or can occur every other period, depending on the probability of issues and the rule in place. Later, we introduce variations of these rules that are both worker-driven and time-dependent, to add constraints on how frequently or rarely coordination occurs. We did not seek to capture the policies in Perlow (1999) perfectly, but the spirit of PP is the same as "quiet time", while that of PC mirrors "interaction time." The policies in Perlow (1999) are hybrids, specifying alternating periods during which production or coordination dominate, which is closer to the worker-driven and time-based variations that we introduce later. Obviously, other rules exist, such as coordinating if and only if at least k workers from a subset $\mathcal{S}$ want it, for some given $\mathcal{S} \subseteq \{1, \ldots, n\}$ and $k \in \{1, \ldots, |\mathcal{S}|\}$, i.e., $A^{\pi}(\mathbf{a}) = C \Leftrightarrow \sum_{i \in \mathcal{S}} \mathbb{1}[a_i = C] \geq k$. This k-out-of-$\mathcal{S}$-votes coordination rule is a generalization of PC, PP, and HS_j. Table 2 lists all coordination scheduling rules considered in the paper.

To avoid trivialities, we only consider rules where coordination is always followed by production, i.e., $A^{\pi}(\mathbf{a}, 0) = P$ for any $\mathbf{a}$ (otherwise workers would never produce after coordinating), and that are monotone in the following sense: for any $\mathbf{a}$, $A^{\pi}(\mathbf{a}, t) = C \Rightarrow A^{\pi}(\mathbf{a}', t') = C$ for all $\mathbf{a}'$ such that for all i, $a_i = C \Rightarrow a'_i = C$, and for all $t' \geq t$. If C is the outcome associated with a set of workers wanting to coordinate t periods since the last coordination period, then C will also occur with any superset of workers wanting to coordinate at any longer time since last coordination. These reasonable requirements are fulfilled by all the rules above.

Table 2 Examples of coordination scheduling rules

Name	Definition	Worker-driven	Time-based
Preemptive Coordination (PC)	$A^{\mathrm{PC}}(\mathbf{a}) = C \Leftrightarrow \sum_{i=1}^n \mathbb{1}[a_i = C] \geq 1$	Y	N
Preemptive Production (PP)	$A^{\mathrm{PP}}(\mathbf{a}) = P \Leftrightarrow \sum_{i=1}^n \mathbb{1}[a_i = P] \geq 1$	Y	N
Hierarchical Structure (HS_j)	$A^{\mathrm{HS}_j}(\mathbf{a}) = P \Leftrightarrow a_j = P$	Y	N
k-out-of-$\mathcal{S}$-Votes Coordination ($k\mathcal{S}$V)	$A^{k\mathcal{S}\mathrm{V}}(\mathbf{a}) = C \Leftrightarrow \sum_{i\in\mathcal{S}} \mathbb{1}[a_i = C] \geq k$	Y	N
Fixed Interval (FI)	$A^{\mathrm{FI}}(t) = C \Leftrightarrow t \geq T-1$	N	Y
PC with Min Cycle Duration (PC-$\mathrm{C^{min}}$)	$A^{\mathrm{PC\text{-}C^{min}}}(\mathbf{a},t) = C \Leftrightarrow t \geq T-1$ and $\sum_{i=1}^n \mathbb{1}[a_i = C] \geq 1$	Y	Y
PP with Max Cycle Duration (PP-$\mathrm{C^{max}}$)	$A^{\mathrm{PP\text{-}C^{max}}}(\mathbf{a},t) = P \Leftrightarrow t < T-1$ and $\sum_{i=1}^n \mathbb{1}[a_i = P] \geq 1$	Y	Y
HS with Min Cycle Duration (HS_j-$\mathrm{C^{min}}$)	$A^{\mathrm{HS}_j\text{-}\mathrm{C^{min}}}(\mathbf{a},t) = C \Leftrightarrow t \geq T-1$ and $a_j = C$	Y	Y
HS with Max Cycle Duration (HS_j-$\mathrm{C^{max}}$)	$A^{\mathrm{HS}_j\text{-}\mathrm{C^{max}}}(\mathbf{a},t) = P \Leftrightarrow t < T-1$ and $a_j = P$	Y	Y

Worker i's future discounted value (or "value-to-go") under rule π in state $(\mathbf{x},t)$, denoted $V_i^\pi(\mathbf{x},t)$, equals $\delta V_i^\pi(\mathbf{0},0)$ if the outcome is to coordinate and $f_i(x_i) + \delta\mathbb{E}[V_i^\pi(\mathbf{x}+\boldsymbol{\xi},t+1)]$ if the outcome is to produce, in which $\mathbb{E}[V_i^\pi(\mathbf{x}+\boldsymbol{\xi},t+1)] \doteq \sum_{\boldsymbol{\xi}\geq\mathbf{0}} \mathbb{P}[\boldsymbol{\xi}] V_i^\pi(\mathbf{x}+\boldsymbol{\xi},t+1)$.

We focus on Markov-perfect equilibria. Although there exist in principle many such equilibria (Maskin and Tirole 2001), we focus on those in which workers play dominant strategies. Specifically, for each worker i, a *dominant strategy* $a_i^\pi(\mathbf{x},t) \in \{C,P\}$ is a mapping from a state $(\mathbf{x},t)$ to worker i's preferred outcome irrespective of the rule in place in the current period, but assuming that rule π applies subsequently. In case worker i is indifferent between coordinating and producing, we assume (without loss of generality) that she prefers coordinating. Accordingly,

$$a_i^\pi(\mathbf{x},t) = C \Leftrightarrow \delta V_i^\pi(\mathbf{0},0) \geq f_i(x_i) + \delta\mathbb{E}\left[V_i^\pi(\mathbf{x}+\boldsymbol{\xi},t+1)\right]. \tag{5}$$

As a result, the equilibrium $\mathbf{a}^\pi(\mathbf{x},t) \doteq (a_1^\pi(\mathbf{x},t),\ldots,a_n^\pi(\mathbf{x},t))$ can be computed by solving n independent dynamic programs. We show later that they can sometimes be solved in closed form. Under coordination scheduling rule π, worker i's value-to-go in state $(\mathbf{x},t)$ is:

$$V_i^\pi(\mathbf{x},t) = \begin{cases} \delta V_i^\pi(\mathbf{0},0) & \text{if } A^\pi(\mathbf{a}^\pi(\mathbf{x},t),t) = C \\ f_i(x_i) + \delta\mathbb{E}\left[V_i^\pi(\mathbf{x}+\boldsymbol{\xi},t+1)\right] & \text{if } A^\pi(\mathbf{a}^\pi(\mathbf{x},t),t) = P. \end{cases} \tag{6}$$

The total value-to-go of all workers is $V^\pi(\mathbf{x},t) \doteq \sum_{i=1}^n V_i^\pi(\mathbf{x},t)$. We use the shorthand $V_i^\pi \doteq V_i^\pi(\mathbf{0},0)$ and $V^\pi \doteq \sum_{i=1}^n V_i^\pi$. With a slight abuse of notation, let $A^\pi(\mathbf{x},t) \doteq A^\pi(\mathbf{a}^\pi(\mathbf{x},t),t)$ be the *policy* induced by rule π in equilibrium, i.e., the mapping from state $(\mathbf{x},t)$ to outcomes in $\{C,P\}$. We use the term "policy" in the dynamic programming sense, not implying an organizational policy.

We aim to identify a simple coordination scheduling rule π that maximizes the total discounted value V^π subject to (5) and (6), only using knowledge of the workers' stated preferences $\mathbf{a}$ and the elapsed time since coordination t. As a benchmark, we consider the "first-best" (FB) policy, which maximizes the total value, under full knowledge of the accumulated number of issues. The FB policy turns out to be only dependent on $\mathbf{x}$, not on t, and we denote it as $A^{\mathrm{FB}}(\mathbf{x})$. Accordingly,

$$A^{\mathrm{FB}}(\mathbf{x}) = C \Leftrightarrow \delta V^{\mathrm{FB}}(\mathbf{0}) \geq \sum_{i=1}^n f_i(x_i) + \delta\mathbb{E}\left[V^{\mathrm{FB}}(\mathbf{x}+\boldsymbol{\xi})\right], \tag{7}$$

in which $V^{\mathrm{FB}}(\mathbf{x})$ is given by

$$V^{\mathrm{FB}}(\mathbf{x}) = \begin{cases} \delta V^{\mathrm{FB}}(\mathbf{0}) & \text{if } A^{\mathrm{FB}}(\mathbf{x}) = C \\ \sum_{i=1}^{n} f_i(x_i) + \delta \mathbb{E}\left[V^{\mathrm{FB}}(\mathbf{x}+\boldsymbol{\xi})\right] & \text{if } A^{\mathrm{FB}}(\mathbf{x}) = P. \end{cases} \tag{8}$$

Similar to the machine maintenance planning problem (Sasieni 1956), the FB policy is a threshold policy. In particular, it is optimal to produce if and only if $\sum_{i=1}^{n} f_i(\mathbf{x}) > \phi$ for some ϕ.

Proposition 1. *There exists a threshold* $\phi \in [0, \sum_{i=1}^{n} f_i(0))$ *such that* $A^{FB}(\mathbf{x}) = P \Leftrightarrow \sum_{i=1}^{n} f_i(x_i) > \phi$, *and* $V^{FB} = \phi/(\delta(1-\delta))$.

All proofs appear in Electronic Companion §EC.2. Although the FB policy is simple to describe, it may not be practical because it requires full knowledge of the workers' needs, which may be impossible to verify. To simplify the analysis, we make the following assumptions:

Assumption 1. *(i) The transition probabilities are independent, i.e.,* $\mathbb{P}[\boldsymbol{\xi}] = \prod_{i=1}^{n} \mathbb{P}_i[\xi_i]$, *and, for each worker, at most one issue arises per period i.e.,* $\mathbb{P}_i[1] = 1 - \mathbb{P}_i[0]$; *let* $p_i \doteq \mathbb{P}_i[1]$.
(ii) $\overline{x}_i = 1$ *for all* $i = 1, \ldots, n$, *i.e.,* $f_i(x_i) = 0$ *for all* $x_i \geq 1$ *for all* $i = 1, \ldots, n$; *let* $v_i \doteq f_i(0)$.

Under Assumption 1(ii), it is optimal for worker i to want to produce if and only if she is fully productive (Lemmas EC.1-EC.2 in §EC.2). In our model, each worker continues producing, even if their productivity has declined to 0. In a more general model, workers could decide to stop working once they are unproductive; that is payoff-equivalent to our model under Assumption 1(ii). Accordingly, worker i's stated preference, which solves the dynamic program (5), simplifies to:

$$a_i^{\pi}(\mathbf{x}, t) = P \Leftrightarrow x_i = 0 \quad \forall i. \tag{9}$$

Hence, under Assumption 1(ii), there is a one-to-one relationship between workers' needs and their stated preferences, which are independent of both the time since coordination and the other workers' needs. We refer to the likelihood of worker i encountering an issue, p_i, as her *coordination demands*. Workers differ in terms of their coordination demands p_i and productivity v_i.

We relax Assumption 1(ii) in Appendix B. When the productivity function is binary but has a base value, i.e., when $f_i(x_i) = b_i < v_i$ when $x_i > 0$, with $b_i > 0$, worker i may want to produce even if she has accumulated issues, provided that b_i is large enough, contrary to Lemma EC.1.

An alternative approach could be to reward workers for their team performance. In Appendix C, we consider replacing the in-period reward in (5) and (6) from $f_i(x_i)$ to $\gamma f_i(x_i) + (1-\gamma)\sum_{j \neq i} f_j(x_j)$, with $1/2 \leq \gamma \leq 1$. With team incentives, Lemmas EC.1 and EC.2 may no longer hold since a worker may wish to produce even if she has accumulated issues (if her coworkers are productive) and to coordinate even if she is fully productive (if her coworkers have accumulated issues). When $\gamma = 1/2$, workers' strategies are fully aligned and the workers always agree to implement the FB policy.

4. Teams of Two Workers

We first consider a team of only two workers to motivate the PC, PP, and HS worker-driven coordination scheduling rules (1)-(3). Throughout this section, let $-i \doteq 3-i$ for $i \in \{1,2\}$.

4.1. First-Best Policy

By Proposition 1, the FB policy prescribes production if and only if $f_1(x_1)+f_2(x_2)$ exceeds some threshold ϕ. For binary productivity functions (Assumption 1), where $f_i(0)=v_i$, assuming, without loss of generality, that $v_1 \geq v_2$, this gives rise to three ranges of interest for ϕ: $(0,v_2]$, $(v_2,v_1]$, and $(v_1,v_1+v_2]$. To characterize the FB policy we define the following functions:

$$\alpha(p_1,p_2,\delta) \doteq \frac{\delta}{1-\delta(1-p_1)(1-p_2)} \text{ and } \beta(p_1,p_2,\delta) \doteq \frac{1+\delta p_1}{\delta} - \frac{1-\delta(1-p_1)}{\delta}\alpha(p_1,p_2,\delta). \qquad (10)$$

It turns out that $\alpha(p_1,p_2,\delta) \geq 1$ if and only if $\beta(p_1,p_2,\delta) \leq 1$. Hence, if $v_1/v_2 \geq 1$, then $v_1/v_2 \leq \alpha(p_1,p_2,\delta) \Rightarrow v_1/v_2 \geq \beta(p_1,p_2,\delta)$. The FB policy can be expressed in terms of three ranges for v_1/v_2.

Proposition 2. *Under Assumption 1 when $n=2$, the FB policy is the following: When $v_1 \geq v_2$:*

- *If $\frac{v_1}{v_2} \leq \alpha(p_1,p_2,\delta)$, Produce in $(0,0)$, Coordinate otherwise (if anybody has an issue);*
- *If $\frac{v_1}{v_2} \leq \beta(p_1,p_2,\delta)$, Produce in $(0,0)$, $(0,x_2)$ for any $x_2 \geq 1$, and $(x_1,0)$ for any $x_1 \geq 1$, Coordinate otherwise (if everybody has an issue);*
- *If $\frac{v_1}{v_2} \geq \alpha(p_1,p_2,\delta)$ and $\frac{v_1}{v_2} \geq \beta(p_1,p_2,\delta)$, Produce in $(0,0)$, $(0,x_2)$ for any $x_2 \geq 1$, Coordinate otherwise (if worker 1 has an issue).*

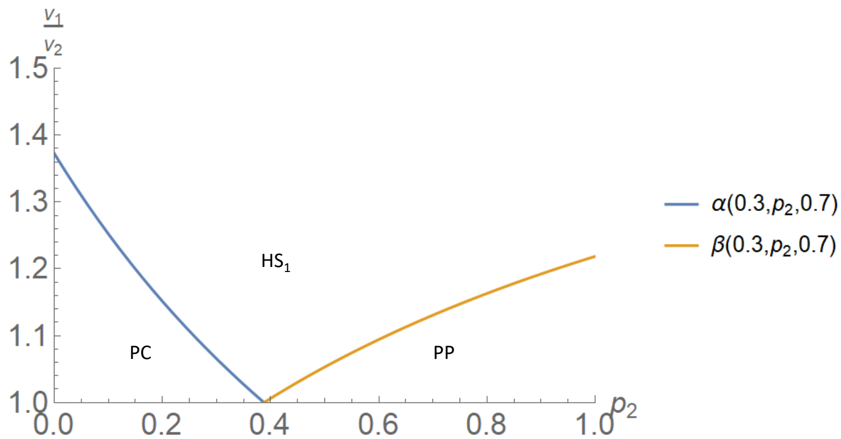
This means that (when $v_1 \geq v_2$) the FB can be achieved by PC if $\frac{v_1}{v_2} \leq \alpha(p_1,p_2,\delta)$, by PP if $\frac{v_1}{v_2} \leq \beta(p_1,p_2,\delta)$, and by HS_1 if $\frac{v_1}{v_2} \geq \alpha(p_1,p_2,\delta)$ and $\frac{v_1}{v_2} \geq \beta(p_1,p_2,\delta)$. Given that PC, PP and HS achieve the FB outcome for certain regions when $n=2$ and under Assumption 1, they are natural candidates in more general settings, e.g., larger teams or general productivity functions.

Figure 2 illustrates Proposition 2 by depicting where each rule is optimal when $v_1 \geq v_2$, $p_1=0.3$, and $\delta=0.7$. The space $(p_2, v_1/v_2)$ is divided into three regions by the functions $\alpha(p_1,p_2,\delta)$ and $\beta(p_1,p_2,\delta)$ defined in (10). Although the magnitude of each region is parameter-specific, the overall layout is quite general since $\alpha(p_1,p_2,\delta)$ is decreasing in p_2, $\beta(p_1,p_2,\delta)$ is increasing in p_2, and $\alpha(p_1,p_2,\delta)=\beta(p_1,p_2,\delta)$ if and only if $\alpha(p_1,p_2,\delta)=1$.

To interpret Figure 2, assume first that δ, p_1, and p_2 are fixed and consider what happens when v_1/v_2 increases (along the vertical axis). As worker 1 becomes more productive relative to worker 2, it eventually becomes optimal to allocate the decision rights to her to avoid her being held up by the least productive worker. Thus, HS_1 ends up being optimal if v_1/v_2 is large enough.

Now keep δ, p_1, and v_1/v_2 fixed, with v_1/v_2 just above 1, and consider what happens when p_2 increases (along the horizontal axis). Initially, each worker's likelihood of encountering issues is small; they will thus each remain productive for a long time before any of them encounters an issue,

Figure 2 **Optimal basic worker-driven coordination scheduling rules under Assumption 1 with two workers ($n = 2$) and $v_1 \geq v_2$**



Note. Here, $p_1 = 0.3$ and $\delta = 0.7$. Functions $\alpha(p_1, p_2, \delta)$ and $\beta(p_1, p_2, \delta)$ are defined in (10).

at which point it is optimal to coordinate, i.e., PC is optimal. As worker 2 encounters issues more frequently, adopting PC becomes undesirable, because worker 2's frequent issues will keep disrupting worker 1. Thus, PC becomes dominated by HS_1 to ensure that the most productive worker decides when to coordinate. As p_2 increases further, worker 2 almost always wants to coordinate. In the unlikely event that worker 2 has no issue, but worker 1 does, it becomes counterproductive to force worker 2 to coordinate, since coordination will happen soon enough. Thus, HS_1 becomes dominated by PP, under which worker 1 can still usually coordinate when she wants to, but without disrupting worker 2 when the latter is actually productive.

In Appendix A and Electronic Companion §EC.1, we simulate the production cycles under FB, PC, PP, and HS, and calculate how often a worker disagrees with the outcome. Almost homogeneous workers may consider 8-15% of meetings a waste of time. As workers are more heterogenous, these rates can increase. It is more common though that a worker is forced to produce even when she wants to coordinate; in these experiments, "too few meetings" is thus more likely than "too many."

4.2. Robustness of Basic Worker-Driven Coordination Scheduling Rules

Before characterizing the performance of PC, PP, and HS in more general settings, we investigate their robustness across all possible values of parameters $(v_1/v_2, p_1, p_2, \delta)$. As shown in Proposition 3 and Table 3, no rule is truly robust; there could be a substantial optimality loss from choosing the wrong one when workers have asymmetric probabilities and values.

PROPOSITION 3. *Under Assumption 1 when $n = 2$,*

$$\frac{V^{PC}}{V^{FB}} \geq \frac{1}{1+\delta}, \qquad \frac{V^{PP}}{V^{FB}} \geq 1-\delta^2, \text{ and } \qquad \frac{V^{HS_j}}{V^{FB}} \geq \min\left\{1-\delta^2, \frac{1}{1+\delta}\right\} \quad \forall j \in \{1,2\},$$

and the bounds are tight when either $v_1 = 0$ or $v_2 = 0$, when $p_1 \in \{0,1\}$, and when $p_2 \in \{0,1\}$.

Table 3 **Robustness of PC, PP, and HS rules**

$V^{\mathrm{PC}}/V^{\mathrm{FB}}$

PC	v_1/v_2				
p_2	1/9	1/3	1	3	9
0	86%	92%	**100%**	100%	100%
0.25	90%	94%	**100%**	95%	93%
0.5	94%	96%	99%	91%	86%
0.75	97%	98%	97%	86%	80%
1	100%	100%	94%	81%	75%

$V^{\mathrm{PP}}/V^{\mathrm{FB}}$

PP	v_1/v_2				
p_2	1/9	1/3	1	3	9
0	100%	100%	96%	84%	76%
0.25	92%	95%	99%	93%	89%
0.5	86%	91%	**100%**	97%	95%
0.75	81%	89%	**100%**	99%	98%
1	79%	89%	**100%**	100%	100%

$V^{\mathrm{HS}_1}/V^{\mathrm{FB}}$

HS_1	v_1/v_2				
p_2	1/9	1/3	1	3	9
0	86%	92%	100%	**100%**	**100%**
0.25	82%	89%	99%	**100%**	**100%**
0.5	80%	87%	100%	**100%**	**100%**
0.75	79%	88%	100%	**100%**	**100%**
1	79%	89%	100%	**100%**	**100%**

Note: Here, $p_1 = 0.3$ and $\delta = 0.7$, as in Figure 2. In the table for each rule $\pi \in \{\mathrm{PC}, \mathrm{PP}, \mathrm{HS}_1, \mathrm{FB}\}$, the bold cells correspond to the values of $(p_2, v_1/v_2)$ for which π is known to implement the FB policy, per Proposition 2, and the values $(p_2, v_1/v_2)$ that achieve the lowest relative performance are highlighted in gray. To save space, the table for HS_2 is not reproduced here as it mirrors that for HS_1.

Table 3 shows that PC, PP, and HS do particularly poorly in the corners (highlighted in gray), i.e., when there is a large asymmetry in the workers' coordination demands (p_2 relative to p_1) or productivity (v_1/v_2), which can cause one worker's output to be disproportionately negatively affected by the other worker's choice. Consider a case where worker 2 has no coordination demands (i.e., $p_2 = 0$). PC allows worker 1 to interrupt worker 2, which is the most costly when worker 2 is much more productive than worker 1. Under PP, worker 2 would never coordinate, which could be hugely detrimental if worker 1 is much more productive. Adding safeguards (time-based controls) to guarantee a minimum level of coordination or production can mitigate this.

4.3. Worker-Driven Coordination Scheduling Rules Enhanced with Time-Based Controls

The experiments in Perlow (1999) assigned specific periods every week during which production or coordination should dominate, to ensure that the team was not held hostage by a single worker. We explore comparable enhancements to the basic coordination scheduling rules:

- Preemptive Coordination with Minimum Cycle Duration (PC-$\mathrm{C}^{\min}$): if coordination took place at time 0, then no coordination is allowed until at least time $T^{\min} \geq 2$, after which the Preemptive Coordination rule applies. Formally,

$$A^{\text{PC-C}^{\min}}(\mathbf{a}, t) = C \Leftrightarrow t \geq T^{\min} - 1 \text{ and } \sum_{i=1}^{n} \mathbb{1}[a_i = C] \geq 1. \tag{11}$$

- Preemptive Production with Maximum Cycle Duration (PP-$\mathrm{C}^{\max}$): if coordination took place at time 0, then coordination occurs when all workers want to but definitely no later than time $T^{\max} \geq 2$. Formally,

$$A^{\text{PP-C}^{\max}}(\mathbf{a}, t) = P \Leftrightarrow t < T^{\max} - 1 \text{ and } \sum_{i=1}^{n} \mathbb{1}[a_i = P] \geq 1. \tag{12}$$

- Hierarchical Structure delegated to worker j with Minimum Cycle Duration (HS_j-$\mathrm{C}^{\min}$): if coordination took place at time 0, then no coordination occurs until at least time $T^{\min} \geq 2$ and then only when worker j wants to. Formally,

$$A^{\mathrm{HS}_j\text{-C}^{\min}}(\mathbf{a}, t) = C \Leftrightarrow t \geq T^{\min} - 1 \text{ and } a_j = C. \tag{13}$$

- Hierarchical Structure delegated to worker j with Maximum Cycle Duration (HS$_j$-C$^{\max}$): if coordination took place at time 0, then coordination occurs when worker j wants to but definitely no later than time $T^{\max} \geq 2$. Formally,

$$A^{\mathrm{HS}_j\text{-C}^{\max}}(\mathbf{a}, t) = P \Leftrightarrow t < T^{\max} - 1 \text{ and } a_j = P. \tag{14}$$

Are these enhancements worthwhile? Optimally chosen time-based controls will always (weakly) improve on the basic rules (which are special cases of the enhanced rules with a minimum cycle of $T^{\min} = 2$ or a maximum cycle of $T^{\max} \to \infty$). Comparing Tables 3 and 4 reveals that these enhancements can indeed considerably reduce the suboptimality loss of the basic rules, especially when workers have asymmetric coordination demands and productivity (i.e., in the corners). To see why, consider again a worker 2 with $p_2 = 0$. If worker 1 is less productive, letting her, under PC, constantly interrupt worker 2 (who never wants to coordinate) can be very disruptive and costly. A minimum cycle duration during which worker 1 is not allowed to disrupt worker 2 would mitigate this. Similarly, under PP, worker 2 with $p_2 = 0$ always wants to produce, which can be very costly if worker 1 is more productive but has an issue. Setting a maximum cycle duration would limit worker 2's ability to dismiss worker 1's coordination needs endlessly.

Table 5 shows the optimal minimum or maximum cycle durations, optimized over $\{2, \ldots, 50\}$, for each combination of (p_2, v_2) when $p_1 = 0.3$ and $\delta = 0.7$. Naturally, whenever a basic rule (i.e., without time-based control) is optimal (as shown in Proposition 2), the optimal enhancement is to not impose any time restriction, i.e., to set $T^{\min} = 2$ as the minimum cycle duration or $T^{\max}$ as large as possible (50, here) as the maximum cycle duration. Even when a specific basic rule is not optimal, Table 5 shows that the optimal minimum or maximum cycles can be as small or as large as permitted, so one should *a priori* not restrict attention to a particular set of values.

5. Large Teams

Although the basic worker-driven rules (PC, PP, and HS) are not necessarily ever optimal in larger teams (unlike in teams of two; see Proposition 2), their simplicity makes them natural candidates to explore. Most analytical results below refer to a possibly arbitrarily large team size, but numerically they already apply when team size exceeds 5-10 workers.

5.1. First-Best Policy in Large Teams

The FB policy is characterized in Proposition 1: it is optimal to produce if and only if $\sum_{i=1}^{n} f_i(\mathbf{x}) > \phi$ for some ϕ. Although simple to describe, it may be complicated to implement with large teams, as it requires being explicit about every worker's productivity. This may be uncomfortable if shared, or if not shared it may lead to coordination decisions that are perceived as arbitrary, as we discuss further in Appendix A. As before, we seek simple worker-driven coordination scheduling rules, starting with the basic rules above, then proposing more advanced versions.

Table 4 Robustness of PC-Cmax, PP-Cmax, HS-Cmin, and HS-Cmax rules

$V^{\text{PC-C}^{\min}}/V^{\text{FB}}$

PC-$C^{\min}$ p_2 \ v_1/v_2	1/9	1/3	1	3	9
0	100%	100%	**100%**	100%	100%
0.25	90%	94%	**100%**	95%	93%
0.5	94%	96%	99%	91%	87%
0.75	97%	98%	97%	88%	85%
1	100%	100%	94%	86%	84% (gray)

$V^{\text{PP-C}^{\max}}/V^{\text{FB}}$

PP-$C^{\max}$ p_2 \ v_1/v_2	1/9	1/3	1	3	9
0	100%	100%	96%	88%	85% (gray)
0.25	92%	95%	99%	93%	89%
0.5	88%	92%	**100%**	97%	95%
0.75	90%	93%	**100%**	99%	98%
1	100%	100%	**100%**	100%	100%

$V^{\text{HS}_1\text{-C}^{\min}}/V^{\text{FB}}$

HS_1-$C^{\min}$ p_2 \ v_1/v_2	1/9	1/3	1	3	9
0	100%	100%	100%	**100%**	**100%**
0.25	84%	89%	99%	**100%**	**100%**
0.5	81%	87%	100%	**100%**	**100%**
0.75	79% (gray)	88%	100%	**100%**	**100%**
1	79% (gray)	89%	100%	**100%**	**100%**

$V^{\text{HS}_1\text{-C}^{\max}}/V^{\text{FB}}$

HS_1-$C^{\max}$ p_2 \ v_1/v_2	1/9	1/3	1	3	9
0	86%	92%	100%	**100%**	**100%**
0.25	83% (gray)	89%	99%	**100%**	**100%**
0.5	85%	89%	100%	**100%**	**100%**
0.75	90%	92%	100%	**100%**	**100%**
1	100%	100%	100%	**100%**	**100%**

Note: Here, $p_1 = 0.3$ and $\delta = 0.7$, as in Table 3. Each table corresponds to a rule $\pi \in \{\text{PC-C}^{\min}, \text{PP-C}^{\max}, \text{HS}_1\text{-C}^{\min}, \text{HS}_1\text{-C}^{\max}\}$. The bold cells correspond to the values of $(p_2, v_1/v_2)$ for which π is known to be optimal, per Proposition 2, and the combination of values $(p_2, v_1/v_2)$ that achieves the worst relative performance is highlighted in gray. The tables for HS_2-$C^{\min}$ and HS_2-$C^{\max}$ are not shown as they mirror those for HS_1-$C^{\min}$ and HS_1-$C^{\max}$.

Table 5 Optimal minimum or maximum cycle length duration of PC-Cmax, PP-Cmax, HS-Cmin, and HS-Cmax rules optimized over $\{2, \ldots, 50\}$

$T^{\min}$

PC-$C^{\min}$ p_2 \ v_1/v_2	1/9	1/3	1	3	9
0	50	50	**2**	2	2
0.25	2	2	**2**	2	2
0.5	2	2	2	2	3
0.75	2	2	2	3	3
1	2	2	2	3	3

$T^{\max}$

PP-$C^{\max}$ p_2 \ v_1/v_2	1/9	1/3	1	3	9
0	50	50	8	4	4
0.25	50	50	9	6	6
0.5	3	4	**50**	50	50
0.75	2	3	**50**	50	50
1	2	2	**50**	50	50

$T^{\min}$

HS_1-$C^{\min}$ p_2 \ v_1/v_2	1/9	1/3	1	3	9
0	50	50	2	**2**	**2**
0.25	4	3	2	**2**	**2**
0.5	3	2	2	**2**	**2**
0.75	2	2	2	**2**	**2**
1	2	2	2	**2**	**2**

$T^{\max}$

HS_1-$C^{\max}$ p_2 \ v_1/v_2	1/9	1/3	1	3	9
0	50	50	50	**50**	**50**
0.25	4	5	12	**50**	**50**
0.5	3	3	50	**50**	**50**
0.75	2	3	50	**50**	**50**
1	2	2	50	**50**	**50**

Note: Here, $p_1 = 0.3$ and $\delta = 0.7$, as in Table 3. Each table corresponds to a rule $\pi \in \{\text{PC-C}^{\min}, \text{PP-C}^{\max}, \text{HS}_1\text{-C}^{\min}, \text{HS}_1\text{-C}^{\max}\}$. The bold cells correspond to the values of $(p_2, v_1/v_2)$ for which the basic version of π, without time-based controls, is known to be optimal, per Proposition 2. The tables for HS_2-$C^{\min}$ and HS_2-$C^{\max}$ are not shown as they mirror those for HS_1-$C^{\min}$ and HS_1-$C^{\max}$.

5.2. Worker-Driven Coordination Scheduling Rules

We first consider the basic coordination scheduling rules PC, PP, and HS, which all give the right to an individual to trigger coordination or force production, and then later consider a more advanced version of these policies, which aggregates the opinions of k individuals.

5.2.1. Basic Worker-Driven Coordination Scheduling Rules. We know from Proposition 2 that PC, PP, and HS can each be optimal when $n = 2$ for a given set of parameters, but how do they perform with larger teams? We first compare them relative to one another, then to FB. In what follows, we assume that workers have symmetric coordination demands, i.e., $p_i = p$ for all i, but do not restrict productivity values v_i to be equal. For the HS rule, we assume without loss of generality that worker 1 decides when to coordinate and refer to that rule without subscript.

Proposition 4. *Under Assumption 1, when $p_i = p$ for all i,*

- *There exists a threshold $\hat{n}$ such that $V^{HS} \geq \max\{V^{PP}, V^{PC}\}$ if and only if $n \geq \hat{n}$.*
- *There exist two thresholds n_L and n_U such that $V^{PP} \geq V^{PC}$ if and only if $n \in [n_L, n_U)$.*

Proposition 4 shows that no rule among PC, PP, and HS uniformly dominates the others. When the team is large, allocating the decision rights to a single worker (HS) outperforms PP and PC, irrespective of the relative value of that worker. A large team would almost never produce under PC, and would almost never coordinate under PP, because any worker has a veto right. Under HS, the production cycles have an intermediate duration; even though dictated by one worker, it is probably more in line with most of the other workers' needs. This does depend on our assumption that $p_i = p\ \forall i$. If decision rights are assigned to a worker with very high or low p_i relative to the others, this benefit is reduced. Hence, worker-driven coordination scheduling rules may only work for moderately-sized teams. Between PP and PC, there is no uniform ranking as shown in the second part of Proposition 4; the set of team sizes where PP dominates PC is convex.

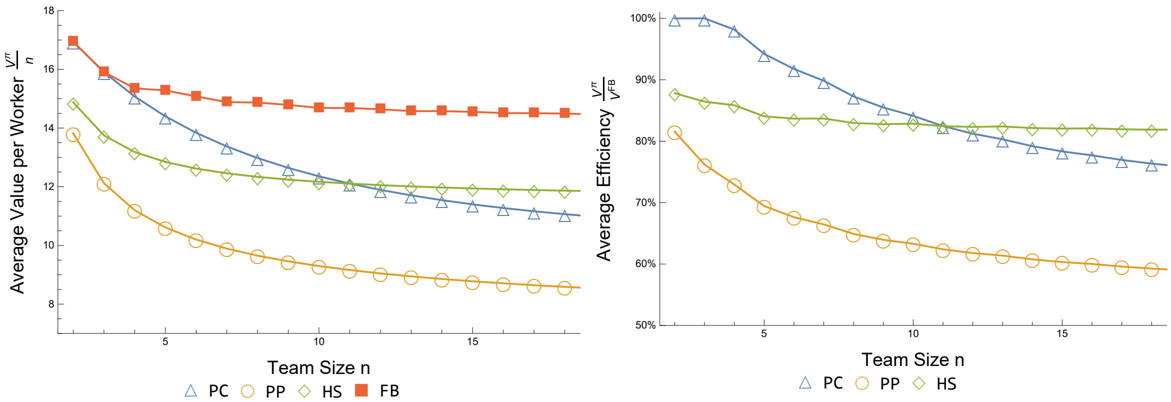
We next discuss the efficiency of PC, PP, and HS relative to FB. The left panel of Figure 3a depicts the average value per worker for all three basic worker-driven rules and the FB policy. Specifically, it depicts V^{π}/n, for $\pi \in \{\text{FB}, \text{PC}, \text{PP}, \text{HS}\}$. Both V^{PP}/n and V^{PC}/n are decreasing in n because in larger teams there is a higher chance that a worker would exert her veto power. V^{HS}/n is also decreasing in n because as n increases less weight is put on worker 1, whose value is maximized under HS_1, and more weight is put on the other workers, who have no decision rights. Even V^{FB}/n is decreasing in n because the multiplicative nature of the workers' interactions (coordination involves all workers) outweighs the additive nature of their productivity. Because the average values of each rule (V^{π}/n) and the average FB value (V^{FB}/n) are both decreasing in n, the average efficiency of each rule (V^{π}/V^{FB}) may not necessarily evolve monotonically (although they do with the parameters in the upper right panel of Figure 3a), even though it tends to decline as n grows. This motivates us to consider more advanced worker-driven rules (§5.2.2), as well as enhancing PC, PP, and HS with time-based controls (§5.3).

5.2.2. Advanced Worker-Driven Coordination Rules. The k-out-of-$\mathcal{S}$-votes rule, where coordination occurs if at least k workers from a given set $\mathcal{S}$ want to, generalizes PC, PP, and HS. Because of its additional degrees of freedom (k and $\mathcal{S}$), this more advanced rule outperforms the basic rules. In fact, under Assumption 1 when $v_i = v$ for all i, this rule attains the FB outcome. To see this, note that the FB policy (Proposition 1) then simplifies to: $A^{\text{FB}}(\mathbf{x}) = P \Leftrightarrow \sum_{i=1}^{n} \mathbb{1}[x_i = 0] > \phi/v$. Setting $\mathcal{S} = \{1, \ldots, n\}$ and $k = \phi/v$ thus achieves the FB policy.

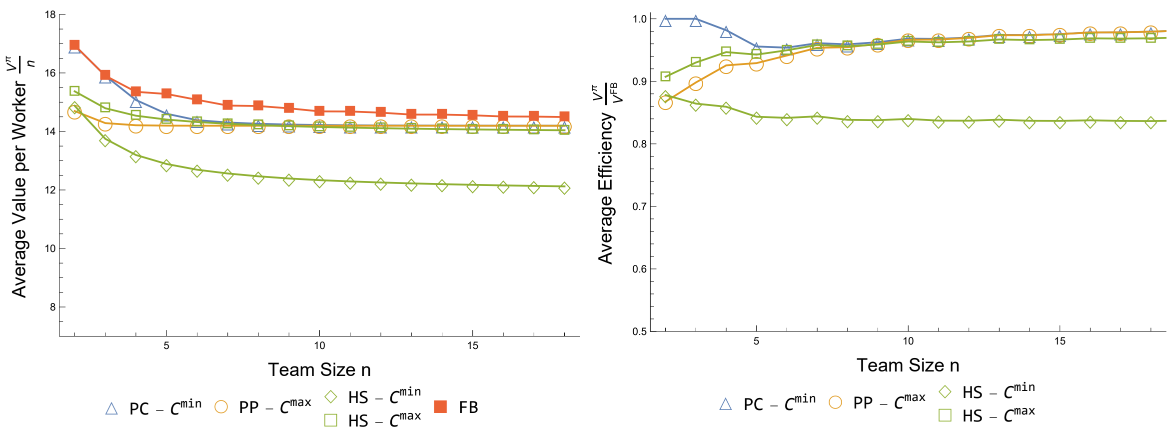
In this simple context, what is the optimal k? The left panel of Figure 4 indicates that the best k is roughly one-third of the team size and grows linearly. This suggests that a simple majority

Figure 3 Average value per worker in absolute terms (left) or relative to FB (right).

(a) Basic worker-driven coordination scheduling rules

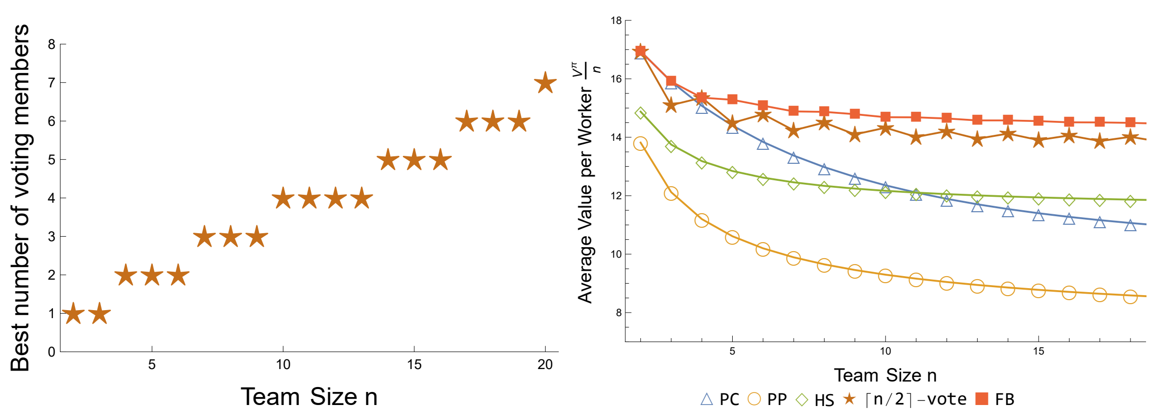


(b) Worker-driven coordination scheduling rules with time-based controls



Note. Here, $v_i = 1$ and $p_i = 0.1$ for all i and $\delta = 0.95$. The left panels show V^π/n, the right panels V^π/V^{FB}, for (a) $\pi \in \{\text{PC}, \text{PP}, \text{HS}, \text{FB}\}$ (above) and (b) $\pi \in \{\text{PC-C}^{\text{min}}, \text{PP-C}^{\text{max}}, \text{HS-C}^{\text{min}}, \text{HS-C}^{\text{max}}, \text{FB}\}$ (below).

Figure 4 Best number of voting members in the k-out-of-$\mathcal{S}$-votes rule when $\mathcal{S} = \{1, \ldots, n\}$ workers (left) and performance of a simple majority rule, with $k = \lceil n/2 \rceil$ and $\mathcal{S} = \{1, \ldots, n\}$ (right)



Note. Here, $v_i = 1$ and $p_i = 0.1$ for all i and $\delta = 0.95$. The values in the right panel are V^π/V^{FB}, for $\pi \in \{\text{PC}, \text{PP}, \text{HS}, \lceil n/2 \rceil\text{-out-of-}\{1, \ldots, n\}\text{ votes}, \text{FB}\}$.

rule, i.e., setting $k = \lceil n/2 \rceil$, could perform quite well. Indeed, the right panel of Figure 4 shows that such a majority rule is near-optimal in all instances.

While a simple majority rule performs well in large teams of homogenous workers, our results from §4.2 suggest that may not hold when workers are heterogeneous, especially with respect to productivity; in such cases it may be desirable to allow only a subset of workers to vote. The majority rule may also be more complicated to implement in practice given that it requires tallying votes at the beginning of each period, although the use of common team management software (e.g., Slack or Microsoft Teams) could automate that function. We leave it for future research to investigate the performance of such majority rules in more heterogeneous teams.

5.3. Worker-Driven Coordination Scheduling Rules with Time-Based Controls

Optimally chosen time-based controls again naturally improve each rule when the team is large, but they are especially useful for PP and PC. We have a complete reversal of the ordering among the rules when the team size is large: without enhancements HS dominated PP and PC (Proposition 4), but the enhanced versions of PP and PC now dominate the enhanced version of HS.

Proposition 5. *Under Assumption 1, when $p_i = p$ for all $i > 1$, there exists a threshold $\tilde{n}$ such that for all $n \geq \tilde{n}$, $\min\{V^{PC\text{-}C^{min}}, V^{PP\text{-}C^{max}}\} \geq \max\{V^{HS\text{-}C^{min}}, V^{HS\text{-}C^{max}}\}$.*

The left panel of Figure 3b depicts the average values of the enhanced coordination scheduling rules, to be compared with the basic rules in the left panel of Figure 3a. Clearly, the enhancements improve the performance of each basic rule. Adding a minimum cycle duration has only a marginal impact on the performance of HS, but results in a substantial improvement for PC, especially at large team sizes. Adding a maximum cycle duration results in a substantial performance improvement for both HS and PP, but PP outperforms HS when the team becomes very large. The right panel of Figure 3b depicts the relative efficiency of all enhanced coordination scheduling rules relative to V^{FB}. (Here, we see that the evolution is not always monotone due to V^{FB}/n being decreasing in n.) Although the efficiency of HS-C^{min} follows a general declining trend, similar to the basic rules, the efficiency of both HS-C^{max} and PP-C^{max} improves with team size. The efficiency of PC-C^{min} follows a similar upward trajectory, after an initial steep drop. Overall, the relative efficiency of PP-C^{max}, HS-C^{max}, and to a smaller extent PC-C^{min}, appears to improve with n, in contrast to the basic rules: it is precisely when the basic rules perform the worst that these enhanced rules perform the best, similar to when $n = 2$ in §4.3.

What explains the dominance of PC-C^{min} and PP-C^{max} over HS-C^{min} and HS-C^{max} when the team size becomes large? As the team gets larger, both PP-C^{max} and PC-C^{min} converge to a fixed-interval coordination scheduling rule. Under PC-C^{min}, the cycle duration will never be smaller than the set minimum (by definition), but it will also rarely be larger because most likely some team member will exert her preemptive right to coordinate. Similarly, the cycle under PP-C$^{\max}$ will rarely be shorter than the set maximum because most likely some team member would still want to produce. By contrast, the cycle durations under HS-C$^{\min}$ and HS-C$^{\max}$ remain stochastic since they depend on a specific worker (namely, worker 1) rather than on whether anyone of an increasingly large set wants to coordinate or produce. Hence, when the team size grows large, a fixed-interval rule may perform well, which we investigate next.

6. Fixed Interval Coordination Scheduling Rules

In contrast to the rules considered so far, which were based on the workers' stated preferences, we now consider a coordination scheduling rule that is only based on the time since coordination happened last, namely FI rule (4). Let $V^{\mathrm{FI}}(T)$ be the value function under FI when the cycle duration is T. Let $T^* = \arg\max_{T\in\{2,3,\ldots\}} V^{\mathrm{FI}}(T)$.

6.1. Comparison to FB Policy

FI rules are optimal in a deterministic stationary environment, but how do they perform when issues arise stochastically? How robust are they to misspecification of the cycle duration? Unlike most results so far, we do not need Assumption 1 to address these two questions:

Proposition 6.

$$\frac{V^{FI}(T^*)}{V^{FB}} \geq \frac{16+\sqrt{13}}{27} \approx 0.7261,$$

and the bound is tight with one worker (or alternatively, with a team of n identical workers with perfectly correlated probabilities of issues) with $f(0)=v$ and $f(x)=0$ if $x>0$, $\mathbb{P}[1]=1-\mathbb{P}[0]=(5-\sqrt{13})/6$, and $\delta\to 1$. Moreover,

$$\frac{V^{FI}(T^*+1)}{V^{FI}(T^*)} \geq \frac{T^*}{T^*+1} \geq \frac{2}{3}$$

and the bound is tight with $f_i(0)=v_i$ and $f_i(x)=0$ if $x>0$ for all $i=1,\ldots,n$, $\mathbb{P}[\mathbf{1}]=1$, and $\delta\to 1$.

Proposition 6 shows that when the periodicity is chosen optimally, the suboptimality loss of FI relative to FB is no greater than 28% across all team sizes. Although 28% may be perceived as significant, it is quite remarkable that the efficiency loss is bounded by a constant. Moreover, the FI rule appears quite robust, at least locally: choosing a periodicity that is one period longer than optimal results in an optimality loss of $1/(T^*+1)$. Hence, if the optimal periodicity recommends meeting every 4.5 days, rounding up to weekly meetings only results in a small loss of optimality.

The worst-case analysis also indicates when FI is likely to perform poorly: when workers quickly become unproductive once they accumulate issues (binary productivity function, as in Assumption

1), when the team size is small, when issues are likely to occur simultaneously, and when the length of a period is short ($\delta \to 1$). The opposite scenario suggests that FI attains good performance in large teams, when issues are likely to occur independently, with minimal impact on productivity.

To explore this conjecture, Table 6 assesses the local robustness and relative efficiency of the FI rule considering, as a base case, two workers, a short period ($\delta = 0.99$), perfectly correlated probabilities ($\mathbb{P}[1,1] = 1 - \mathbb{P}[0,0] = 0.1$), and binary productivity functions ($f_i(x) = v$ if $x = 0$ and $f_i(x) = 0$ otherwise). For this case, the FI rule is (locally) robust, but rather inefficient, as shown in the top row. It achieves 76% of the FB outcome, close to the lower bound of 72.6% from Proposition 6. The table then shows the robustness and efficiency of the FI rule for variations when (i) workers can accumulate a certain number of issues ($\theta_i > 0$) before their productivity drops to zero, (ii) workers' productivity declines linearly as they accumulate issues ($\lambda = 0$), (iii) workers have a higher chance of encountering issues, (iv) workers' probabilities of having an issue become less correlated (but still symmetric), (v) workers' probabilities of having an issue become asymmetric, and (vi) the time periods become so large that the future becomes more heavily discounted (smaller δ). Overall, FI performs much better than the bounds derived in Proposition 6 suggest. Efficiency improves significantly if workers remain somewhat productive with a few issues (higher θ_i); the binary productivity function in Assumption 1(ii) heavily penalizes suboptimal rules. Efficiency is relatively insensitive to probabilities, provided they are positively correlated, but improves significantly as correlation decreases. With perfectly correlated issues, if the outcome of FI is undesirable for one worker it is also undesirable for the other; independent probabilities make it more likely that the outcome is desirable for at least one worker. Efficiency also improves with heavier discounting, since future actions bear less weight and both FI and FB necessarily produce in the first period.

6.2. Comparison to Basic Worker-Driven Coordination Scheduling Rules

Combining the results from §5.2 and §6.1, we have shown that the basic worker-driven coordination scheduling rules (PP, PC, and HS) perform well in small teams, but become less efficient in large teams, and that FI tends to perform well in large teams. The next proposition formalizes this.

Proposition 7. *Under Assumption 1, if $p_i = p$ for all $i > 1$ and $\sum_{i=2}^{\infty} v_i = \infty$, there exists a threshold $\tilde{n}$ such that for all $n \geq \tilde{n}$, $V^{FI} \geq V^{HS} \geq \max\{V^{PP}, V^{PC}\}$.*

In large teams, a simple FI rule dominates allocating the decision rights to one worker, and the latter dominates giving decision rights to all workers (Proposition 4). Decentralizing decision rights gives too much veto power to any worker, who can then hold the rest of the team hostage. In practice, smaller teams often adopt fluid, i.e., worker-driven, coordination scheduling rules (e.g., "let's meet whenever you get stuck") while larger teams follow more rigid rules (e.g., weekly meetings).

Table 6 **Robustness and suboptimality of FI rule when $n=2$ when $f_i(x)=\lambda v+(1-\lambda)2v(\theta_i+1-x)/(\theta_i+2)$ for all $x\le\theta_i$ and zero otherwise**

Scenario	δ	$\mathbb{P}[1,0]$	$\mathbb{P}[0,1]$	$\mathbb{P}[1,1]$	$\mathbb{P}[0,0]$	θ_1	θ_2	λ	$\frac{V^{\text{FI}}(T^*+1)}{V^{\text{FI}}(T^*)}$	$\frac{V^{\text{FI}}(T^*)}{V^{\text{FB}}}$
Baseline	0.99	0	0	0.1	0.9	0	0	1	99%	76%
Higher threshold on accumulated issues	0.99	0	0	0.1	0.9	1	1	1	100%	89%
before productivity drops to zero	0.99	0	0	0.1	0.9	2	2	1	100%	93%
(higher θ_i for $i=1,2$)	0.99	0	0	0.1	0.9	3	3	1	100%	95%
Perfectly correlated coordination demands	0.99	0	0	0.1	0.9	4	4	1	100%	97%
Linear productivity decline ($\lambda=0$);	0.99	0	0	0.1	0.9	1	1	0	100%	89%
Perfectly correlated coordination demands	0.99	0	0	0.1	0.9	2	2	0	100%	93%
	0.99	0	0	0.1	0.9	3	3	0	100%	95%
	0.99	0	0	0.1	0.9	4	4	0	100%	97%
Higher coordination demands (higher $\mathbb{P}[1,1]$);	0.99	0	0	0.2	0.8	0	0	1	97%	74%
Perfectly correlated coordination demands	0.99	0	0	0.3	0.7	0	0	1	97%	74%
	0.99	0	0	0.4	0.6	0	0	1	92%	75%
	0.99	0	0	0.5	0.5	0	0	1	88%	75%
Imperfectly correlated coordination demands	0.99	0.1	0.1	0.1	0.7	0	0	1	97%	80%
($\mathbb{P}[1,0]\neq 0,\mathbb{P}[0,1]\neq 0$)	0.99	0.1	0.1	0.2	0.6	0	0	1	97%	80%
	0.99	0.2	0.2	0.1	0.5	0	0	1	97%	85%
	0.99	0.2	0.2	0.2	0.4	0	0	1	92%	86%
Asymmetric coordination demands	0.99	0.1	0	0.1	0.8	0	0	1	99%	78%
($\mathbb{P}[1,0]\neq\mathbb{P}[0,1]$)	0.99	0.2	0	0.1	0.7	0	0	1	98%	80%
	0.99	0.3	0	0.1	0.6	0	0	1	96%	82%
	0.99	0.4	0	0.1	0.5	0	0	1	99%	85%
Longer time periods (lower δ)	0.9	0	0	0.1	0.9	0	0	1	100%	80%
	0.8	0	0	0.1	0.9	0	0	1	100%	84%
	0.7	0	0	0.1	0.9	0	0	1	100%	89%

To illustrate Proposition 7, the left panel of Figure 5 superimposes the average value per worker under FI on the left panel of Figure 3a. Because FI affects each worker equally, its average value is flat. For small teams, FI is dominated by some basic worker-driven rules (here, PC and HS); for large teams, FI dominates all basic worker-driven rules. In this example, the FI rule already dominates all basic worker-driven rules for teams of 6 or more.

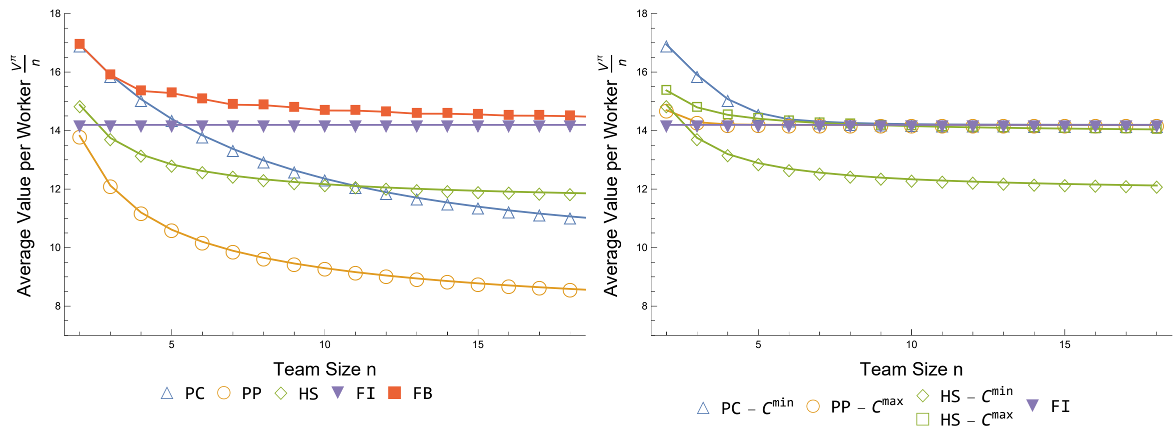
6.3. Comparison to Worker-Driven Coordination Scheduling Rules Enhanced with Time-Based Controls

We showed that FI weakly dominates all basic worker-driven rules with large teams. However, FI is not optimal: it is still dominated for any team size by PC-C$^{\min}$ and PC-C$^{\max}$.

Proposition 8. *Under Assumption 1,*

- *For any n,* $\min\{V^{PC\text{-}C^{min}},V^{PP\text{-}C^{max}}\}\geq V^{FI}$.
- *When $p_i=p$ for all $i>1$, there exists a threshold $\tilde{n}$ such that for all $n\geq\tilde{n}$, $V^{FI}\geq$* $\min\{V^{HS\text{-}C^{min}},V^{HS\text{-}C^{max}}\}$.
- *Suppose that $v_i\leq\overline{v}$ for all i and $0<\underline{p}\leq p_i\leq\overline{p}<1$ for all i. For any $\epsilon>0$, $\exists\tilde{n}$ such that $\forall n\geq\tilde{n}$,* $\max\{V^{PC\text{-}C^{min}},V^{PP\text{-}C^{max}}\}-V^{FI}<\epsilon$.

Figure 5 **Average value per worker with FI and with basic worker-driven coordination scheduling rules (left) and worker-driven coordination scheduling rules enhanced with time-based controls (right)**



Note. Here, $v_i = 1$ and $p_i = 0.1$ for all i and $\delta = 0.95$. The values are V^{π}/n, with $\pi \in \{\text{PC}, \text{PP}, \text{HS}, \text{FI}, \text{FB}\}$ in the left panel and $\pi \in \{\text{PC-C}^{\text{min}}, \text{PP-C}^{\text{max}}, \text{HS-C}^{\text{min}}, \text{HS-C}^{\text{max}}, \text{FI}\}$ in the right panel.

To illustrate Proposition 8, the right panel of Figure 5 superimposes the average value per worker under FI on the left panel of Figure 3b. By Proposition 5, for large teams PC-C^{min} and PP-C^{max} dominate HS-C^{min} and HS-C^{max}. The first two points of Proposition 8 extend this by showing that FI lies between these two. However, as the team becomes very large, PC-C^{min} and PP-C^{max} effectively become equivalent to FI (the third point of Proposition 8). Given that the worker-driven coordination scheduling rules enhanced with time-based controls are more complex to implement than FI rules, Proposition 8 makes a case for a purely time-based rule for large teams. For small teams, a more fluid worker-driven approach is preferable, ideally with safeguards to prevent the team from being held up by a single member.

7. Discussion and Conclusions

Workers in knowledge-intensive industries often complain of having too many meetings; yet, businesses rarely think about how to set meeting scheduling rules. Here we investigate the efficiency and robustness of various coordination scheduling rules, using a stylized game-theoretic model.

We consider a situation where workers are engaged in a common activity, which can be divided into independent, parallel tasks, but necessitates occasional in-depth coordination that cannot be accomplished by email or in a short casual meeting. Coordination is useful because it helps address the issues workers have encountered, but it takes time away from production.

Table 7 summarizes the high-level prescriptions suggested by our analysis.

- When the team is small, the most basic worker-driven coordination scheduling rules (PC, PP, HS) perform well (Proposition 2) unless workers are heterogeneous (Proposition 3), in which case one needs to build in time-based controls (Table 4).

Table 7 Key suggested prescriptions

Team Size	Worker Productivity Heterogeneity	
	Low	High
Small	PC, PP, HS are adequate; which rule is preferred depends on parameters	PC-C^{min}, PP-C^{max} HS-C^{min}, HS-C^{max}
Intermediate (workers have equal coordination demands)	PC-C^{min}, PP-C^{max}; HS is adequate	PC-C^{min}, PP-C^{max}
Large (workers have equal coordination demands)	PC-C^{min}, PP-C^{max}; FI is nearly as good	PC-C^{min}, PP-C^{max}; FI is nearly as good

- For intermediate team sizes, HS dominates PC and PP (Proposition 4), but adding time-based controls to the latter two rules is the most robust choice, especially when workers are quite heterogeneous (Proposition 5).
- When the team size becomes large, an FI rule generally performs well (Proposition 7), even though it is marginally dominated by PC-C^{min} and PP-C^{max} (Proposition 8).

Our results rely on several simplifying assumptions, so further research is needed to test their robustness. Throughout, we assume a binary productivity function. Exploration of more general productivity functions in Appendix B yields the following observations. First, with two workers, the general structure of the equilibrium still holds. That structure hinges on the binary nature of the workers' decisions (coordinate vs. produce), so the exact productivity function is less relevant. We observe numerically that the sensitivity of the basic worker-driven rules to the problem parameters remains similar to that with a binary productivity function, suggesting that the first takeaway carries over. Second, the main driver of the dynamics in large teams is that one worker can hold all others hostage under PC and PP. Hence, the second and third takeaways do not depend directly on the productivity function. Third, additional analytical complications arise under PP even with two workers, rendering any formal characterization very challenging.

Our model makes numerous other assumptions, including: the frequency with which issues arise is exogenous, issues never get resolved by themselves, meetings should involve all workers, meetings resolve all issues, meeting duration is independent of the number of issues, workers do not incur a setup time to go back to production, the prospect of an upcoming meeting does not boost a worker's productivity, the productivity function is separable, and the activity does not have a finite deadline. Relaxing these would bring the model closer to reality. Some of these extensions, such as allowing the time needed for coordination to vary with the number of issues to be resolved, might be analytically challenging as the FB policy may no longer be a threshold policy (it may be optimal to coordinate preemptively to avoid long meetings or when the number of accumulated issues is large, but not in between). Some of our results are generalizable to account for worker unavailability, random coordination meeting durations, or not requiring all workers to be present for coordination. Moreover, our stylized model could be embedded into a broader organizational design model, e.g., to study who should be required to attend a coordination meeting, trading off their opportunity cost of time with their ability to get issues resolved. A generalization of the model could study collaborative dynamics in addition to coordination dynamics by considering a non-separable production function $f(x_1, \ldots, x_n)$ shared among workers according to some rule. We limited ourselves to coordination meetings aimed at resolving issues that prevented a worker from being productive. Other types of coordination meeting are daily scrum meetings or huddles, or meetings to discuss project management policies or team norms. The issues we model are experienced by individual workers, and resolving them only increases that worker's productivity. We do not necessarily capture all issues that relate to the project content, or exclude all those that relate to its management. For instance, a meeting to address a worker talking too loud in the next cubicle causing their neighbor to be unproductive could fit our model, while a content-focused meeting consisting of a performance review would not.

With Microsoft's launch of Viva Insights, which suggests blocking off focus time for deep work and identifies one's key collaborators, current technology seems ripe for more effective time management in organizations. Yet, it remains ignorant of the negative externality such time blocking may create on team members since it is based only on individual preferences. We hope our work will lead to further research on more coordinated time blocking, including further analysis of the hybrid policies in the experiments in Perlow (1999), where some specific time slots are set aside during which production or coordination dominates.

References

Bertsekas DP (2001) *Dynamic Programming and Optimal Control* (Belmont, MA: Athena Scientific), vol 2, 2nd edition.

Bevins F, De Smet A (2013) Making time management the organization's priority. *McKinsey Quarterly* 1:26–41.

Bonatti A, Hörner J (2011) Collaborating. *Am Econ Rev* 101(2):632–63.

Brooks FP (1975) *The Mythical Man-Month: Essays on Software Engineering* (New York, NY: Addison-Wesley).

Crama P, Sting FJ, Wu Y (2019) Encouraging help across projects. *Manage Sci* 65(3):1408–1429.

Cross R, Rebele R, Grant A (2016) Collaborative overload. *Harvard Bus Rev* 94(1):74—-79.

Drucker PF (1967) *The Effective Executive* (New York, NY: Harper&Row).

Faraj S, Sproull L (2000) Coordinating expertise in software development teams. *Manage Sci* 46(12):1554–1568.

Georgiadis G (2015) Projects and team dynamics. *Rev Econ Stud* 82(1):187–218.

Gittell JH (2002) Coordinating mechanisms in care provider groups: Relational coordination as a mediator and input uncertainty as a moderator of performance effects. *Manage Sci* 48(11):1408–1426.

Griessman BE (1994) *Time Tactics of Very Successful People* (McGraw-Hill).

Gurvich I, O'Leary KJ, Wang L, Van Mieghem JA (2020) Collaboration, interruptions, and changeover times: Workflow model and empirical study of hospitalist charting. *M&SOM* 22(4):754–774.

Gurvich I, Van Mieghem JA (2015) Collaboration and multitasking in networks: Architectures, bottlenecks, and capacity. *M&SOM* 17(1):16–33.

Gurvich I, Van Mieghem JA (2018) Collaboration and multitasking in networks: Prioritization and achievable capacity. *Manage Sci* 64(5):2390–2406.

Ha AY, Porteus EL (1995) Optimal timing of reviews in concurrent design for manufacturability. *Manage Sci* 41(9):1431–1447.

Harrison M (2018) How a US health care system uses 15-minute huddles to keep 23 hospitals aligned. *Harvard Bus Rev* https://hbr.org/2018/11/how-a-u-s-health-care-system-uses-15-minute-huddles-to-keep-23-hospitals-aligned.

Heath C, Staudenmayer N (2000) Coordination neglect: How lay theories of organizing complicate coordination in organizations. *Res Organ Behav* 22:153–191.

Huckman RS, Staats BR (2011) Fluid tasks and fluid teams: The impact of diversity in experience and team familiarity on team performance. *M&SOM* 13(3):310–328.

Jett QR, George JM (2003) Work interrupted: A closer look at the role of interruptions in organizational life. *Acad Manage Rev* 28(3):494–507.

Kotlarsky J, van Fenema PC, Willcocks LP (2008) Developing a knowledge-based perspective on coordination: The case of global software projects. *Information & Management* 45(2):96–108.

Krishnan V, Eppinger SD, Whitney DE (1997) A model-based framework to overlap product development activities. *Manage Sci* 43(4):437–451.

Loch CH, Terwiesch C (1998) Communication and uncertainty in concurrent engineering. *Manage Sci* 44(8):1032–1048.

MacKenzie RA (1972) *The Time Trap: Managing Your Way Out* (New York, NY: Amacom).

Maskin E, Tirole J (2001) Markov perfect equilibrium: I. oObservable actions. *J Econ Theory* 100(2):191–219.

McGrath JE, Kelly JR (1986) *Time and Human Interaction: Toward a Social Psychology of Time* (Guilford Press).

McGrath JE, Tschan F (2004) *Temporal Matters in Social Psychology: Examining the Role of Time in the Lives of Groups and Individuals* (American Psychological Association).

Metiu A, Rothbard NP (2013) Task bubbles, artifacts, shared emotion, and mutual focus of attention: A comparative study of the microprocesses of group engagement. *Organ Sci* 24(2):455–475.

Mihm J, Loch C, Huchzermeier A (2003) Problem–solving oscillations in complex engineering projects. *Manage Sci* 49(6):733–750.

Mihm J, Loch CH, Wilkinson D, Huberman BA (2010) Hierarchical structure and search in complex organizations. *Manage Sci* 56(5):831–848.

Mohammed S, Nadkarni S (2011) Temporal diversity and team performance: The moderating role of team temporal leadership. *Acad Manage J* 54(3):489–508.

Narayanan S, Balasubramanian S, Swaminathan JM (2011) Managing outsourced software projects: An analysis of project performance and customer satisfaction. *Prod Oper Manag* 20(4):508–521.

Newport C (2016) *Deep Work: Rules for Focused Success in a Distracted World* (London, UK: Piatkus).

Newport C (2021) *A World Without Email: Find Focus and Transform the Way You Work Forever* (UK: Penguin Random House).

Özkan-Seely GF, Gaimon C, Kavadias S (2015) Dynamic knowledge transfer and knowledge development for product and process design teams. *M&SOM* 17(2):177–190.

Parke MR, Weinhardt JM, Brodsky A, Tangirala S, DeVoe SE (2018) When daily planning improves employee performance: The importance of planning type, engagement, and interruptions. *J Appl Psychol* 103(3):300–312.

Perlow LA (1999) The time famine: Toward a sociology of work time. *Admin Sci Quart* 44(1):57–81.

Perlow LA, Hadley CN, Eun E (2017) Stop the meeting madness. *Harvard Bus Rev* 95(4):62–69.

Radner R, Rothschild M (1975) On the allocation of effort. *J Econ Theory* 10(3):358–376.

Rahmani M, Roels G, Karmarkar US (2017) Collaborative work dynamics in projects with co-production. *Prod Oper Manag* 26(4):686–703.

Rahmani M, Roels G, Karmarkar US (2018) Team leadership and performance: Combining the roles of direction and contribution. *Manage Sci* 64(11):5234–5249.

Roemer TA, Ahmadi R (2004) Concurrent crashing and overlapping in product development. *Oper Res* 52(4):606–622.

Sasieni MW (1956) A Markov chain process in industrial replacement. *J Oper Res Soc* 7(4):148–155.

Saunders EG (2017) How to establish a meeting-free day each week. *Harvard Bus Rev* https://hbr.org/2017/02/how-to-establish-a-meeting-free-day-each-week.

Schwarz R (2017) Is your team coordinating too much, or not enough? *Harvard Bus Rev* https://hbr.org/2017/03/is-your-team-coordinating-too-much-or-not-enough.

Seshadri S, Shapira Z (2001) Managerial allocation of time and effort: The effects of interruptions. *Manage Sci* 47(5):647–662.

Siemsen E, Balasubramanian S, Roth AV (2007) Incentives that induce task-related effort, helping, and knowledge sharing in workgroups. *Manage Sci* 53(10):1533–1550.

Simon HA (1947) *Administrative Behavior* (New York, NY: Mcmillan).

Sosa ME, Eppinger SD, Rowles CM (2004) The misalignment of product architecture and organizational structure in complex product development. *Manage Sci* 50(12):1674–1689.

Staats BR (2018) *Never Stop Learning: Stay Relevant, Reinvent Yourself, and Thrive* (Harvard Business Press).

Staats BR, Brunner DJ, Upton DM (2011) Lean principles, learning, and knowledge work: Evidence from a software services provider. *J Oper Manag* 29(5):376–390.

Staats BR, Milkman KL, Fox CR (2012) The team scaling fallacy: Underestimating the declining efficiency of larger teams. *Org Behav Hum Dec* 118(2):132–142.

Sting FJ, Loch CH, Stempfhuber D (2015) Accelerating projects by encouraging help. *MIT Sloan Manage Rev* 56(3):33.

Sting FJ, Mihm J, Loch CH (2020) Collaborative search: The role of joint problem solving. https://ssrn.com/abstract=1850607.

Thomke S, Bell DE (2001) Sequential testing in product development. *Manage Sci* 47(2):308–323.

Thompson JD (1967) *Organizations in Action: Social Science Bases of Administrative Theory* (New York, NY: McGraw-Hill).

Tucker AL, Spear SJ (2006) Operational failures and interruptions in hospital nursing. *Health Serv Res* 41(3p1):643–662.

Wiesche M (2018) Toward a model of managing interruptions in agile IT projects. *International Research Workshop on IT Project Management*, volume 10.

Yoo OS, Corbett CJ, Roels G (2016) Optimal time allocation for process improvement for growth-focused entrepreneurs. *M&SOM* 18(3):361–375.

Appendix A: Average Cycle Durations and Individual Workers' Perceptions

Here we examine the effect of the coordination scheduling rules on individual workers. When are there "too many meetings," and when too few? A rule will be easier to enforce if it does not dictate outcomes workers are not in favor of. We consider the FB policy in this appendix and consider the four basic rules PC, PP, HS_1, and FI in an electronic appendix §EC.1. For each rule $\pi \in \{\mathrm{FB}, \mathrm{PC}, \mathrm{PP}, \mathrm{HS}_1, \mathrm{FI}\}$, we compute, through simulation, the average cycle duration, and the inverse of meeting frequency. As in Figure 2 and Table 3, we set $p_1 = 0.3$, $\delta = 0.7$, and vary $p_2 \in \{0, 0.25, 0.5, 0.75, 1\}$ and $v_1/v_2 \in \{1/9, 1/3, 1, 3, 9\}$. For each rule and set of values p_2 and v_1/v_2, we first compute the value functions $V_i^\pi(\mathbf{x}, t)$ for all $i = 1, 2$, $\mathbf{0} \leq \mathbf{x} \leq (10, 10)$, $0 \leq t \leq 30$ by value iteration. Using these value-to-go functions, we then simulate 20,000 periods for each rule and report in Tables A-1 - EC.4 the mean cycle durations, the frequency of coordination periods while worker i

Table A-1 FB average cycle durations and frequencies of outcomes involving some disagreement

Average Cycle Duration

FB	v_1/v_2				
p_2	1/9	1/3	1	3	9
0	∞	∞	4.3	4.4	4.3
0.25	5.0	5.0	3.1	4.4	4.3
0.5	3.0	3.0	4.8	4.4	4.4
0.75	2.3	2.3	4.5	4.4	4.3
1	2.0	2.0	4.3	4.3	4.3

Worker preferring opposite outcome

Outcome: Coordination — Worker 1: $\mathbb{P}[A^{\text{FB}}(\mathbf{x}) = C \text{ and } a_1^{\text{FB}}(\mathbf{x}) = P]$

FB	v_1/v_2				
p_2	1/9	1/3	1	3	9
0	0%	0%	0%	0%	0%
0.25	7%	8%	12%	0%	0%
0.5	18%	18%	0%	0%	0%
0.75	27%	27%	0%	0%	0%
1	35%	35%	0%	0%	0%

Outcome: Coordination — Worker 2: $\mathbb{P}[A^{\text{FB}}(\mathbf{x}) = C \text{ and } a_2^{\text{FB}}(\mathbf{x}) = P]$

FB	v_1/v_2				
p_2	1/9	1/3	1	3	9
0	0%	0%	23%	23%	23%
0.25	0%	0%	15%	11%	11%
0.5	0%	0%	0%	5%	5%
0.75	0%	0%	0%	2%	2%
1	0%	0%	0%	0%	0%

Outcome: Production — Worker 1: $\mathbb{P}[A^{\text{FB}}(\mathbf{x}) = P \text{ and } a_1^{\text{FB}}(\mathbf{x}) = C]$

FB	v_1/v_2				
p_2	1/9	1/3	1	3	9
0	100%	100%	0%	0%	0%
0.25	38%	38%	0%	0%	0%
0.5	16%	15%	10%	0%	0%
0.75	5%	5%	3%	0%	0%
1	0%	0%	0%	0%	0%

Outcome: Production — Worker 2: $\mathbb{P}[A^{\text{FB}}(\mathbf{x}) = P \text{ and } a_2^{\text{FB}}(\mathbf{x}) = C]$

FB	v_1/v_2				
p_2	1/9	1/3	1	3	9
0	0%	0%	0%	0%	0%
0.25	0%	0%	0%	29%	28%
0.5	0%	0%	37%	41%	42%
0.75	0%	0%	48%	50%	48%
1	0%	0%	54%	54%	54%

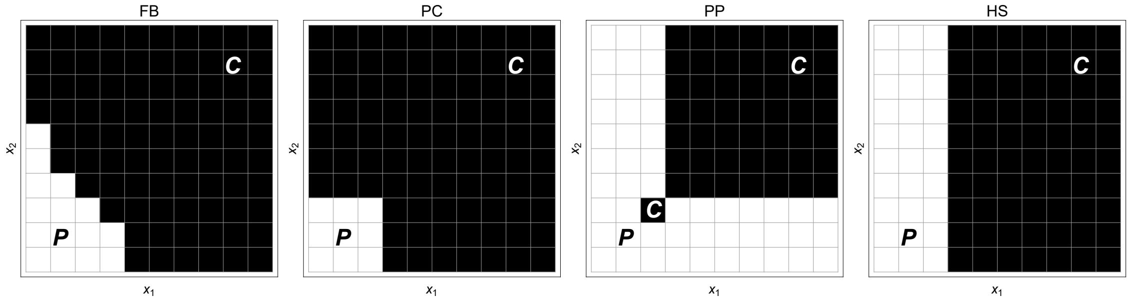
Note: Here, $p_1 = 0.3$ and $\delta = 0.7$ as in Figure 2. The average cycle durations and frequencies of outcomes are estimated from simulation over 20,000 transitions. The highlighted cells are those with the most symmetric workers.

wanted to produce ($\mathbb{P}[A^\pi(\mathbf{x},t) = C \text{ and } a_i^\pi(\mathbf{x},t) = P]$), and the frequency of production periods while worker i wanted to coordinate ($\mathbb{P}[A^\pi(\mathbf{x},t) = P \text{ and } a_i^\pi(\mathbf{x},t) = C]$).

Consider first the FB policy, in Table A-1. Naturally, the mean cycle duration is sensitive to worker 2's coordination demands p_2 whenever $v_2 > v_1$. When $v_2 > v_1$ and worker 2 never encounters issues ($p_2 = 0$), the optimal cycle duration is infinite, whereas when worker 2 always encounters issues ($p_2 = 1$), the optimal cycle duration is as small as possible, i.e., 2 periods. The non-monotone character of the mean cycle duration of the FB policy with respect to p_2 and v_1/v_2 is due to the change in the structure of the FB policy, alternating between PC, PP, HS_1, and HS_2 as these parameters change (Proposition 2).

Examining the outcomes involving disagreement shows that in about half of the scenarios, worker i never disagrees with the outcome, but also that for about a third of the scenarios, worker i often disagrees. For the latter case, consider $v_1/v_2 \in \{1/9, 1/3\}$ and $p_2 = 1$, so that coordination happens every other period. Since $p_1 = 0.3$, in 70% of the cases when coordination is triggered worker 1 wants to produce. This happens every other period, so the likelihood that coordination occurs but worker 1 wants to produce equals $70\%/2 = 35\%$; worker 1 will feel that 35% of meetings are wasted. Conversely, when $p_2 = 0$, worker 2 never wants to coordinate, so every time coordination takes place (once every 4.3 periods, i.e., 23% of the time, when

Figure B-1 Equilibrium policies under FB, PC, PP, and HS with general productivity functions



Note. Here, $p_1 = 0.3$, $p_2 = 0.1$, $\delta = 0.75$, $f_i(x_i) = v_i(1 - x_i/7)$ for $i = 1,2$, $v_1 = 1.4$ and $v_2 = 1$. For any $\pi \in \{\text{FB}, \text{PC}, \text{PP}, \text{HS}\}$, a particular cell $\mathbf{x}$ is colored in black when $A^\pi(\mathbf{x}) = C$, and in white when $A^\pi(\mathbf{x}) = P$.

$v_1/v_2 \in \{1,3,9\}$), worker 2 disagrees with the outcome. Almost equal workers ($p_1 = 0.3$, $p_2 = 0.25$, $v_1/v_2 = 1$) will feel that 12% (worker 1) or 15% (worker 2) of meetings are wasted. Overall, Table A-1 shows that even under the FB policy, workers will feel that some meetings are wasted.

Appendix B: Equilibrium Characterization under General Productivity Functions

Here we characterize the equilibrium outcomes with two workers under PC, PP, and HS when Assumption 1(ii) is relaxed. In §B.1 we consider a more general productivity function $f_i(x)$, without a base productivity value, and in §B.2 a binary productivity function with a base productivity value. These two cases can have opposing effects. The general productivity function may induce workers to want to coordinate early, even if they have not accumulated many issues, so as to operate at full productivity. Conversely, a base productivity value may reduce the need to coordinate even with many accumulated issues.

B.1. General Productivity Function with No Base Value

We first consider a general productivity function $f_i(x)$ with no base value, i.e., such that $f'(x) \leq 0$, $f_i(x) = 0$ for all $x \geq \overline{x}_i$, and $f_i(0) > 0$. The structures of the equilibrium policies under PC and HS characterized in Proposition 2 still hold. The equilibrium policy under PP is quite similar to that in Proposition 2, with additional peculiarities that makes a full analytical characterization challenging. Proofs appear in §EC.2.5.

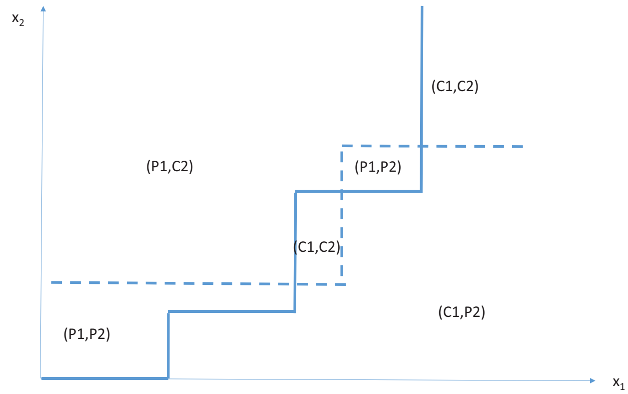
Figure B-1 illustrates the equilibrium policies under FB and the PC, PP, and HS rules. By Proposition 1, the FB policy policy involves producing if and only if $\sum_i f_i(x_i) > \phi$ for some ϕ, so production occurs in a connected region anchored at the origin, the white area in the leftmost panel of Figure B-1.

Under PC, the equilibrium policy yields production in the lower-left of the nonnegative area $\mathbf{x} \geq \mathbf{0}$ and coordination otherwise (second panel in Figure B-1). This generalizes Lemmas EC.1-EC.2 for a binary productivity function, where production was optimal if and only if $\mathbf{x} = \mathbf{0}$.

Proposition B-1. *Suppose that $n = 2$ and that Assumption 1(i) holds. There exists a threshold state $\hat{\mathbf{x}}^{PC}$ satisfying $f_i(\hat{x}_i^{PC} - 1) > \delta(1-\delta)V_i^{PC} \geq f_i(\hat{x}_i^{PC})$ for $i = 1,2$ such that $A^{PC}(\mathbf{x}) = P$ if and only if $\mathbf{x} < \hat{\mathbf{x}}^{PC}$.*

Call $\hat{\mathbf{x}}^{\text{PC}}$ the coordination trigger point: coordination occurs in equilibrium as soon as $\mathbf{x} \geq \hat{\mathbf{x}}^{\text{PC}}$. The PC coordination trigger point is lower than that in the FB. Specifically, if $\mathbf{x} \geq \hat{\mathbf{x}}^{\text{PC}}$, then $A^{\text{PC}}(\mathbf{x}) = C$, so $\delta(1-\delta)\left(V_1^{\text{PC}}(\mathbf{0}) + V_2^{\text{PC}}(\mathbf{0})\right) \geq f_1(\hat{x}_1^{\text{PC}}) + f_2(\hat{x}_2^{\text{PC}})$ by Proposition B-1. Therefore, $\delta(1-\delta)V^{\text{FB}}(\mathbf{0}) \geq f_1(\hat{x}_1^{\text{PC}}) + f_2(\hat{x}_2^{\text{PC}})$. By Proposition 1, this implies that $A^{\text{FB}}(\mathbf{x}) = C$.

Figure B-2 **Equilibrium strategies under PP with general productivity function**



Note. Worker 1's strategy is to produce (P1) in all states lying above the solid line and coordinate (C1) otherwise. Worker 2's strategy is to produce (P2) in all states lying below the dashed line and coordinate (C2) otherwise. $A^{\text{PP}}(\mathbf{x}) = C$ if and only if $a_1^{\text{PP}}(\mathbf{x}) = a_2^{\text{PP}}(\mathbf{x}) = C$.

In the PP equilibrium outcome, similar to Proposition 2, coordination happens in an upper-right area of the nonnegative orthant and production happens in both the corresponding upper-left and lower-right areas. However, unlike Proposition 2, production may not happen everywhere in the lower-left area. The third panel in Figure B-1 shows P occurring in states (3,2) and (2,3) but not in (3,3).

Proposition B-2. *Suppose that $n = 2$ and that Assumption 1(i) holds. There exists a state $\hat{\mathbf{x}}^{PP}$, satisfying $f_i(\hat{x}_i^{PP} - 1) > \delta(1-\delta)V_i^{PP} \geq f_i(\hat{x}_i^{PP})$ for $i = 1, 2$ such that $A^{PP}(\mathbf{x}) = C$ if $\mathbf{x} \geq \hat{\mathbf{x}}^{PP}$; $A^{PP}(\mathbf{x}) = P$ if $x_i \geq \hat{x}_i^{PP}$ and $x_{-i} < \hat{x}_{-i}^{PP}$ for $i = 1, 2$; for states $\mathbf{x} < \hat{\mathbf{x}}^{PP}$, if $A^{PP}(\mathbf{x}) = P$, then $A^{PP}(\mathbf{y}) = P$ if $y_i \leq x_i$ and $y_{-i} \geq x_{-i}$ for either $i = 1$ or $i = 2$ or both.*

In the lower-left, i.e., $\mathbf{x} < \hat{\mathbf{x}}^{\text{PP}}$, the structure of the equilibrium is now more complicated. If production happens in some state $\mathbf{y}$, then it also happens in all states weakly above and to the left or below and to the right of $\mathbf{y}$. However, coordination can emerge in some states $\mathbf{x} < \hat{\mathbf{x}}^{\text{PP}}$, due to the behavior of the value function of any worker with respect to the other worker's stock of issues. Workers never benefit from having issues themselves; $V_i^\pi(x_i, x_{-i})$ is nonincreasing in x_i for $\pi \in \{\text{PC}, \text{PP}\}$. As the other worker $(-i)$ accumulates more issues, it becomes more likely that she will want to coordinate soon. Under PC, worker i then loses control over which outcome will occur, but under PP she then gains control. Worker i never benefits from worker $-i$'s issues under PC ($V_i^{\text{PC}}(x_i, x_{-i})$ is nonincreasing in x_{-i}; Lemma EC.14), but she benefits from them under PP ($V_i^{\text{PP}}(x_i, x_{-i})$ is nondecreasing in x_{-i}; Lemma EC.17). By (5), worker i's decision consists of comparing $\delta V_i^\pi(\mathbf{0})$ to $f_i(x_i) + \delta\mathbb{E}[V_i^\pi(\mathbf{x} + \boldsymbol{\xi})]$ and the latter is decreasing in x_i and decreasing (resp. increasing) in x_i when $\pi =$PC (resp. when $\pi =$PP). Hence, under PP worker i's "indifference curve" between coordinating and producing is increasing in $\mathbf{x}$ as shown in Figure B-2, potentially leading to 'pockets' where both workers want to coordinate even though both would choose to produce if they each had one more issue.

In the HS equilibrium, as before, the worker who has all decision rights wants to coordinate when her stock of issues is large, and produce otherwise; see the rightmost panel in Figure B-1.

Table B-1 Robustness of PC, PP, and HS rules with piecewise-constant productivity functions

$V^{\mathrm{PC}}/V^{\mathrm{FB}}$

PC	v_1/v_2				
p_2	1/9	1/3	1	3	9
0	100%	100%	100%	100%	100%
0.25	100%	100%	100%	100%	100%
0.5	100%	100%	100%	100%	100%
0.75	100%	100%	100%	100%	100%
1	100%	100%	100%	100%	99%

$V^{\mathrm{PP}}/V^{\mathrm{FB}}$

PP	v_1/v_2				
p_2	1/9	1/3	1	3	9
0	100%	100%	100%	100%	100%
0.25	100%	100%	100%	100%	100%
0.5	99%	99%	100%	100%	100%
0.75	93%	94%	97%	99%	100%
1	81%	85%	91%	97%	100%

$V^{\mathrm{HS_1}}/V^{\mathrm{FB}}$

HS_1	v_1/v_2				
p_2	1/9	1/3	1	3	9
0	100%	100%	100%	100%	100%
0.25	100%	100%	100%	100%	100%
0.5	99%	99%	100%	100%	100%
0.75	93%	94%	97%	99%	100%
1	81%	85%	91%	97%	100%

Note: Here, $p_1 = 0.3$, $\delta = 0.95$, and $f_i(x_i) = v_i$ if $x_i \leq 6$ and $f_i(x_i) = 0$ otherwise for $i = 1, 2$. The combination of values $(p_2, v_1/v_2)$ that achieves the lowest relative performance is highlighted in gray.

Table B-2 Robustness of PC, PP, and HS rules with linear productivity functions

$V^{\mathrm{PC}}/V^{\mathrm{FB}}$

PC	v_1/v_2				
p_2	1/9	1/3	1	3	9
0	95%	97%	99%	100%	100%
0.25	97%	98%	99%	98%	98%
0.5	98%	99%	99%	99%	98%
0.75	99%	99%	99%	97%	95%
1	100%	100%	98%	95%	92%

$V^{\mathrm{PP}}/V^{\mathrm{FB}}$

PP	v_1/v_2				
p_2	1/9	1/3	1	3	9
0	100%	100%	99%	97%	95%
0.25	98%	99%	99%	98%	97%
0.5	95%	96%	98%	99%	98%
0.75	90%	93%	97%	99%	100%
1	84%	89%	95%	99%	100%

$V^{\mathrm{HS_1}}/V^{\mathrm{FB}}$

HS_1	v_1/v_2				
p_2	1/9	1/3	1	3	9
0	95%	97%	99%	100%	100%
0.25	95%	97%	99%	100%	100%
0.5	93%	95%	98%	100%	100%
0.75	89%	92%	97%	99%	100%
1	84%	88%	95%	99%	100%

Note: Here, $p_1 = 0.3$, $\delta = 0.95$, and $f_i(x_i) = v_i(1 - x_i/7)$ for $i = 1, 2$. The combination of values $(p_2, v_1/v_2)$ that achieves the lowest relative performance is highlighted in gray.

Proposition B-3. *Suppose that $n = 2$ and that Assumption 1(i) holds. There exists a state $\hat{x}_1^{HS}$ satisfying $f_1(\hat{x}_1^{HS} - 1) > \delta(1-\delta)V_1^{HS} \geq f_1(\hat{x}_1^{HS})$ such that $A^{HS}(\mathbf{x}) = P$ if and only if $x_1 < \hat{x}_1^{HS}$.*

Tables B-1 and B-2 depict the robustness of the basic worker-driven coordination scheduling rules, for piecewise constant and linear productivity functions. Similar to Table 3, the basic worker-driven rules suffer when workers have asymmetric probabilities and values. Yet, the suboptimality gap is lower than under Assumption 1(ii) because the more general productivity function is more forgiving to suboptimal rules (as also highlighted for Table 6).

B.2. Binary Productivity Function with a Base Value

We now consider a binary productivity function $f_i(x)$ with a base value: $f_i(x) = b_i + (v_i - b_i)]\mathbb{1}[x = 0]$, with $v_i > b_i \geq 0$. When $b_i > 0$, worker i may no longer want to coordinate whenever $x_i > 0$, unlike Lemma EC.1. The equilibrium under PC is the first to change as b_i increases because PC is the rule that favors the most coordination, followed by PP and HS_i. The equilibrium under HS_{-i} is independent of b_i. The proof is in §EC.2.6.

Proposition B-4. *Suppose that $n = 2$, that Assumption 1(i) holds, and that $f_i(x) = b_i + (v_i - b_i)]\mathbb{1}[x = 0]$, with $v_i > b_i \geq 0$ for $i = 1, 2$. For any i, there exist some thresholds $\underline{b}_i$ and $\bar{b}_i$ with $\underline{b}_i \leq \bar{b}_i$ such that*

- *for any $b \leq \underline{b}_i$, $a_i^{PC}(\mathbf{x}) = a_i^{PP}(\mathbf{x}) = a_i^{HS_i}(\mathbf{x}) = a_i^{HS_{-i}}(\mathbf{x}) = C \Leftrightarrow x_i \geq 1$;*
- *for any $b \leq \bar{b}_i$, $a_i^{PP}(\mathbf{x}) = a_i^{HS_i}(\mathbf{x}) = a_i^{HS_{-i}}(\mathbf{x}) = C \Leftrightarrow x_i \geq 1$;*
- *for any $b > \bar{b}_i$, $a_i^{HS_{-i}}(\mathbf{x}) = C \Leftrightarrow x_i \geq 1$.*

Appendix C: Team Incentives

Assume that, while producing, each worker i is paid an incremental wage $\gamma f_i(x_i) + (1-\gamma)\sum_{j\neq i} f_j(x_j)$, with $1/2 \le \gamma \le 1$. No wage is paid when coordinating. We assume two workers, i.e., $n=2$. This analysis is preliminary as we ignore team moral hazard issues.

With team incentives, workers' preferred outcomes are more aligned. When $\gamma < 1$, worker i may no longer choose to coordinate whenever $x_i > 0$ and $x_{-i} = 0$, unlike Lemma EC.1, and to produce whenever $x_i = 0$ and $x_{-i} > 0$, unlike Lemma EC.2. Hence, as $(1-\gamma)/\gamma$ increases, the equilibrium policy under PC, PP, and HS may be different from that in Proposition 2. Let $A^{\pi}_{\gamma}(\mathbf{x})$ be the equilibrium policy under rule π when weight $(1-\gamma)$ is put on team incentives. When $\gamma = 1/2$, workers' preferred outcomes are perfectly aligned: for any rule π, $a^{\pi}_1(\mathbf{x}, t) = a^{\pi}_2(\mathbf{x}, t)$. If π is either PC, PP, or HS as defined by (1), (2), and (3), then $\lim_{\gamma\to 1/2} A^{\pi}_{\gamma}(\mathbf{x}) = A^{\mathrm{FB}}(\mathbf{x})$.

The robustness of the equilibrium policies induced by PC, PP, and HS to team incentives depends on the parameters, through $\alpha(p_1, p_2, \delta)$ defined in (10). The proof appears is in §EC.2.6.

Proposition C-1. *Suppose that $n=2$, that Assumption 1 holds, that workers receive $\gamma f_i(x_i) + (1-\gamma) f_{-i}(x_{-i})$ for $i = 1, 2$, with $1/2 \le \gamma \le 1$, and that $v_1 \ge v_2$. Then, for any γ,*

- *If $\alpha(p_1, p_2, \delta) \ge 1$, then $\left(A^{HS_2}_{\gamma}(\mathbf{x}) = P \Leftrightarrow x_2 = 0\right) \Longrightarrow \left(A^{PP}_{\gamma}(\mathbf{x}) = C \Leftrightarrow \mathbf{x} \ge \mathbf{1}\right) \Longrightarrow \left(A^{PC}_{\gamma}(\mathbf{x}) = P \Leftrightarrow \mathbf{x} = \mathbf{0}\right)$ and $\left(A^{HS_2}_{\gamma}(\mathbf{x}) = P \Leftrightarrow x_2 = 0\right) \Longrightarrow \left(A^{HS_1}_{\gamma}(\mathbf{x}) = P \Leftrightarrow x_1 = 0\right)$;*
- *If $\alpha(p_1, p_2, \delta) \le 1$, then $\left(A^{PC}_{\gamma}(\mathbf{x}) = P \Leftrightarrow \mathbf{x} = \mathbf{0}\right) \Longrightarrow \left\{\left(A^{HS_1}_{\gamma}(\mathbf{x}) = P \Leftrightarrow x_1 = 0\right) \text{ and } \left(A^{HS_2}_{\gamma}(\mathbf{x}) = P \Leftrightarrow x_2 = 0\right)\right\}$ and $\left\{\left(A^{HS_1}_{\gamma}(\mathbf{x}) = P \Leftrightarrow x_1 = 0\right) \text{ or } \left(A^{HS_2}_{\gamma}(\mathbf{x}) = P \Leftrightarrow x_2 = 0\right)\right\} \Longrightarrow \left(A^{PP}_{\gamma}(\mathbf{x}) = C \Leftrightarrow \mathbf{x} \ge \mathbf{1}\right)$.*

Proposition C-1 shows that, when $\alpha(p_1, p_2, \delta) \ge 1$ and $v_1 \ge v_2$, our characterization of the equilibrium policies under PC in our base model (when $\gamma = 1$) is more robust to a decrease in γ than that under PP, and that those under rules PC, PP and HS_1 are all more robust than that under HS_2. This is intuitive since, by Proposition 2, when $\alpha(p_1, p_2, \delta) \ge 1$, either PC or HS_1 already achieves FB without team incentives, so adding them will thus not change anything. Either $A^{\mathrm{PC}}_{\gamma}(\mathbf{x})$ or $A^{\mathrm{HS}_1}_{\gamma}(\mathbf{x})$ is completely insensitive to γ, for any $\gamma \in [1/2, 1]$, depending on whether $v_1/v_2 \le \alpha(p_1, p_2, \delta)$. $A^{\mathrm{HS}_2}_{\gamma}(\mathbf{x})$ is the least robust: as soon as γ decreases from 1, worker 2 becomes more likely to coordinate even if she is fully productive, if worker 1 has accumulated issues.

Conversely, when $\alpha(p_1, p_2, \delta) \le 1$ and $v_1 \ge v_2$, it is our characterization of the equilibrium policies when $\gamma = 1$ under PP and potentially either HS_1 or HS_2 that are the most robust to a decrease in γ. Again, this is intuitive since, by Proposition 2, when $\alpha(p_1, p_2, \delta) \le 1$, either PP or HS_1 already achieves the FB policy without team incentives. Surprisingly, HS_2 (never optimal when $v_1 \ge v_2$) may be more robust than HS_1, in particular when both workers' productivities are comparable and worker 2 has high coordination demands (so that PP is optimal; see Figure 2). In this case, worker 1 may be tempted to produce even if she has accumulated issues whenever worker 2 is fully productive, whereas in the symmetric scenario, worker 2 would prefer to coordinate. $A^{\mathrm{PC}}(\mathbf{x})$ is the least robust: as soon as γ decreases from 1, worker 2 becomes more likely to produce even if she has accumulated issues, if worker 1 is fully productive.

In sum, the first equilibrium policies that will be affected by team incentives are $A^{\mathrm{PC}}(\mathbf{x})$ if $\alpha(p_1, p_2, \delta) \le 1$, $A^{\mathrm{HS}_2}(\mathbf{x})$ if $\alpha(p_1, p_2, \delta) \ge 1$ and $v_1 \ge v_2$, and $A^{\mathrm{HS}_1}(\mathbf{x})$ if $\alpha(p_1, p_2, \delta) \ge 1$ and $v_2 \ge v_1$.

Electronic Companion

Appendix EC.1: Average Cycle Durations and Individual Workers' Perceptions under Basic Coordination Rules

In this electronic appendix, we pursue our analysis of the effect of coordination scheduling rules on individual workers, started in Appendix A for the FB policy, by considering here the following basic rules $\pi \in \{\text{FB}, \text{PC}, \text{PP}, \text{HS}_1, \text{FI}\}$. For each rule, we compute, through simulation, the average cycle duration, and the inverse of meeting frequency. We set $p_1 = 0.3$, $\delta = 0.7$, and vary $p_2 \in \{0, 0.25, 0.5, 0.75, 1\}$ and $v_1/v_2 \in \{1/9, 1/3, 1, 3, 9\}$. For each rule and set of values p_2 and v_1/v_2, we first compute the value functions $V_i^\pi(\mathbf{x}, t)$ for all $i = 1, 2$, $\mathbf{0} \leq \mathbf{x} \leq (10, 10)$, $0 \leq t \leq 30$ by value iteration. Using these value-to-go functions, we then simulate 20,000 periods for each rule and report in Tables A-1 - EC.4 the mean cycle durations, the frequency of coordination periods while worker i wanted to produce ($\mathbb{P}[A^\pi(\mathbf{x}, t) = C$ and $a_i^\pi(\mathbf{x}, t) = P]$), and the frequency of production periods while worker i wanted to coordinate ($\mathbb{P}[A^\pi(\mathbf{x}, t) = P$ and $a_i^\pi(\mathbf{x}, t) = C]$).

Table EC.1 **PC average cycle durations and frequencies of outcomes involving some disagreement**

Average Cycle Duration

PC	v_1/v_2				
p_2	1/9	1/3	1	3	9
0	4.3	4.3	4.4	4.3	4.3
0.25	3.1	3.1	3.1	3.1	3.1
0.5	2.6	2.5	2.5	2.5	2.5
0.75	2.2	2.2	2.2	2.2	2.2
1	2.0	2.0	2.0	2.0	2.0

Worker preferring opposite outcome

Outcome	Worker	PC p_2	v_1/v_2 = 1/9	1/3	1	3	9
Coordination: $\mathbb{P}[A^{\text{PC}}(\mathbf{x}) = C$ and $a_1^{\text{PC}}(\mathbf{x}) = P]$	Worker 1	0	0%	0%	0%	0%	0%
		0.25	12%	11%	12%	12%	12%
		0.5	21%	21%	21%	21%	21%
		0.75	29%	29%	29%	29%	29%
		1	35%	34%	35%	35%	35%
Coordination: $\mathbb{P}[A^{\text{PC}}(\mathbf{x}) = C$ and $a_2^{\text{PC}}(\mathbf{x}) = P]$	Worker 2	0	23%	23%	23%	23%	23%
		0.25	15%	15%	15%	15%	15%
		0.5	9%	9%	9%	9%	9%
		0.75	4%	4%	4%	4%	4%
		1	0%	0%	0%	0%	0%
Production: $\mathbb{P}[A^{\text{PC}}(\mathbf{x}) = P$ and $a_1^{\text{PC}}(\mathbf{x}) = C]$	Worker 1	0, 0.25, 0.5, 0.75, 1	N/A; always 0%				
Production: $\mathbb{P}[A^{\text{PC}}(\mathbf{x}) = P$ and $a_2^{\text{PC}}(\mathbf{x}) = C]$	Worker 2	0, 0.25, 0.5, 0.75, 1	N/A; always 0%				

Note: Here, $p_1 = 0.3$ and $\delta = 0.7$ as in Figure 2. The average cycle durations and frequencies of outcomes are estimated from simulation over 20,000 transitions. The highlighted cells are those with the most symmetric workers.

Consider first the PC rule, in Table EC.1. Since the sojourn times in each state $\mathbf{x}$ are now independent of

Table EC.2 PP average cycle durations and frequencies of outcomes involving some disagreement

Average Cycle Duration

PP	v_1/v_2				
p_2	1/9	1/3	1	3	9
0	∞	∞	∞	∞	∞
0.25	6.2	6.3	6.2	6.2	6.2
0.5	4.7	4.8	4.8	4.8	4.8
0.75	4.5	4.6	4.4	4.4	4.4
1	4.4	4.3	4.3	4.3	4.3

Worker preferring opposite outcome

Coordination — Worker 1: $\mathbb{P}[A^{\text{PP}}(\mathbf{x}) = C \text{ and } a_1^{\text{PP}}(\mathbf{x}) = P]$

PP	v_1/v_2				
p_2	1/9	1/3	1	3	9
0, 0.25, 0.5, 0.75, 1	N/A; always 0%				

Coordination — Worker 2: $\mathbb{P}[A^{\text{PP}}(\mathbf{x}) = C \text{ and } a_2^{\text{PP}}(\mathbf{x}) = P]$

PP	v_1/v_2				
p_2	1/9	1/3	1	3	9
0, 0.25, 0.5, 0.75, 1	N/A; always 0%				

Production — Worker 1: $\mathbb{P}[A^{\text{PP}}(\mathbf{x}) = P \text{ and } a_1^{\text{PP}}(\mathbf{x}) = C]$

PP	v_1/v_2				
p_2	1/9	1/3	1	3	9
0	100%	100%	100%	100%	100%
0.25	32%	31%	29%	30%	29%
0.5	9%	9%	10%	10%	9%
0.75	3%	3%	3%	3%	3%
1	0%	0%	0%	0%	0%

Production — Worker 2: $\mathbb{P}[A^{\text{PP}}(\mathbf{x}) = P \text{ and } a_2^{\text{PP}}(\mathbf{x}) = C]$

PP	v_1/v_2				
p_2	1/9	1/3	1	3	9
0	0%	0%	0%	0%	0%
0.25	18%	20%	21%	21%	21%
0.5	37%	37%	38%	38%	38%
0.75	48%	49%	47%	47%	48%
1	55%	54%	54%	54%	53%

Note: Here, $p_1 = 0.3$ and $\delta = 0.7$ as in Figure 2. The average cycle durations and frequencies of outcomes are estimated from simulation over 20,000 transitions. The highlighted cells are those with the most symmetric workers.

the workers' relative productivities, the reported statistics are roughly invariant with respect to v_1/v_2. The average cycle durations become shorter as p_2 increases, which then increases the likelihood that coordination is triggered while worker 1 wants to produce. The cycle durations are shorter than in the FB since coordination occurs as soon as one worker has an issue. If the workers are almost homogenous, they will find 12% (worker 1) or 15% (worker 2) of meetings a waste of time, similar to the FB. By definition, PC never imposes production if one worker wants to coordinate. Table EC.1 reveals a likelihood of a worker disagreeing with an outcome of at most 35%—a stark contrast with the FB in which this can be 100%.

Consider now the PP rule, in Table EC.2. Similar to PC, the sojourn times in each state $\mathbf{x}$ are independent of the workers' productivities, which makes the reported statistics roughly invariant with respect to v_1/v_2, and the average cycle durations become shorter as p_2 increases. The cycle durations are now longer than in the FB since coordination only occurs when both workers have issues. By definition, PP never imposes coordination if one worker wants to produce, so no worker ever considers a meeting a waste. On the other hand, when $p_2 = 0$, the production cycle duration is infinite even though worker 1 always wants to coordinate.

We now turn to the HS_1 rule, in Table EC.3. The sojourn times in each state $\mathbf{x}$ are again independent of the workers' productivities, which makes the reported statistics roughly invariant with respect to v_1/v_2.

Table EC.3 HS$_1$ average cycle durations and frequencies of outcomes involving some disagreement

Average Cycle Duration

HS$_1$	v_1/v_2				
p_2	1/9	1/3	1	3	9
0	4.3	4.4	4.3	4.4	4.3
0.25	4.3	4.3	4.3	4.3	4.4
0.5	4.4	4.3	4.3	4.3	4.3
0.75	4.4	4.3	4.3	4.4	4.3
1	4.3	4.3	4.2	4.3	4.3

Worker preferring opposite outcome

Coordination — Worker 1: $\mathbb{P}[A^{\text{HS}_1}(\mathbf{x}) = C \text{ and } a_1^{\text{HS}_1}(\mathbf{x}) = P]$

HS$_1$	v_1/v_2				
p_2	1/9	1/3	1	3	9
0	N/A; always 0%				
0.25					
0.5					
0.75					
1					

Coordination — Worker 2: $\mathbb{P}[A^{\text{HS}_1}(\mathbf{x}) = C \text{ and } a_2^{\text{HS}_1}(\mathbf{x}) = P]$

HS$_1$	v_1/v_2				
p_2	1/9	1/3	1	3	9
0	23%	23%	23%	23%	23%
0.25	11%	11%	11%	11%	11%
0.5	5%	5%	5%	6%	6%
0.75	2%	2%	2%	2%	2%
1	0%	0%	0%	0%	0%

Production — Worker 1: $\mathbb{P}[A^{\text{HS}_1}(\mathbf{x}) = P \text{ and } a_1^{\text{HS}_1}(\mathbf{x}) = C]$

HS$_1$	v_1/v_2				
p_2	1/9	1/3	1	3	9
0	N/A; always 0%				
0.25					
0.5					
0.75					
1					

Production — Worker 2: $\mathbb{P}[A^{\text{HS}_1}(\mathbf{x}) = P \text{ and } a_2^{\text{HS}_1}(\mathbf{x}) = C]$

HS$_1$	v_1/v_2				
p_2	1/9	1/3	1	3	9
0	0%	0%	0%	0%	0%
0.25	29%	28%	29%	29%	29%
0.5	42%	51%	41%	41%	41%
0.75	49%	49%	48%	50%	50%
1	54%	54%	53%	54%	54%

Note: Here, $p_1 = 0.3$ and $\delta = 0.7$ as in Figure 2. The average cycle durations and frequencies of outcomes are estimated from simulation over 20,000 transitions. The highlighted cells are those with the most symmetric workers.

The mean cycle durations are also independent of p_2 since worker 1 is the sole decision-maker, and hence always agrees with the outcome. Worker 2 may disagree up to 54% of the time, similar to the worst case in FB and PP. From a worst-case perspective, worker 2 is no worse off under HS$_1$ than under FB, and worker 1 is (naturally) better off. When both workers are almost equal, worker 2 will consider 11% of meetings called by worker 1 a waste of time.

Finally, consider FI in Table EC.4. For each set of values for p_2 and v_1/v_2, the cycle duration is optimized over $\{2,\ldots,29\}$. Due to the discrete nature of this optimization problem, the frequencies of outcomes involving some disagreement may not be monotonic. Although the (optimized) cycle durations follow a similar pattern to the FB average cycle durations (see Table A-1), they sometimes deviate from simply rounding those off to obtain a discrete fixed interval. For instance, when $p_2 = 0$ and $v_1/v_2 = 1$, the optimal FI cycle is 8 periods, almost twice as large as the corresponding FB average cycle duration of 4.3. Based on Proposition 6, we know that $V^{\text{FI}}(T^*+1)/V^{\text{FI}}(T^*) \geq T^*/(T^*+1)$, so when $T^* = 8$, we expect $V^{\text{FI}}(T)$ to be quite robust to suboptimal choices of T (indeed, we find that $V^{\text{FI}}(4)/V^{\text{FI}}(8) \approx 97\%$), but it is interesting that rounding off the FB solution does not necessarily serve as a good baseline for T^*. In stark contrast with all other policies and rules, for almost any set of values of p_2 and v_1/v_2, both workers may disagree with the implemented

Table EC.4 FI average cycle durations and frequencies of outcomes involving some disagreement

Average Cycle Duration

FI	v_1/v_2				
p_2	1/9	1/3	1	3	9
0	29.0	29.0	8.0	4.0	4.0
0.25	4.0	4.0	4.0	4.0	4.0
0.5	3.0	3.0	3.0	3.0	3.0
0.75	2.0	2.0	3.0	3.0	3.0
1	2.0	2.0	2.0	3.0	3.0

Worker preferring opposite outcome

Outcome: Coordination — Worker 1: $\mathbb{P}[A^{\mathrm{FI}}(t) = C \text{ and } a_1^{\mathrm{FI}}(\mathbf{x}) = P]$

FI	v_1/v_2				
p_2	1/9	1/3	1	3	9
0	0%	0%	1%	9%	9%
0.25	9%	9%	8%	9%	9%
0.5	16%	16%	16%	17%	16%
0.75	35%	35%	17%	16%	16%
1	35%	35%	35%	16%	16%

Outcome: Coordination — Worker 2: $\mathbb{P}[A^{\mathrm{FI}}(t) = C \text{ and } a_2^{\mathrm{FI}}(\mathbf{x}) = P]$

FI	v_1/v_2				
p_2	1/9	1/3	1	3	9
0	3%	3%	13%	25%	25%
0.25	11%	11%	10%	10%	10%
0.5	9%	8%	8%	8%	8%
0.75	12%	13%	2%	2%	2%
1	0%	0%	0%	0%	0%

Outcome: Production — Worker 1: $\mathbb{P}[A^{\mathrm{FI}}(t) = P \text{ and } a_1^{\mathrm{FI}}(\mathbf{x}) = C]$

FI	v_1/v_2				
p_2	1/9	1/3	1	3	9
0	85%	85%	50%	20%	21%
0.25	20%	20%	20%	20%	20%
0.5	10%	10%	10%	10%	10%
0.75	0%	0%	9%	10%	10%
1	0%	0%	0%	10%	10%

Outcome: Production — Worker 2: $\mathbb{P}[A^{\mathrm{FI}}(t) = P \text{ and } a_2^{\mathrm{FI}}(\mathbf{x}) = C]$

FI	v_1/v_2				
p_2	1/9	1/3	1	3	9
0	0%	0%	0%	0%	0%
0.25	17%	17%	17%	17%	17%
0.5	16%	17%	17%	17%	16%
0.75	0%	0%	25%	25%	25%
1	0%	0%	0%	33%	33%

Note: Here, $p_1 = 0.3$ and $\delta = 0.7$ as in Figure 2. The average cycle durations and frequencies of outcomes are estimated from simulation over 20,000 transitions. For each set of values for p_2 and v_1/v_2, the cycle duration is optimized over $\{2, \ldots, 29\}$. The highlighted cells are those with the most symmetric workers.

outcome, because the FI rule is time-based and not worker-driven. Nevertheless, the frequency of outcomes involving disagreement is bounded by 85%, in contrast to the 100% possible under FB and PP, but it is higher than the worst case under PC (35%) and HS_1 (54%). Nearly equal workers will consider 8% (worker 1) or 10% (worker 2) of meetings wasted. Overall, when workers are almost homogenous, they may consider up to 15% of meetings to be wasted, depending on the scenario. When workers are not homogenous, the less valuable worker may find up to 35% of meetings a waste.

Appendix EC.2: Proofs of Statements

This electronic appendix is organized as follows. We provide the proofs of the results of §3 in §EC.2.1, of those of §4 in §EC.2.2, of those of §5 in §EC.2.3, of those of §6 in §EC.2.4, of those from Appendix B.1 in §EC.2.5, and of those from Appendices B.2 and C in §EC.2.6.

Throughout the proofs, we adopt the following mathematical conventions: For any function $g(.)$, $\sum_{t=0}^{-1} g(t) \doteq 0$, and $\prod_{k=j}^{m} g(k) \doteq 1$ if $j > m$. In addition, we denote expectations of random variables as follows: $\mathbb{E}[g(\tilde{\mathbf{x}})] \doteq \sum_{\boldsymbol{\xi} \geq \mathbf{0}} g(\mathbf{x} + \boldsymbol{\xi})\mathbb{P}[\boldsymbol{\xi}]$ for any function $g(.)$.

EC.2.1. Preliminaries

Proposition 1 establishes that the FB policy is a threshold policy.

Proof of Proposition 1. The proof is similar to Example 1.2.1 in Bertsekas (2001). Applying the value iteration algorithm (Bertsekas 2001, Proposition 1.2.1) shows that $V^{\text{FB}}(\mathbf{x})$ is nonincreasing in $\mathbf{x}$ given that the per-period reward function $\sum_{i=1}^n f_i(x_i)$ is nonincreasing in $\mathbf{x}$ and that the transition probabilities $\mathbb{P}[\boldsymbol{\xi}]$ are stationary. Consider any state $\mathbf{y}$ such that $A^{\text{FB}}(\mathbf{y}) = C$. By (7), $\sum_{i=1}^n f_i(y_i) + \delta\mathbb{E}[V^{\text{FB}}(\mathbf{y}+\boldsymbol{\xi})] \leq \delta V^{\text{FB}}(\mathbf{0})$. Hence for any state $\mathbf{z} \geq \mathbf{y}$, because $f_i(z_i) \leq f_i(y_i)$ and $V^{\text{FB}}(\mathbf{z}) \leq V^{\text{FB}}(\mathbf{y})$, we obtain that $\sum_{i=1}^n f_i(z_i) + \delta\mathbb{E}[V^{\text{FB}}(\mathbf{z}+\boldsymbol{\xi})] \leq \delta V^{\text{FB}}(\mathbf{0})$, i.e., $A^{\text{FB}}(\mathbf{z}) = C$. That is, problem (7)-(8) is an optimal stopping problem. Define $\mathbf{x}$ as one of the largest (componentwise) states in which it is optimal to produce; i.e., for all i, if worker i has one more issue then it becomes optimal to coordinate. Accordingly, by (8), $V^{\text{FB}}(\mathbf{x}+\boldsymbol{\xi}) = \delta V^{\text{FB}}(\mathbf{0})$ for any $\boldsymbol{\xi} \geq \mathbf{0}$ such that $\boldsymbol{\xi} \neq \mathbf{0}$. Hence, by (7)-(8), $\sum_{i=1}^n f_i(x_i) > \delta V^{\text{FB}}(\mathbf{0})(1-\delta) \doteq \phi$.

We next show that $\phi < \sum_{i=1}^n f_i(0)$. If we had that $A^{\text{FB}}(\mathbf{0}) = C$, we would then have $V^{\text{FB}}(\mathbf{0}) = \delta V^{\text{FB}}(\mathbf{0})$, a contradiction since $\delta < 1$ and $\sum_i f_i(\mathbf{0}) > 0$. We finally show that $\phi \geq 0$. If we had that $A^{\text{FB}}(\overline{\mathbf{x}}) = P$, we would then have, by (7), $\delta V^{\text{FB}}(\mathbf{0}) < 0 + \delta\mathbb{E}[V^{\text{FB}}(\overline{\mathbf{x}}+\boldsymbol{\xi})]$, a contradiction since $V^{\text{FB}}(\mathbf{x})$ is nonincreasing. □

The next two lemmas show that each worker's dynamic program to state their preferred outcome (5) simplies under Assumption 1(ii) to (9). First, Lemma EC.1 shows that worker i prefers to coordinate when she have more than $\overline{x}_i$ issues.

LEMMA EC.1. *For any $\mathbf{x}$ such that $x_i \geq \overline{x}_i$, $a_i^\pi(\mathbf{x},t) = C$, for any t.*

Proof. Fix t and $\mathbf{x}$ such that $x_i \geq \overline{x}_i$. Consider any sample path of transitions $\sigma = (\boldsymbol{\xi}_1^\sigma, \boldsymbol{\xi}_2^\sigma, \ldots)$. Let $\theta_\sigma^\pi(\mathbf{x},t) \geq 1$ the number of periods of production under coordination scheduling rule π on σ if production takes place in $(\mathbf{x},t)$ until coordination happens (if it ever happens). That is, for any $1 \leq \tau < \theta_\sigma^\pi(\mathbf{x},t)$, $A^\pi(\mathbf{x} + \sum_{s=1}^\tau \boldsymbol{\xi}_s^\sigma, t+\tau) = P$ and $A^\pi(\mathbf{x} + \sum_{s=1}^{\theta_\sigma^\pi(\mathbf{x},t)} \boldsymbol{\xi}_s^\sigma, t+\theta_\sigma^\pi(\mathbf{x},t)) = C$. Since $f(x_i+\xi_i) = 0$ for all $\xi_i \geq 0$ since $x_i \geq \overline{x}_i$ and since transitions happen only upwards, worker i's value-to-go in state $(\mathbf{x},t)$ if the outcome were to produce would be equal to $0 + \delta^{\theta_\sigma^\pi(\mathbf{x},t)+1} V_i^\pi(\mathbf{0},0) \leq \delta^2 V_i^\pi(\mathbf{0},0) \leq \delta V_i^\pi(\mathbf{0},0)$. Hence, by (5), $a_i^\pi(\mathbf{x},t) = C$. □

Second, Lemma EC.2 shows that, if worker i's productivity drops to 0 if she has an issue (Assumption 1(ii)), she prefers to produce when she has no issue.

LEMMA EC.2. *Under Assumption 1(ii), for any $\mathbf{x}$ such that $x_i = 0$, $a_i^\pi(\mathbf{x},t) = P$.*

Proof. The proof is by policy iteration. Suppose that, in any future time period, $a_j^\pi(\mathbf{x},t) = P \Leftrightarrow x_j = 0$ for all $(\mathbf{x},t)$ and for all $j = 1,\ldots,n$. We show that the same applies in the current period. Specifically, fixing a state $(\mathbf{x},t)$ such that $x_i = 0$, we show that, on each sample path, worker i's value-to-go obtained from producing in $(\mathbf{x},t)$ is higher than her value-to-go obtained from coordinating in $(\mathbf{x},t)$, i.e., $\delta V_i^\pi(\mathbf{0},0)$.

Fix any T and suppose coordination must happen by time $T+1$. Since T is chosen arbitrarily, this is without loss of generality. Consider any sample path of transitions $\sigma = (\boldsymbol{\xi}_1^\sigma, \boldsymbol{\xi}_2^\sigma, \ldots, \boldsymbol{\xi}_{T-1}^\sigma)$, which happens with probability $\mathbb{P}_\sigma \doteq \prod_{s=1}^{T-1} \mathbb{P}[\boldsymbol{\xi}_s^\sigma]$. Let $\mathcal{U}_T$ be the set of all possible sample paths. If production were to take place in $(\mathbf{x},t)$, let $1 \leq \theta_\sigma^\pi(\mathbf{x},t) \leq T$ be the number of periods of production under π on σ until coordination happens. That is, for any $1 \leq \tau < \theta_\sigma^\pi(\mathbf{x},t)$, $A^\pi(\mathbf{x} + \sum_{s=1}^\tau \boldsymbol{\xi}_s^\sigma, t+\tau) = P$ and $A^\pi(\mathbf{x} + \sum_{s=1}^{\theta_\sigma^\pi(\mathbf{x},t)} \boldsymbol{\xi}_s^\sigma, t+\theta_\sigma^\pi(\mathbf{x},t))) = C$.

Accordingly, worker i's value-to-go from $(\mathbf{x},t)$ on sample path σ equals $\sum_{\tau=0}^{\theta^\pi_\sigma(\mathbf{x},t)-1}\delta^\tau f_i\left(x_i+\sum_{s=1}^{\tau}\xi^\sigma_{s,i}\right)+\delta^{\theta^\pi_\sigma(\mathbf{x},t)+1}V^\pi_i(\mathbf{0},0)$ in which $\xi^\sigma_{s,i}$ is the ith component of vector $\boldsymbol{\xi}^\sigma_s$.

Consider now state $(\mathbf{0},0)$. By assumption, coordination is always followed by production, so $A^\pi(\mathbf{a},0)=P$ for any $\mathbf{a}$. Therefore, worker i's value-to-go from $(\mathbf{0},0)$ on sample path σ can be similarly defined as $\sum_{\tau=0}^{\theta^\pi_\sigma(\mathbf{0},0)-1}\delta^\tau f_i\left(\sum_{s=1}^{\tau}\xi^\sigma_{s,i}\right)+\delta^{\theta^\pi_\sigma(\mathbf{0},0)+1}V^\pi_i(\mathbf{0},0)$, with $1\le\theta^\pi_\sigma(\mathbf{0},0)\le T$.

Since, in the future, $a^\pi_j(\mathbf{x},t)=P\Leftrightarrow x_j=0$ for all $(\mathbf{x},t)$ and for all $j=1,\dots,n$, $a^\pi_j(\mathbf{x},t)$ is thus independent of t, for all $j=1,\dots,n$. Therefore, under the monotonicity assumptions that for any $\mathbf{a}$, $A^\pi(\mathbf{a},t)=C\Rightarrow A^\pi(\mathbf{a}',t')=C$ for all $\mathbf{a}'$ such that for all i, $a_i=C\Rightarrow a'_i=C$, and for all $t'\ge t$, the rule in the future is such that $A^\pi(\mathbf{a}^\pi(\mathbf{x},t),t)=C\Rightarrow A^\pi(\mathbf{a}^\pi(\mathbf{x}',t'),t')=C$ for all $t'\ge t$ and $\mathbf{x}'\ge\mathbf{x}$. As a result, $\theta^\pi_\sigma(\mathbf{x},t)\le\theta^\pi_\sigma(\mathbf{0},0)$, i.e., coordination occurs sooner when starting from $(\mathbf{x},t)$ than from $(0,0)$.

Let $\chi_{\sigma,i}\doteq\min\{s\le T+1|\xi^\sigma_{s,i}\ge 1\}$ be the first time on the sample path σ that worker i encounters an issue if she encounters one, or $T+1$ otherwise. That is, the ith component of $\mathbf{x}+\sum_{s=0}^{\tau}\boldsymbol{\xi}^\sigma_s$ is equal to zero for all $\tau<\chi_{\sigma,i}$ and greater than or equal to one afterwards. We distinguish two types of sample paths: let $\mathcal{S}^\pi(\mathbf{x},t)\doteq\{\sigma|\chi_{\sigma,i}<\theta^\pi_\sigma(\mathbf{x},t)\}$ and $\mathcal{T}^\pi(\mathbf{x},t)\doteq\{\sigma|\chi_{\sigma,i}\ge\theta^\pi_\sigma(\mathbf{x},t)\}$. That is, on sample paths $\sigma\in\mathcal{T}^\pi(\mathbf{x},t)$, starting from $(\mathbf{x},t)$, worker i earns v_i until coordination takes place, unlike on sample paths $\sigma\in\mathcal{S}^\pi(\mathbf{x},t)$, on which worker i earns zero for some periods before coordination takes place. Obviously, $\mathcal{S}^\pi(\mathbf{x},t)\cup\mathcal{T}^\pi(\mathbf{x},t)=\mathcal{U}_T$.

For any $\sigma\in\mathcal{S}^\pi(\mathbf{x},t)$, $\sum_{\tau=0}^{\theta^\pi_\sigma(\mathbf{x},t)-1}\delta^\tau f_i\left(x_i+\sum_{s=1}^{\tau}\xi^\sigma_{s,i}\right)=\sum_{\tau=0}^{\chi_{\sigma,i}-1}\delta^\tau v_i=\sum_{\tau=0}^{\theta^\pi_\sigma(\mathbf{0},0)-1}\delta^\tau f_i\left(0+\sum_{s=1}^{\tau}\xi^\sigma_{s,i}\right)$. The second equality follows because $\theta^\pi_\sigma(\mathbf{x},t)\le\theta^\pi_\sigma(\mathbf{0},0)$; hence, if worker i earns v for $\chi_{\sigma,i}$ on sample path σ starting from $(\mathbf{x},t)$, the same will happen starting from $(\mathbf{0},0)$. Consequently, for any $\sigma\in\mathcal{S}^\pi(\mathbf{x},t)$, worker i's value-to-go on sample path σ if production happens in state $(\mathbf{x},t)$ is greater than or equal to her value-to-go on the same sample path, but starting from state $(\mathbf{0},0)$, i.e.,

$$\left(\sum_{\tau=0}^{\theta^\pi_\sigma(\mathbf{x},t)-1}\delta^\tau f_i\left(x_i+\sum_{s=1}^{\tau}\xi^\sigma_{s,i}\right)+\delta^{\theta^\pi_\sigma(\mathbf{x},t)+1}V^\pi_i(\mathbf{0},0)\right)\ge\left(\sum_{\tau=0}^{\theta^\pi_\sigma(\mathbf{0},0)-1}\delta^\tau f_i\left(\sum_{s=1}^{\tau}\xi^\sigma_{s,i}\right)+\delta^{\theta^\pi_\sigma(\mathbf{0},0)+1}V^\pi_i(\mathbf{0},0)\right).$$

Since $\delta<1$, $V^\pi_i(\mathbf{0},0)>0$, $f_i(0)>0$, and $\theta^\pi_\sigma(\mathbf{0},0)\ge 1$, the above inequality is strict:

$$\left(\sum_{\tau=0}^{\theta^\pi_\sigma(\mathbf{x},t)-1}\delta^\tau f_i\left(x_i+\sum_{s=1}^{\tau}\xi^\sigma_{s,i}\right)+\delta^{\theta^\pi_\sigma(\mathbf{x},t)+1}V^\pi_i(\mathbf{0},0)\right)>\delta\left(\sum_{\tau=0}^{\theta^\pi_\sigma(\mathbf{0},0)-1}\delta^\tau f_i\left(\sum_{s=1}^{\tau}\xi^\sigma_{s,i}\right)+\delta^{\theta^\pi_\sigma(\mathbf{0},0)+1}V^\pi_i(\mathbf{0},0)\right)$$
$$\forall\sigma\in\mathcal{S}^\pi(\mathbf{x},t). \tag{EC.1}$$

Consider next any sample path $\sigma\in\mathcal{T}^\pi(\mathbf{x},t)$. Simplifying

$$V^\pi_i(\mathbf{0},0)=\sum_{\sigma'\in\mathcal{U}_T}\mathbb{P}_{\sigma'}\left(\sum_{\tau=0}^{\min\{\chi_{\sigma'},\theta^\pi_{\sigma'}(\mathbf{0},0)\}-1}\delta^\tau v_i+\delta^{\theta^\pi_{\sigma'}(\mathbf{0},0)+1}V^\pi_i(\mathbf{0},0)\right)$$

yields

$$V^\pi_i(\mathbf{0},0)=\frac{\sum_{\sigma'\in\mathcal{U}_T}\mathbb{P}_{\sigma'}\left(\sum_{\tau=0}^{\min\{\chi_{\sigma'},\theta^\pi_{\sigma'}(\mathbf{0},0)\}-1}\delta^\tau v_i\right)}{1-\sum_{\sigma'\in\mathcal{U}_T}\mathbb{P}_{\sigma'}\delta^{\theta^\pi_{\sigma'}(\mathbf{0},0)+1}}.$$

From this we obtain:

$$\left(\sum_{\tau=0}^{\theta^\pi_\sigma(\mathbf{x},t)-1}\delta^\tau f_i\left(x_i+\sum_{s=1}^{\tau}\xi^\sigma_{s,i}\right)+\delta^{\theta^\pi_\sigma(\mathbf{x},t)+1}V^\pi_i(\mathbf{0},0)\right)-\delta\left(\sum_{\tau=0}^{\theta^\pi_\sigma(\mathbf{0},0)-1}\delta^\tau f_i\left(\sum_{s=1}^{\tau}\xi^\sigma_{s,i}\right)+\delta^{\theta^\pi_\sigma(\mathbf{0},0)+1}V^\pi_i(\mathbf{0},0)\right)$$

$$= \left(\sum_{\tau=0}^{\theta^\pi_\sigma(\mathbf{x},t)-1} \delta^\tau v_i + \delta^{\theta^\pi_\sigma(\mathbf{x},t)+1} V_i^\pi(\mathbf{0},0) \right) - \delta \left(\sum_{\tau=0}^{\min\{\chi_{\sigma,i},\theta^\pi_\sigma(\mathbf{0},0)\}-1} \delta^\tau v_i + \delta^{\theta^\pi_\sigma(\mathbf{0},0)+1} V_i^\pi(\mathbf{0},0) \right)$$
$$= \frac{v_i}{1-\delta}\left(1-\delta^{\theta^\pi_\sigma(\mathbf{x},t)} - \delta + \delta^{\min\{\chi_{\sigma,i},\theta^\pi_\sigma(\mathbf{0},0)\}+1}\right) + \left(\delta^{\theta^\pi_\sigma(\mathbf{x},t)+1} - \delta^{\theta^\pi_\sigma(\mathbf{0},0)+2}\right) V_i^\pi(\mathbf{0},0)$$
$$= \frac{v_i}{1-\delta}\left(1-\delta^{\theta^\pi_\sigma(\mathbf{x},t)} - \delta + \delta^{\min\{\chi_{\sigma,i},\theta^\pi_\sigma(\mathbf{0},0)\}+1}\right) + \left(\delta^{\theta^\pi_\sigma(\mathbf{x},t)+1} - \delta^{\theta^\pi_\sigma(\mathbf{0},0)+2}\right) \frac{v_i}{1-\delta} \frac{1-\sum_{\sigma'\in\mathcal{U}_T} \mathbb{P}_{\sigma'} \delta^{\min\{\chi_{\sigma'},\theta^\pi_{\sigma'}(\mathbf{0},0)\}}}{1-\sum_{\sigma'\in\mathcal{U}_T} \mathbb{P}_{\sigma'} \delta^{\theta^\pi_{\sigma'}(\mathbf{0},0)+1}}$$
$$= \frac{v_i}{(1-\delta)\left(1-\sum_{\sigma'\in\mathcal{U}_T} \mathbb{P}_{\sigma'} \delta^{\theta^\pi_{\sigma'}(\mathbf{0},0)+1}\right)} \times F,$$

in which

$$F \doteq \left(1-\delta^{\theta^\pi_\sigma(\mathbf{x},t)} - \delta + \delta^{\min\{\chi_{\sigma,i},\theta^\pi_\sigma(\mathbf{0},0)\}+1}\right)\left(1 - \sum_{\sigma'\in\mathcal{U}_T} \mathbb{P}_{\sigma'} \delta^{\theta^\pi_{\sigma'}(\mathbf{0},0)+1}\right)$$
$$+ \left(\delta^{\theta^\pi_\sigma(\mathbf{x},t)+1} - \delta^{\theta^\pi_\sigma(\mathbf{0},0)+2}\right)\left(1 - \sum_{\sigma'\in\mathcal{S}^\pi(\mathbf{x},t)\cup\mathcal{T}^\pi(\mathbf{x},t)} \mathbb{P}_{\sigma'} \delta^{\min\{\chi_{\sigma'},\theta^\pi_{\sigma'}(\mathbf{0},0)\}}\right).$$

The rest of the proof consists in showing that $F > 0$ across all problem instances. For any $\sigma' \neq \sigma$, F is nondecreasing in $\chi_{\sigma'}$. Hence, a lower bound on F is obtained by replacing $\chi_{\sigma'}$ by its lower bound, namely, 1 if $\sigma' \in \mathcal{S}^\pi(\mathbf{x},t)$ and $\theta^\pi_{\sigma'}(\mathbf{x},t)$ if $\sigma' \in \mathcal{T}^\pi(\mathbf{x},t)$. Since, when $\sigma' \in \mathcal{T}^\pi(\mathbf{x},t)$, the term $\theta^\pi_{\sigma'}(\mathbf{x},t)$ does not appear in F, this can be further tightened by replacing it with its lower bound, equal to 1. In contrast, F is nonincreasing in χ_σ since the derivative of F with respect to $\delta^{\min\{\chi_{\sigma,i},\theta^\pi_\sigma(\mathbf{0},0)\}}$ equals $\delta(1-\sum_{\sigma'\neq\sigma} \mathbb{P}_{\sigma'} \delta^{\theta^\pi_{\sigma'}(\mathbf{0},0)+1} - \mathbb{P}_\sigma \delta^{\theta^\pi_\sigma(\mathbf{x},t)})$, which is nonnegative. Hence, a lower bound on F is obtained by replacing $\chi_{\sigma,i}$ with its upper bound, i.e., T. Since T was chosen arbitrarily, a looser upper bound consists in taking the limit when $\chi_{\sigma,i} \to \infty$. Implementing these two changes, we obtain

$$F \geq F' \doteq \left(1-\delta^{\theta^\pi_\sigma(\mathbf{x},t)} - \delta + \delta^{\theta^\pi_\sigma(\mathbf{0},0)+1}\right)\left(1 - \sum_{\sigma'\in\mathcal{U}_T} \mathbb{P}_{\sigma'} \delta^{\theta^\pi_{\sigma'}(\mathbf{0},0)+1}\right)$$
$$+ \left(\delta^{\theta^\pi_\sigma(\mathbf{x},t)+1} - \delta^{\theta^\pi_\sigma(\mathbf{0},0)+2}\right)\left(1 - \sum_{\sigma'\in\mathcal{U}_T\setminus\{\sigma\}} \mathbb{P}_{\sigma'} \delta - \mathbb{P}_\sigma \delta^{\theta^\pi_\sigma(\mathbf{0},0)}\right).$$

We consider two cases. First, if $1-\delta^{\theta^\pi_\sigma(\mathbf{x},t)} - \delta + \delta^{\theta^\pi_\sigma(\mathbf{0},0)+1} > 0$, then $F' > 0$. Second, suppose that $1 - \delta^{\theta^\pi_\sigma(\mathbf{x},t)} - \delta + \delta^{\theta^\pi_\sigma(\mathbf{0},0)+1} \leq 0$. Under this assumption, F' is nondecreasing in $\delta^{\theta^\pi_{\sigma'}(\mathbf{0},0)}$ for any $\sigma' \neq \sigma$. Hence a lower bound on F' is achieved by replacing $\delta^{\theta^\pi_{\sigma'}(\mathbf{0},0)}$ by its lower bound, namely 0, for any $\sigma' \neq \sigma$. Accordingly,

$$F' \geq F'' \doteq \left(1-\delta^{\theta^\pi_\sigma(\mathbf{x},t)} - \delta + \delta^{\theta^\pi_\sigma(\mathbf{0},0)+1}\right)\left(1 - \mathbb{P}_\sigma \delta^{\theta^\pi_\sigma(\mathbf{0},0)+1}\right)$$
$$+ \left(\delta^{\theta^\pi_\sigma(\mathbf{x},t)+1} - \delta^{\theta^\pi_\sigma(\mathbf{0},0)+2}\right)\left(1 - (1-\mathbb{P}_\sigma)\delta - \mathbb{P}_\sigma \delta^{\theta^\pi_\sigma(\mathbf{0},0)}\right).$$

Since F'' is nonincreasing in $\mathbb{P}_\sigma$ given that $\theta^\pi_\sigma(\mathbf{x},t) \leq \theta^\pi_\sigma(\mathbf{0},0)$, a lower bound on F'' is achieved by replacing $\mathbb{P}_\sigma$ with its upper bound, i.e., 1, yielding:

$$F'' \geq F''' \doteq \left(1-\delta^{\theta^\pi_\sigma(\mathbf{x},t)} - \delta + \delta^{\theta^\pi_\sigma(\mathbf{0},0)+1}\right)\left(1 - \delta^{\theta^\pi_\sigma(\mathbf{0},0)+1}\right) + \left(\delta^{\theta^\pi_\sigma(\mathbf{x},t)+1} - \delta^{\theta^\pi_\sigma(\mathbf{0},0)+2}\right)\left(1 - \delta^{\theta^\pi_\sigma(\mathbf{0},0)}\right).$$

Since F''' is nonincreasing in $\delta^{\theta^\pi_\sigma(\mathbf{x},t)}$, a lower bound on F''' is achieved by replacing $\delta^{\theta^\pi_\sigma(\mathbf{x},t)}$ with its upper bound, i.e., $\delta^{\theta^\pi_\sigma(\mathbf{0},0)}$, yielding:

$$F''' \geq (1-\delta^{\theta^\pi_\sigma(\mathbf{0},0)})(1-\delta) > 0.$$

Combining these inequalities, we obtain:

$$\left(\sum_{\tau=0}^{\theta_\sigma^\pi(\mathbf{x},t)-1}\delta^\tau f_i\left(x_i+\sum_{s=1}^{\tau}\xi_{s,i}^\sigma\right)+\delta^{\theta_\sigma^\pi(\mathbf{x},t)+1}V_i^\pi(\mathbf{0},0)\right)>\delta\left(\sum_{\tau=0}^{\theta_\sigma^\pi(\mathbf{0},0)-1}\delta^\tau f_i\left(\sum_{s=1}^{\tau}\xi_{s,i}^\sigma\right)+\delta^{\theta_\sigma^\pi(\mathbf{0},0)+1}V_i^\pi(\mathbf{0},0)\right)$$
$$\forall\sigma\in\mathcal{T}^\pi(\mathbf{x},t). \tag{EC.2}$$

Combining (EC.1) and (EC.2), we obtain that

$$\begin{aligned}V_i^\pi(\mathbf{x},t|a^\pi(\mathbf{x},t)=P)&=\sum_\sigma\left(\sum_{\tau=0}^{\theta_\sigma^\pi(\mathbf{x},t)-1}\delta^\tau f_i\left(x_i+\sum_{s=1}^{\tau}\xi_{s,i}^\sigma\right)+\delta^{\theta_\sigma^\pi(\mathbf{x},t)+1}V_i^\pi(\mathbf{0},0)\right)\\&>\delta\sum_\sigma\left(\sum_{\tau=0}^{\theta_\sigma^\pi(\mathbf{0},0)-1}\delta^\tau f_i\left(\sum_{s=1}^{\tau}\xi_{s,i}^\sigma\right)+\delta^{\theta_\sigma^\pi(\mathbf{0},0)+1}V_i^\pi(\mathbf{0},0)\right)\\&=\delta V_i^\pi(\mathbf{0},0)\end{aligned}$$

Hence, $a_i^\pi(\mathbf{x},t)=P$. □

EC.2.2. Teams of Two Workers

Proposition 2 characterizes the FB policy for teams of two workers, using the generic characterization of the FB policy (Proposition 1).

Proof of Proposition 2. By Proposition 1, $A^{\text{FB}}(\mathbf{x})=P$ if and only if $\sum_{i=1}^2 f_i(x_i)>\phi$, for some ϕ and $V^{\text{FB}}(\mathbf{0})=\phi/(\delta(1-\delta))$. Under Assumption 1, $\sum_{i=1}^2 f_i(x_i)\in\{0,v_2,v_1,v_1+v_2\}$. Because $\phi\in[0,v_1+v_2)$ and because $v_1\geq v_2$, $\phi\in[0,v_2)\cup[v_2,v_1)\cup[v_1,v_1+v_2)$. We first evaluate $V^{\text{FB}}(\mathbf{0})$ for each of these three cases and then compare these three values to obtain optimality conditions in each case. To simplify the notation, we henceforth omit the superscript 'FB'. We also denote by $V^{\text{FB}}(\mathbf{x}|A)$ the value function in state $\mathbf{x}$ when action $A\in\{C,P\}$ is taken in the current period and the FB policy is followed subsequently.

- If $\phi\in[v_1,v_1+v_2)$, $A(\mathbf{x})=P$ if and only if $\mathbf{x}=\mathbf{0}$. Accordingly,

$$\begin{aligned}V(\mathbf{0})&=v_1+v_2+\delta(1-p_1)(1-p_2)V(\mathbf{0})+\delta p_1(1-p_2)V((1,0)|C)\\&\quad+\delta(1-p_1)p_2V((0,1)|C)+\delta p_1p_2V((1,1)|C),\text{which reduces to}\\V(\mathbf{0})&\left(1-\delta(1-p_1)(1-p_2)\right)=v_1+v_2+\delta\left(1-(1-p_1)(1-p_2)\right)\delta V(\mathbf{0})\\\Leftrightarrow V(\mathbf{0})&(1-\delta)\left(1+\delta-\delta(1-p_1)(1-p_2)\right)=v_1+v_2\\\Leftrightarrow V(\mathbf{0})&(1-\delta)=\frac{v_1+v_2}{1+\delta-\delta(1-p_1)(1-p_2)}.\end{aligned}\tag{EC.3}$$

- If $\phi\in[v_2,v_1)$, $A(\mathbf{x})=P$ if and only if $x_1=0$. Accordingly,

$$\begin{aligned}V(\mathbf{0})&=v_1+v_2+\delta(1-p_1)(1-p_2)V(\mathbf{0})+\delta p_1(1-p_2)V((1,0)|C)\\&\quad+\delta(1-p_1)p_2V((0,1)|P)+\delta p_1p_2V((1,1)|C),\text{which reduces to}\\V(\mathbf{0})&\left(1-\delta(1-p_1)(1-p_2)\right)=v_1+v_2+\delta p_2(1-p_1)\left(\frac{v_1+\delta p_1\delta V(\mathbf{0})}{1-\delta(1-p_1)}\right)+\delta p_1\delta V(\mathbf{0})\\\Leftrightarrow V(\mathbf{0})&(1-\delta)\left(1-\delta(1-p_1)(1-p_2)\right)\frac{1+\delta p_1}{1-\delta(1-p_1)}=\frac{1-\delta(1-p_1)(1-p_2)}{1-\delta(1-p_1)}v_1+v_2\\\Leftrightarrow V(\mathbf{0})&(1-\delta)=\frac{v_1}{1+\delta p_1}+\frac{v_2(1-\delta(1-p_1))}{(1+\delta p_1)(1-\delta(1-p_1)(1-p_2))}.\end{aligned}\tag{EC.4}$$

- If $\phi \in [0, v_2)$, $A(\mathbf{x}) = P$ if and only if $x_1 = 0$ or $x_2 = 0$. Accordingly,

$$\begin{aligned}
&V(\mathbf{0}) = v_1 + v_2 + \delta(1-p_1)(1-p_2)V(\mathbf{0}) + \delta p_1(1-p_2)V((1,0)|P) \\
&\qquad + \delta(1-p_1)p_2V((0,1)|P) + \delta p_1 p_2 V((1,1)|C), \text{which reduces to} \\
&V(\mathbf{0})\left(1-\delta(1-p_1)(1-p_2)\right) \\
&\quad = v_1 + v_2 + \delta p_2(1-p_1)\left(\frac{v_1 + \delta p_1 \delta V(\mathbf{0})}{1-\delta(1-p_1)}\right) + \delta p_1(1-p_2)\left(\frac{v_2 + \delta p_2 \delta V(\mathbf{0})}{1-\delta(1-p_2)}\right) + \delta p_1 p_2 \delta V(\mathbf{0}) \\
\Leftrightarrow\ &V(\mathbf{0})(1-\delta)\frac{\delta^2(p_1^2 + p_2^2 + p_1p_2(1-\delta p_1 p_2)) + \delta(1-\delta)(2-\delta p_1p_2)(p_1+p_2) + (1-\delta)^2(1-\delta p_1p_2)}{(1-\delta(1-p_1))(1-\delta(1-p_2))} \\
&\quad = (1-\delta(1-p_1)(1-p_2))\left(\frac{v_1}{1-\delta(1-p_1)} + \frac{v_2}{1-\delta(1-p_2)}\right) \\
\Leftrightarrow\ &V(\mathbf{0})(1-\delta) = \frac{\left(v_1(1-\delta(1-p_2)) + v_2(1-\delta(1-p_1))\right)(1-\delta(1-p_1)(1-p_2))}{\delta^2(p_1^2+p_2^2+p_1p_2(1-\delta p_1p_2)) + \delta(1-\delta)(2-\delta p_1p_2)(p_1+p_2) + (1-\delta)^2(1-\delta p_1 p_2)}.
\end{aligned} \tag{EC.5}$$

The rest of the proof consists in comparing these three value functions. Comparing (EC.3) to (EC.4), we obtain

$$\begin{aligned}
&\frac{v_1+v_2}{1+\delta-\delta(1-p_1)(1-p_2)} \geq \frac{v_1}{1+\delta p_1} + \frac{v_2(1-\delta(1-p_1))}{(1+\delta p_1)(1-\delta(1-p_1)(1-p_2))} \\
\Leftrightarrow\ &\frac{v_1}{v_2} \leq \frac{\delta}{1-\delta(1-p_1)(1-p_2)}.
\end{aligned}$$

Comparing (EC.4) to (EC.5), we obtain

$$\begin{aligned}
&\frac{v_1}{1+\delta p_1} + \frac{v_2(1-\delta(1-p_1))}{(1+\delta p_1)(1-\delta(1-p_1)(1-p_2))} \\
&\geq \frac{\left(v_1(1-\delta(1-p_2)) + v_2(1-\delta(1-p_1))\right)(1-\delta(1-p_1)(1-p_2))}{\delta^2(p_1^2+p_2^2+p_1p_2(1-\delta p_1p_2)) + \delta(1-\delta)(2-\delta p_1p_2)(p_1+p_2) + (1-\delta)^2(1-\delta p_1p_2)} \\
\Leftrightarrow\ &\frac{v_1}{v_2} \geq \frac{(1-\delta(1-p_1))(1-\delta(1-p_1)(1-p_2)) + p_2\delta^2(1-p_1)}{\delta(1-\delta(1-p_1)(1-p_2))}.
\end{aligned}$$

Finally, comparing (EC.3) to (EC.5), we obtain

$$\begin{aligned}
&\frac{v_1+v_2}{1+\delta-\delta(1-p_1)(1-p_2)} \\
&\geq \frac{\left(v_1(1-\delta(1-p_2)) + v_2(1-\delta(1-p_1))\right)(1-\delta(1-p_1)(1-p_2))}{\delta^2(p_1^2+p_2^2+p_1p_2(1-\delta p_1p_2)) + \delta(1-\delta)(2-\delta p_1p_2)(p_1+p_2) + (1-\delta)^2(1-\delta p_1p_2)} \\
&\Leftrightarrow g(p_1,p_2,\delta)v_1 + g(p_2,p_1,\delta)v_2 \geq 0,
\end{aligned}$$

in which

$$g(p_1,p_2,\delta) \doteq -p_2(1-p_1) + \delta(1-p_2)(p_1 + (2-p_1)(1-p_1)p_2) - \delta^2(1-p_1)(1-p_2)(p_1 + p_2(1-p_2)(1-p_1)).$$

Define $\gamma(p_1,p_2,\delta) \doteq -g(p_2,p_1,\delta)/g(p_1,p_2,\delta)$. Hence, the FB policy can be characterized as follows: When $g(p_1,p_2,\delta) >_{(<)} 0$,

- If $\frac{v_1}{v_2} \leq \alpha(p_1,p_2,\delta)$ and $\frac{v_1}{v_2} \geq_{(\leq)} \gamma(p_1,p_2,\delta)$, Produce in $(0,0)$, Coordinate otherwise;
- If $\frac{v_1}{v_2} \geq \alpha(p_1,p_2,\delta)$ and $\frac{v_1}{v_2} \geq \beta(p_1,p_2,\delta)$, Produce in $(0,0)$ and $(0,x_2)$, $\forall x_2 \geq 1$, Coordinate otherwise;
- If $\frac{v_1}{v_2} \leq \beta(p_1,p_2,\delta)$ and $\frac{v_1}{v_2} \leq_{(\geq)} \gamma(p_1,p_2,\delta)$, Produce in $(0,0)$, $(0,x_2)$, $\forall x_2 \geq 1$, and $(x_1,0)$, $\forall x_1 \geq 1$, Coordinate otherwise.

The conditions that involve $\gamma(p_1,p_2,\delta)$ turn out to be redundant. To see this, note that $\alpha(p_1,p_2,\delta)\geq 1$ if and only if $\beta(p_1,p_2,\delta)\leq 1$. Hence if $v_1/v_2\leq\alpha(p_1,p_2,\delta)$, then $\alpha(p_1,p_2,\delta)\geq 1$ since $v_1\geq v_2$, which implies that $\beta(p_1,p_2,\delta)\leq 1$, and therefore that $v_1/v_2\geq\beta(p_1,p_2,\delta)$. Conversely if $v_1/v_2\leq\beta(p_1,p_2,\delta)$, then $\beta(p_1,p_2,\delta)\geq 1$ since $v_1\geq v_2$, which implies that $\alpha(p_1,p_2,\delta)\leq 1$, and therefore $v_1/v_2\geq\alpha(p_1,p_2,\delta)$. As a result, the three cases can be completely described in terms of the conditions on $\alpha(p_1,p_2,\delta)$ and $\beta(p_1,p_2,\delta)$. □

Proposition 3 characterizes the robustness of PC, PP, and HS_j, by deriving a worst-case bound on their optimality gap.

Proof of Proposition 3. The proof uses the expressions for the value functions derived in the proof of Proposition 2 corresponding to PC if $\phi\in[v_1,v_1+v_2)$, namely, (EC.3), HS_1 if $\phi\in[v_2,v_1)$, namely, (EC.4), and PP if $\phi\in[0,v_2)$, namely, (EC.5). For each pair of coordination scheduling rules $\pi_1,\pi_2\in\{\text{PC},\text{HS}_1,\text{HS}_2,\text{PP}\}$ with $\pi_1\neq\pi_2$, we characterize a lower and an upper bound on the ratio V^{π_1}/V^{π_2} by considering a worst-case extreme problem instance. Since $V^{\text{FB}}=\max\{V^{\text{PC}},V^{\text{HS}_1},V^{\text{HS}_2},V^{\text{PP}}\}$ by Proposition 1, for any $\pi_1\in\{\text{PC},\text{HS}_1,\text{HS}_2,\text{PP}\}$, an attainable worst-case bound on V^{π_1}/V^{FB} is defined as $\min_{\pi_2\in\{\text{PC},\text{HS}_1,\text{HS}_2,\text{PP}\}}V^{\pi_1}/V^{\pi_2}$.

First consider the ratio $V^{\text{HS}_1}/V^{\text{PC}}$:

$$\frac{V^{\text{HS}_1}}{V^{\text{PC}}}=\frac{v_1}{v_1+v_2}\frac{1+\delta(1-(1-p_1)(1-p_2))}{1+\delta p_1}+\frac{v_2}{v_1+v_2}\frac{(1-\delta(1-p_1))\,(1+\delta(1-(1-p_1)(1-p_2)))}{(1+\delta p_1)(1-\delta(1-p_1)(1-p_2))}.$$

The function is increasing in v_1/v_2. Hence,

$$\frac{V^{\text{HS}_1}}{V^{\text{PC}}}\leq\frac{1+\delta(1-(1-p_1)(1-p_2))}{1+\delta p_1}\leq\frac{1+\delta}{1+\delta p_1}\leq 1+\delta;$$

here, the first inequality is because the left-hand side is maximized when $v_1/v_2\to\infty$, and the second and third inequalities follow by taking $p_2=1$ and $p_1=0$, respectively.

Similarly,

$$\frac{V^{\text{HS}_1}}{V^{\text{PC}}}\geq\frac{(1-\delta(1-p_1))\,(1+\delta(1-(1-p_1)(1-p_2)))}{(1+\delta p_1)(1-\delta(1-p_1)(1-p_2))}\geq\frac{(1-\delta(1-p_1))\,(1+\delta)}{1+\delta p_1}\geq 1-\delta^2;$$

here, the first inequality is because the left-hand side is minimized when $v_1/v_2\to 0$, and the second and third inequalities follow by taking $p_2=1$ and $p_1=0$, respectively.

Consider next the ratio $V^{\text{PP}}/V^{\text{PC}}$:

$$\frac{V^{\text{PP}}}{V^{\text{PC}}}=\left(\frac{v_1}{v_1+v_2}\frac{1}{1-\delta(1-p_1)}+\frac{v_2}{v_1+v_2}\frac{1}{1-\delta(1-p_2)}\right)\times\frac{(1-\delta)\,(1+\delta(1-(1-p_1)(1-p_2)))}{1-\sum_{t=0}^{\infty}\delta^{t+2}\left(p_1(1-p_1)^t(1-(1-p_2)^{t+1})+p_2(1-p_2)^t(1-(1-p_1)^t)\right)}.\tag{EC.6}$$

Without loss of generality, suppose that $p_2<p_1$. Then, the right-hand side is decreasing in v_1/v_2. Therefore, an upper bound is attained when $v_1=0$; that is,

$$\frac{V^{\text{PP}}}{V^{\text{PC}}}\leq\frac{1}{1-\delta(1-p_2)}\cdot\frac{(1-\delta)\,(1+\delta(1-(1-p_1)(1-p_2)))}{1-\sum_{t=0}^{\infty}\delta^{t+2}\left(p_1(1-p_1)^t(1-(1-p_2)^{t+1})+p_2(1-p_2)^t(1-(1-p_1)^t)\right)}.$$

The right-hand side is decreasing in p_2 if and only if the following function,

$$-(1-\delta(1-p_1))\left(\delta(1-\delta)+p_1(1-\delta)^2+\delta p_1^2(2-\delta-\delta^2)+\delta^2p_1^3(1+\delta)\right)$$

$$- 2\delta(1-\delta(1-p_1))(1-p_1)(\delta+p_1+\delta p_1^2(1+\delta))p_2$$
$$- \delta^2(1-p_1)^2(\delta+p_1(1+\delta)+p_1^2\delta(1+\delta))p_2^2,$$

is negative, which always holds. Thus, an upper bound is attained when $p_2=0$, i.e.,

$$\frac{V^{\mathrm{PP}}}{V^{\mathrm{PC}}} \le 1+\delta p_1 \le 1+\delta.$$

Using a symmetric argument, suppose that $p_2>p_1$. Then, the right-hand side (EC.6) is increasing in v_1/v_2. Substituting $v_1=0$ into the the right-hand side of (EC.6), we obtain that it is decreasing in p_2, similar to the argument above; hence, a lower bound on $V^{\mathrm{PP}}/V^{\mathrm{PC}}$ is attained when $p_2=1$:

$$\frac{V^{\mathrm{PP}}}{V^{\mathrm{PC}}} \ge \frac{(1-\delta)(1+\delta)}{1-\sum_{t=0}^{\infty}\delta^{t+2}p_1(1-p_1)^t} \ge 1-\delta^2.$$

Here, the last inequality is achieved by taking $p_1=0$.

Third, consider the ratio $V^{\mathrm{HS}_1}/V^{\mathrm{PP}}$:

$$\frac{V^{\mathrm{HS}_1}}{V^{\mathrm{PP}}} = \left(\frac{v_1}{\frac{v_1}{1-\delta(1-p_1)}+\frac{v_2}{1-\delta(1-p_2)}} + \frac{v_2(1-\delta(1-p_1))}{\left(\frac{v_1}{1-\delta(1-p_1)}+\frac{v_2}{1-\delta(1-p_2)}\right)(1-\delta(1-p_1)(1-p_2))}\right)$$
$$\times \frac{1}{(1-\delta)(1+\delta p_1)}\left(1-\sum_{t=0}^{\infty}\delta^{t+2}\left(p_1(1-p_1)^t(1-(1-p_2)^{t+1})+p_2(1-p_2)^t(1-(1-p_1)^t)\right)\right).$$

The right-hand side is increasing in v_1. Hence, its upper bound is attained when $v_1/v_2\to\infty$; that is,

$$\frac{V^{\mathrm{HS}_1}}{V^{\mathrm{PP}}} \le \frac{(1-\delta(1-p_1))}{(1+\delta p_1)(1-\delta)}\left(1-\sum_{t=0}^{\infty}\delta^{t+2}\left(p_1(1-p_1)^t(1-(1-p_2)^{t+1})+p_2(1-p_2)^t(1-(1-p_1)^t)\right)\right).$$

Because the right-hand side is decreasing in p_2, an upper bound on $V^{\mathrm{HS}_1}/V^{\mathrm{PP}}$ is achieved when $p_2=0$, i.e.,

$$\frac{V^{\mathrm{HS}_1}}{V^{\mathrm{PP}}} \le \frac{1-\delta(1-p_1)}{(1+\delta p_1)(1-\delta)} \le \frac{1}{1-\delta^2}.$$

Conversely, a lower bound on $V^{\mathrm{HS}_1}/V^{\mathrm{PP}}$ is attained when $v_1=0$; hence,

$$\frac{V^{\mathrm{HS}_1}}{V^{\mathrm{PP}}} \ge \frac{(1-\delta(1-p_1))(1-\delta(1-p_2))}{(1-\delta(1-p_1)(1-p_2))}$$
$$\times \frac{1}{(1+\delta p_1)(1-\delta)}\left(1-\sum_{t=0}^{\infty}\delta^{t+2}\left(p_1(1-p_1)^t(1-(1-p_2)^{t+1})+p_2(1-p_2)^t(1-(1-p_1)^t)\right)\right).$$

The right-hand side is increasing in p_2 if and only if the following function,

$$G(p_1,p_2,\delta) \doteq 1-\delta(1-p_1)(2-\delta^2+p_1\delta(1+\delta))+\delta p_2(1-p_1)(1+\delta(1-\delta)+\delta p_1(1+\delta)),$$

is positive. Since $G(p_1,p_2,\delta)$ is linear increasing in p_2 and since $G(p_1,1,\delta)\ge 0$, $G(p_1,p_2,\delta)$ will either be always positive or will cross zero once depending on its value at $p_2=0$. We thus need to consider two cases:

1. If $G(p_1,0,\delta)\ge 0$, then $G(p_1,p_2,\delta)$ is nonnegative for all p_2, and a lower bound on $V^{\mathrm{HS}_1}/V^{\mathrm{PP}}$ is attained when $p_2=0$, i.e.,

$$\frac{V^{\mathrm{HS}_1}}{V^{\mathrm{PP}}} \ge \frac{1}{1+\delta p_1} \ge \frac{1}{1+\delta}.$$

2. If $G(p_1,0,\delta)<0$, then $G(p_1,p_2,\delta)$ crosses zero once as p_2 increases and the crossing is from below. Hence, a lower bound on $V^{\mathrm{HS}_1}/V^{\mathrm{PP}}$ is attained when $G(p_1,p_2,\delta)=0$, i.e., when $p_2=-(1-\delta(1-p_1)(2-\delta^2+p_1\delta(1+\delta)))/(\delta(1-p_1)(1+\delta(1-\delta)+\delta p_1(1+\delta)))$. Accordingly,

$$\frac{V^{\mathrm{HS}_1}}{V^{\mathrm{PP}}}\geq\frac{4(1-\delta)\delta+(-1-2\delta+9\delta^2-2\delta^3-\delta^4)p_1-2(1-\delta)\delta(1+3\delta+\delta^2)p_1^2-\delta^2(1+\delta)^2p_1^3}{4\delta(1-\delta(1-p_1))(1-p_1)(1+\delta p_1)}.$$

The derivative of the right-hand side with respect to p_1 is negative if and only if the following function,

$$(1-\delta)(1-\delta(1-\delta))+3(1-\delta)^2\delta p_1+4(1-\delta)\delta^2p_1^2+2\delta^3p_1^3$$

is positive, which is always the case. Hence, a lower bound on $V^{\mathrm{HS}_1}/V^{\mathrm{PP}}$ in this case is attained when p_1 is as large as possible while $G(p_1,0,\delta)<0$, i.e., while $p_1<(-1+\delta+\delta^2)/(\delta(1+\delta))$. As a result, it is optimal in this case to take p_1 such that $G(p_1,0,\delta)\to 0$, and this case is dominated by the former one.

Finally, consider the ratio $V^{\mathrm{HS}_1}/V^{\mathrm{HS}_2}$:

$$\frac{V^{\mathrm{HS}_1}}{V^{\mathrm{HS}_2}}=\frac{v_1(1-\delta(1-p_1)(1-p_2))+v_2(1-\delta(1-p_1))}{v_2(1-\delta(1-p_1)(1-p_2))+v_1(1-\delta(1-p_2))}\cdot\frac{1+\delta p_2}{1+\delta p_1}.$$

Because the right-hand side is increasing in v_1, a lower bound is attained when $v_1=0$, i.e.,

$$\frac{V^{\mathrm{HS}_1}}{V^{\mathrm{HS}_2}}\geq\frac{1-\delta(1-p_1)}{1-\delta(1-p_1)(1-p_2)}\cdot\frac{1+\delta p_2}{1+\delta p_1}.$$

The right-hand side is increasing in p_2. Hence, a lower bound is attained when $p_2=0$, i.e.,

$$\frac{V^{\mathrm{HS}_1}}{V^{\mathrm{HS}_2}}\geq\frac{1}{1+\delta p_1}\geq\frac{1}{1+\delta}.$$

Given the symmetry between V^{HS_1} and V^{HS_2}, the upper bound is derived similarly.

To summarize:

$$\begin{array}{ll} 1-\delta^2\leq\frac{V^{\mathrm{HS}_1}}{V^{\mathrm{PC}}}\leq 1+\delta, & 1-\delta^2\leq\frac{V^{\mathrm{PP}}}{V^{\mathrm{PC}}}\leq 1+\delta,\\ \frac{1}{1+\delta}\leq\frac{V^{\mathrm{HS}_1}}{V^{\mathrm{PP}}}\leq\frac{1}{1-\delta^2}, & \frac{1}{1+\delta}\leq\frac{V^{\mathrm{HS}_1}}{V^{\mathrm{HS}_2}}\leq 1+\delta. \end{array}$$

By Proposition 2, $V^{\mathrm{FB}}=\max\{V^{\mathrm{PC}},V^{\mathrm{PP}},V^{\mathrm{HS}_1},V^{\mathrm{HS}_2}\}$. Accordingly,

$$\begin{aligned} V^{\mathrm{FB}}&\leq\max\{1,1+\delta,1+\delta,1+\delta\}V^{\mathrm{PC}}=(1+\delta)V^{\mathrm{PC}}\\ V^{\mathrm{FB}}&\leq\max\left\{\frac{1}{1-\delta^2},1,\frac{1}{1-\delta^2},\frac{1}{1-\delta^2}\right\}V^{\mathrm{PP}}=\frac{1}{1-\delta^2}V^{\mathrm{PP}}\\ V^{\mathrm{FB}}&\leq\max\left\{\frac{1}{1-\delta^2},1+\delta,1,1+\delta\right\}V^{HS_i}=\max\left\{\frac{1}{1-\delta^2},1+\delta\right\}V^{HS_i}\quad\forall i\in\{1,2\}, \end{aligned}$$

completing the proof. □

EC.2.3. Large Teams

Lemma EC.3 derives closed-form expressions for the value functions associated with the worker-driven coordination scheduling rules enhanced with time-based controls. As a corollary to this lemma, Lemma EC.4 will then derive closed-form expressions for the value functions associated with the basic worker-driven coordination scheduling rules, by taking $T^{\min}=2$ and $T^{\max}\to\infty$.

Lemma EC.3. *Suppose that Assumption 1 holds.*

- *Under PC-C^{min} with a minimum cycle duration of $T^{min} \geq 2$ periods,*

$$V_i^{PC\text{-}C^{min}} = \frac{v_i \left(\frac{1-(\delta(1-p_i))^{T^{min}-1}}{1-\delta(1-p_i)} + \frac{\left(\prod_{j=1}^n (1-p_j)\right)^{T^{min}-1} \delta^{T^{min}-1}}{1-\delta \prod_{j=1}^n (1-p_j)} \right)}{1 - \left(\prod_{j=1}^n (1-p_j)\right)^{T^{min}-1} \delta^{T^{min}+1} \frac{1-\prod_{j=1}^n (1-p_j)}{1-\delta \prod_{j=1}^n (1-p_j)} - \left(1 - \left(\prod_{j=1}^n (1-p_j)\right)^{T^{min}-1}\right) \delta^{T^{min}}}.$$

- *Under PP-C^{max} with a maximum cycle duration of $T^{max} \geq 2$ periods,*

$$V_i^{PP\text{-}C^{max}} = \frac{v_i \frac{1-(\delta(1-p_i))^{T^{max}-1}}{1-\delta(1-p_i)}}{1 - \sum_{t=0}^{T^{max}-3} (\delta^{t+2} - \delta^{T^{max}}) \left(\sum_{j=1}^n p_j (1-p_j)^t \left(\prod_{k=1}^{j-1} (1-(1-p_k)^t) \right) \cdot \left(\prod_{k=j+1}^n (1-(1-p_k)^{t+1}) \right) \right) - \delta^{T^{max}}}.$$

- *Under HS-C^{min} with a minimum cycle duration of $T^{min} \geq 2$ periods,*

$$V_i^{HS\text{-}C^{min}} = \begin{cases} \dfrac{v_1 \left(\frac{1-(\delta(1-p_1))^{T^{min}-1}}{1-\delta(1-p_1)} + \frac{(1-p_1)^{T^{min}-1} \delta^{T^{min}-1}}{1-\delta(1-p_1)} \right)}{1-(1-p_1)^{T^{min}-1} \delta^{T^{min}+1} \frac{p_1}{1-\delta(1-p_1)} - \left(1-(1-p_1)^{T^{min}-1}\right) \delta^{T^{min}}} & \text{if } i = 1 \\ \dfrac{v_i \left(\frac{1-(\delta(1-p_i))^{T^{min}-1}}{1-\delta(1-p_i)} + \frac{(1-p_1)^{T^{min}-1} (1-p_i)^{T^{min}-1} \delta^{T^{min}-1}}{1-\delta(1-p_1)(1-p_i)} \right)}{1-(1-p_1)^{T^{min}-1} \delta^{T^{min}+1} \frac{p_1}{1-\delta(1-p_1)} - \left(1-(1-p_1)^{T^{min}-1}\right) \delta^{T^{min}}} & \text{if } i \neq 1. \end{cases}$$

- *Under HS-C^{max} with a maximum cycle duration of $T^{max} \geq 2$ periods,*

$$V_i^{HS\text{-}C^{max}} = \begin{cases} v_1 \frac{(1-p_1)\left(1-(\delta(1-p_1))^{T^{max}-1}\right)}{(1-\delta)\left((1-p_1)(1+\delta p_1) - \delta^{T^{max}} (1-p_1)^{T^{max}}\right)} & \text{if } i = 1 \\ v_i \frac{(1-p_1)(1-\delta(1-p_1))\left(1-(\delta(1-p_1)(1-p_i))^{T^{max}-1}\right)}{(1-\delta)((1-p_1)(1+\delta p_1) - \delta^{T^{max}} (1-p_1)^{T^{max}})(1-\delta(1-p_1)(1-p_i))} & \text{if } i \neq 1. \end{cases}$$

Proof. The proof is structured in four parts, separately characterizing the value function for PC-C$^{\min}$, PP-C$^{\max}$, HS-C$^{\min}$, and HS-C$^{\max}$.

First, consider PC-C$^{\min}$ with a minimum cycle duration of $T^{\min} \geq 2$ periods. By (9) and (11), $A^{\text{PC-C}^{\min}}(\mathbf{a}^{\text{PC-C}^{\min}}(\mathbf{x}, t), t) = C$ for all $\mathbf{x} \neq \mathbf{0}$ and $t \geq T^{\min} - 1$, and $A^{\text{PC-C}^{\min}}(\mathbf{a}^{\text{PC-C}^{\min}}(\mathbf{x}, t), t) = P$ otherwise. We henceforth ignore the superscripts for simplicity. Accordingly in the first $T^{\min} - 1$ periods of the cycle, worker i earns an expected value equal to $v_i \sum_{t=0}^{T^{\min}-2} (\delta(1-p_i))^t$. For the continuation value, we consider two scenarios.

- First, suppose that a worker encounters an issue during the first $T^{\min} - 1$ periods, which happens with probability $1 - \left(\prod_{j=1}^n (1-p_j)\right)^{T^{\min}-1}$. In that case, coordination happens in period $T^{\min}$, giving worker i a discounted value of $\delta^{T^{\min}} V_i(\mathbf{0}, 0)$.
- Second, suppose that no worker has an issue during the first $T^{\min} - 1$ periods, which happens with probability $\left(\prod_{j=1}^n (1-p_j)\right)^{T^{\min}-1}$. In that case, production continues until an issue arises. In each period of production after period $T^{\min} - 1$, coordination happens with probability $1 - \prod_{j=1}^n (1-p_j)$ and production continues with probability $\prod_{j=1}^n (1-p_j)$. Hence in that scenario, worker i's discounted expected value, from period $T^{\min} - 1$ onwards, is equal to $\delta^{T^{\min}-1} \left(v_i + \left(1 - \prod_{j=1}^n (1-p_j)\right) \delta^2 V_i(\mathbf{0}, 0) \right) / \left(1 - \delta \prod_{j=1}^n (1-p_j)\right)$.

Combining both scenarios, we obtain

$$V_i(\mathbf{0}, 0) = v_i \sum_{t=0}^{T^{\min}-2} (\delta(1-p_i))^t + \left(1 - \left(\prod_{j=1}^n (1-p_j)\right)^{T^{\min}-1}\right) \delta^{T^{\min}} V_i(\mathbf{0}, 0)$$

$$+\left(\prod_{j=1}^{n}(1-p_j)\right)^{T^{\min}-1}\delta^{T^{\min}-1}\frac{v_i+\left(1-\prod_{j=1}^{n}(1-p_j)\right)\delta^2 V_i(\mathbf{0},0)}{1-\delta\prod_{j=1}^{n}(1-p_j)}.$$

Rearranging the terms gives the desired result.

Second, consider PP-C$^{\max}$ with a maximum cycle duration of $T^{\max}\geq 2$ periods. By (9) and (12), $A^{\text{PP-C}^{\max}}(\mathbf{a}^{\text{PP-C}^{\max}}(\mathbf{x},t),t)=C$ for all $\mathbf{x}\geq\mathbf{1}$ if $t<T^{\max}-1$ and all $\mathbf{x}$ if $t\geq T^{\max}-1$, and $A^{\text{PP-C}^{\max}}(\mathbf{a}^{\text{PP-C}^{\max}}(\mathbf{x},t),t)=P$ otherwise. We henceforth ignore the superscripts for simplicity. Starting from state $(\mathbf{0},0)$, worker i earns v_i each period until she faces an issue or until $T^{\max}-1$ periods have elapsed, whichever comes first. Hence, worker i earns $v_i\sum_{t=0}^{T^{\max}-2}\delta^t(1-p_i)^t=v_i(1-(\delta(1-p_i))^{T^{\max}-1})/(1-\delta(1-p_i))$ within a cycle. For the continuation value, we consider two scenarios

- First, suppose that all workers have encountered an issue by the end of period t, for any $t\leq T^{\max}-3$, so that the cycle duration ($t+1$ periods of production and one period of coordination) is strictly less than $T^{\max}$ periods. Coordination happens after one period if all workers have an issue at the end of the first period, which happens under Assumption 1 with probability $\prod_{j=1}^{n}p_j$; after two periods if at least one worker does not have an issue in the first period but all workers have an issue at the end of the second period; etc. More generally, coordination happens after $t+1$ periods with probability
$$\begin{aligned}&\sum_{j=1}^{n}p_j(1-p_j)^t\left(\prod_{k=1}^{j-1}\left(\sum_{s=0}^{t-1}p_k(1-p_k)^s\right)\right)\cdot\left(\prod_{k=j+1}^{n}\left(\sum_{s=0}^{t}p_k(1-p_k)^s\right)\right)\\&=\sum_{j=1}^{n}p_j(1-p_j)^t\left(\prod_{k=1}^{j-1}(1-(1-p_k)^t)\right)\cdot\left(\prod_{k=j+1}^{n}(1-(1-p_k)^{t+1})\right),\end{aligned}$$
and the continuation value associated with that event is $\delta^{t+2}V_i(\mathbf{0},0)$.
- Second, suppose that not all workers have faced an issue by the end of period $T^{\max}-3$, which happens with probability
$$1-\sum_{t=0}^{T^{\max}-3}\sum_{j=1}^{n}p_j(1-p_j)^t\left(\prod_{k=1}^{j-1}(1-(1-p_k)^t)\right)\cdot\left(\prod_{k=j+1}^{n}(1-(1-p_k)^{t+1})\right).$$
In that case, coordination must happen in period $T^{\max}-1$, and the continuation value associated with that scenario is $\delta^{T^{\max}}V_i(\mathbf{0},0)$.

Combining both scenarios, we obtain:

$$\begin{aligned}V_i(\mathbf{0},0)=&v_i\frac{1-(\delta(1-p_i))^{T^{\max}-1}}{1-\delta(1-p_i)}\\&+\sum_{t=0}^{T^{\max}-3}\delta^{t+2}\left(\sum_{j=1}^{n}p_j(1-p_j)^t\left(\prod_{k=1}^{j-1}(1-(1-p_k)^t)\right)\cdot\left(\prod_{k=j+1}^{n}(1-(1-p_k)^{t+1})\right)\right)V_i(\mathbf{0},0)\\&+\delta^{T^{\max}}\left(1-\sum_{t=0}^{T^{\max}-3}\sum_{j=1}^{n}p_j(1-p_j)^t\left(\prod_{k=1}^{j-1}(1-(1-p_k)^t)\right)\cdot\left(\prod_{k=j+1}^{n}(1-(1-p_k)^{t+1})\right)\right)V_i(\mathbf{0},0).\end{aligned}$$

Rearranging the terms gives the desired result.

Third, consider HS-C$^{\min}$ with a minimum cycle duration of $T^{\min}\geq 2$ periods. By (9) and (13), $A^{\text{HS-C}^{\min}}(\mathbf{a}^{\text{HS-C}^{\min}}(\mathbf{x},t),t)=C$ for all $\mathbf{x}$ such that $x_1>0$ and $t\geq T^{\min}-1$ and $A^{\text{HS-C}^{\min}}(\mathbf{a}^{\text{HS-C}^{\min}}(\mathbf{x},t),t)=P$ otherwise. We henceforth ignore the superscripts for simplicity. Accordingly in the first $T^{\min}-1$ periods of a cycle, worker i earns an expected value equal to $v_i\sum_{t=0}^{T^{\min}-2}(\delta(1-p_i))^t$. For the continuation value, we consider two scenarios.

- First, suppose that worker 1 has an issue during the first $T^{\min}-1$ periods, which happens with probability $1-(1-p_1)^{T^{\min}-1}$. In that case, coordination happens in period $T^{\min}$, giving worker i a discounted value of $\delta^{T^{\min}}V_i(\mathbf{0},0)$.
- Second, suppose that worker 1 has no issue during the first $T^{\min}-1$ periods, which happens with probability $(1-p_1)^{T^{\min}-1}$. In that case, production continues until worker 1 has an issue, which happens with probability p_1. In that scenario, worker 1's discounted expected value, from period $T^{\min}-1$ onwards, is equal to $\delta^{T^{\min}-1}\left(v_1+p_1\delta^2V_i(\mathbf{0},0)\right)/\left(1-\delta(1-p_1)\right)$ and worker i's discounted expected value, from period $T^{\min}-1$ onwards, is equal to $\delta^{T^{\min}-1}\left((1-p_i)^{T^{\min}-1}v_i/(1-\delta(1-p_1)(1-p_i))+p_1\delta^2V_i(\mathbf{0},0)/(1-\delta(1-p_1))\right)$.

Combining both scenarios, we obtain

$$\begin{aligned}V_i(\mathbf{0},0)=&v_i\sum_{t=0}^{T^{\min}-2}(\delta(1-p_i))^t+\left(1-(1-p_1)^{T^{\min}-1}\right)\delta^{T^{\min}}V_i(\mathbf{0},0)\\&+(1-p_1)^{T^{\min}-1}\frac{p_1}{1-\delta(1-p_1)}\delta^{T^{\min}+1}V_i(\mathbf{0},0)+(1-p_1)^{T^{\min}-1}\delta^{T^{\min}-1}\begin{cases}\frac{v_1}{1-\delta(1-p_1)} & \text{if } i=1\\ \frac{(1-p_i)^{T^{\min}-1}v_i}{1-\delta(1-p_1)(1-p_i)} & \text{if } i\neq 1.\end{cases}\end{aligned}$$

Rearranging the terms gives the desired result.

Fourth, consider HS-C$^{\max}$ with a maximum cycle duration of $T^{\max}\geq 2$ periods. By (9) and (14), $A^{\text{HS-C}^{\max}}(\mathbf{a}^{\text{HS-C}^{\max}}(\mathbf{x},t),t)=C$ for all $\mathbf{x}$ such that $x_1>0$ if $t<T^{\max}-1$ and all $\mathbf{x}$ if $t\geq T^{\max}-1$, and $A^{\text{HS-C}^{\max}}(\mathbf{a}^{\text{HS-C}^{\max}}(\mathbf{x},t),t)=P$ otherwise. We henceforth ignore the superscripts for simplicity. We consider in turn, worker 1 and then any other worker. First, consider worker 1. Starting from state $(\mathbf{0},0)$, worker 1 earns v_1 each period until she faces an issue. Hence, worker 1 earns $v_1\sum_{t=0}^{T^{\max}-2}\delta^t(1-p_1)^t=v_1(1-(\delta(1-p_1))^{T^{\max}-1})/(1-\delta(1-p_1))$ within a cycle. Second, consider worker $i\neq 1$. Starting from state $(\mathbf{0},0)$, worker i earns v_i each period until she faces an issue or until worker 1 faces an issue. Hence, worker i earns within that cycle $v_i\sum_{t=0}^{T^{\max}-2}\delta^t(1-p_i)^t(1-p_1)^t=v_i(1-(\delta(1-p_i)(1-p_1))^{T^{\max}-1})/(1-\delta(1-p_i)(1-p_1))$. For the continuation value, we consider two scenarios.

- First, suppose that worker 1 has an issue in period t, for any $t\leq T^{\max}-3$, which happens with probability $(1-p_1)^tp_1$. The continuation value associated with that event is $\delta^{t+2}V_i(\mathbf{0},0)$.
- Second, suppose that worker 1 has not encountered an issue by the end of period $T^{\max}-3$, which happens with probability $(1-p_1)^{T^{\max}-2}$. In that case, coordination must happen in period $T^{\max}-1$ and the continuation value associated with that scenario is $\delta^{T^{\max}}V_i(\mathbf{0},0)$.

Combining both scenarios, we obtain, for any i:

$$V_i(\mathbf{0},0)=\sum_{t=0}^{T^{\max}-3}\delta^{t+2}(1-p_1)^tp_1V_i(\mathbf{0},0)+\delta^{T^{\max}}(1-p_1)^{T^{\max}-2}V_i(\mathbf{0},0)+\begin{cases}v_1\frac{1-(\delta(1-p_1))^{T^{\max}-1}}{1-\delta(1-p_1)} & \text{if } i=1\\ v_i\frac{1-(\delta(1-p_i)(1-p_1))^{T^{\max}-1}}{1-\delta(1-p_i)(1-p_1)} & \text{if } i\neq 1.\end{cases}$$

Rearranging the terms gives the desired result. □

Lemma EC.4. *Suppose that Assumption 1 holds. Then,*

$$V_i^{PC}=\frac{v_i}{(1-\delta)\left(1+\delta\left(1-\prod_{j=1}^n(1-p_j)\right)\right)}$$

$$V_i^{PP} = \frac{v_i}{1-\delta(1-p_i)} \cdot \frac{1}{1-\sum_{t=0}^{\infty}\delta^{t+2}\left(\sum_{j=1}^{n} p_j(1-p_j)^t\left(\prod_{k=1}^{j-1}(1-(1-p_k)^t)\right)\cdot\left(\prod_{k=j+1}^{n}(1-(1-p_k)^{t+1})\right)\right)}$$

$$V_i^{HS} = \begin{cases} \frac{v_1}{(1-\delta)(1+\delta p_1)} & \text{if } i=1 \\ \frac{v_i(1-\delta(1-p_1))}{(1-\delta)(1+\delta p_1)(1-\delta(1-p_1)(1-p_i))} & \text{if } i\neq 1. \end{cases}$$

Proof. The proof follows from Lemma EC.3 in appendix by considering minimum cycles of $T^{\min}=2$ in $V_i^{\text{PC-C}^{\min}}$ and $V_i^{\text{HS-C}^{\min}}$ and maximum cycles $T^{\max}\to\infty$ periods in $V_i^{\text{PP-C}^{\max}}$ and $V_i^{\text{HS-C}^{\max}}$. □

Proposition 4 characterizes how PC, PP, and HS compare to each other as team size n increases.

Proof of Proposition 4. The proof consists of comparing the value functions derived in Lemma EC.4 in appendix. For any other worker i, when $p_j=p$ for all j, V_i^{HS} is independent of n, while V_i^{PC} and V_i^{PP} are decreasing (convex) in n. That V_i^{PP} is decreasing (convex) in n may not be obvious at first, but becomes apparent once we use the following equality: $\sum_{t=0}^{\infty}\delta^{t+2}((1-(1-p)^{1+t})^n-(1-(1-p)^t)^n) = (1-\delta)\sum_{t=0}^{\infty}\delta^{t+1}(1-(1-p)^t)^n$. Hence, as n increases, there is at most one crossing between V_i^{HS} and V_i^{PC} (V_i^{PP}), and if there is one, it is from above.

Moreover,

$$\begin{aligned} V_i^{\text{PP}} \geq V_i^{\text{PC}} &\Leftrightarrow \frac{v_i}{(1-\delta)(1+\delta(1-(1-p)^n))} \leq \frac{v_i}{(1-\delta(1-p))\left(1-(1-\delta)\sum_{t=0}^{\infty}\delta^{t+1}(1-(1-p)^t)^n\right)} \\ &\Leftrightarrow (1-\delta)(1+\delta(1-(1-p)^n)) \geq (1-\delta(1-p))\left(1-(1-\delta)\sum_{t=0}^{\infty}\delta^{t+1}(1-(1-p)^t)^n\right) \\ &\Leftrightarrow F(n) \doteq -(1-p)^n + (1-\delta(1-p))\sum_{t=0}^{\infty}\delta^t(1-(1-p)^t)^n - \frac{p+\delta-1}{1-\delta} \geq 0. \end{aligned}$$

Because

$$\begin{aligned} F''(n)\Big|_{F'(n)=0} &= \left(-(1-p)^n(\ln(1-p))^2 + (1-\delta(1-p))\sum_{t=0}^{\infty}\delta^t(1-(1-p)^t)^n(\ln(1-(1-p)^t))^2\right)_{F'(n)=0} \\ &= (1-\delta(1-p))\sum_{t=0}^{\infty}\delta^t(1-(1-p)^t)^n\ln(1-(1-p)^t)(\ln(1-(1-p)^t)-\ln(1-p)), \end{aligned}$$

we obtain

$$\begin{aligned} \frac{d}{dn}\left(F''(n)\Big|_{F'(n)=0}\right) &= (1-\delta(1-p))\sum_{t=0}^{\infty}\delta^t(1-(1-p)^t)^n(\ln(1-(1-p)^t))^2(\ln(1-(1-p)^t)-\ln(1-p)) \\ &\leq (1-\delta(1-p))\ln\left(1-(1-p)^{\lfloor\frac{\ln(p)}{\ln(1-p)}\rfloor}\right) \\ &\quad \times\sum_{t=0}^{\infty}\delta^t(1-(1-p)^t)^n\ln(1-(1-p)^t)(\ln(1-(1-p)^t)-\ln(1-p)) \\ &= \ln\left(1-(1-p)^{\lfloor\frac{\ln(p)}{\ln(1-p)}\rfloor}\right)\left(F''(n)\Big|_{F'(n)=0}\right); \end{aligned}$$

here, the inequality follows from (i) the following equivalence: $\ln(1-(1-p)^t)-\ln(1-p)\leq 0$ if and only if $t\leq\frac{\ln(p)}{\ln(1-p)}$ and (ii) that the function $\ln(1-(1-p)^t)$ is increasing in t.

Accordingly, having $F''(n)\Big|_{F'(n)=0}\geq 0$ implies that $\frac{d}{dn}\left(F''(n)\Big|_{F'(n)=0}\right)\leq 0$. Hence, the continuous function $F''(n)\Big|_{F'(n)=0}$ crosses zero at most once and the crossing is from above as n increases. Accordingly, $F(n)$

has at most two stationary points, and if it does have two, the first one is a local minimum and the second one is a local maximum. Thus, $F(n)$ crosses zero at most thrice and if it does, it is first from above, then from below, and finally from above. Because $F(0)=1-p$ and $F(1)=0$, $F(n)$ crosses zero at most twice when $n\geq 2$. That is, there are two thresholds, n_L and n_U, with $1<n_L\leq n_U\leq\infty$ such that $F(n)\geq 0$, i.e., such that $V_i^{\mathrm{PP}}\geq V_i^{\mathrm{PC}}$, if and only if $n\in[n_L,n_U]$. Since this applies for every worker i, $V^{\mathrm{PP}}\geq V^{\mathrm{PC}}$ if and only if $n\in[n_L,n_U]$. For instance, consider $\delta=0.75$ and $p=0.27$; in that case, $n_L=6$ and $n_U=24$. □

Proposition 5 states that, for team size n large enough, PC-C$^{\min}$ and PP-C$^{\max}$ dominate HS-C$^{\min}$ and HS-C$^{\max}$.

Proof of Proposition 5. The proof follows from the first two bullet points of Proposition 8, which is shown independently. □

EC.2.4. Fixed Interval Coordination Scheduling Rules

The proof of Proposition 6 uses several preliminary lemmas. First, Lemma EC.5 derives worker i's value function associated with the FI coordination scheduling rule. Second, Lemma EC.6 shows that the optimal cycle duration under FI is decreasing in the coordination demands. Third, Lemma EC.7 derives the first-order optimality conditions for the optimal cycle duration under FI. The next two lemmas, namely Lemmas EC.8 and EC.9, are more technical in nature; they show that certain algebraic expressions are monotone, which will be useful to derive a worst-case bound on the optimality gap of the FI coordination scheduling rule, established in Lemma EC.10.

Lemma EC.5. *Suppose that Assumption 1 holds. Under FI with cycle duration of $T>1$ periods,*

$$V_i^{FI}(T)=\frac{v_i}{1-\delta(1-p_i)}\cdot\frac{1-\delta^{T-1}(1-p_i)^{T-1}}{1-\delta^T}.$$

Proof. Under FI, workers coordinate every $T>1$ periods, yielding $V_i^{\mathrm{FI}}(T)=\sum_{\tau=0}^{\infty}\delta^{T\tau}\sum_{t=0}^{T-2}v_i\delta^t(1-p)^t$. □

Lemma EC.6. *For any $p\in(0,1)$ and $\delta\in(0,1)$, $\arg\max_{T\in\{2,3,\ldots\}}(1-(\delta(1-p))^{T-1})/(1-\delta^T)$ is nonincreasing in $p\in(0,1)$.*

Proof. Define

$$F(p,\delta,T)\doteq\frac{1-(\delta(1-p))^{T-1}}{(1-\delta^T)}.$$

We first show that $F(p,\delta,T)$ is pseudo-concave in T. Note that

$$\frac{\partial F(p,\delta,T)}{\partial T}=\frac{1-(\delta(1-p))^{T-1}}{1-\delta^T}\left(\frac{\delta^T\ln(\delta)}{1-\delta^T}-\frac{(\delta(1-p))^{T-1}\ln(\delta(1-p))}{1-(\delta(1-p))^{T-1}}\right).$$

We consider two cases. First, suppose that $\delta^T<(\delta(1-p))^{T-1}$. Because the function $x\ln(x)/(1-x)$ is decreasing when $x\in[0,1]$, we obtain

$$\frac{T-1}{T}\frac{\delta^T\ln(\delta^T)}{1-\delta^T}-\frac{(\delta(1-p))^{T-1}\ln\left((\delta(1-p))^{T-1}\right)}{1-(\delta(1-p))^{T-1}}>\left(\frac{T-1}{T}-1\right)\frac{(\delta(1-p))^{T-1}\ln\left((\delta(1-p))^{T-1}\right)}{1-(\delta(1-p))^{T-1}}\geq 0,$$

i.e., $\partial F(p,\delta,T)/\partial T > 0$.

Second, suppose that $\delta \geq (1-p)^{T-1}$. When $\partial F(p,\delta,T)/\partial T = 0$,

$$\begin{aligned}\frac{\partial^2 F(p,\delta,T)}{\partial T^2}\Big|_{\frac{\partial F(p,\delta,T)}{\partial T}=0} &= \frac{1-(\delta(1-p))^{T-1}}{1-\delta^T}\left(\frac{\delta^T(\ln(\delta))^2}{(1-\delta^T)^2} - \frac{(\delta(1-p))^{T+1}(\ln(\delta(1-p)))^2}{(\delta(1-p)-(\delta(1-p))^T)^2}\right)_{\frac{\partial F(p,\delta,T)}{\partial T}=0} \\ &= -\frac{1-(\delta(1-p))^{T-1}}{1-\delta^T} \times \frac{(\delta(1-p))^T(\delta(1-p)-(1-p)^T)(\ln(\delta(1-p))^2}{(\delta(1-p)-(\delta(1-p))^T)^2} \\ &\leq 0,\end{aligned}$$

in which the inequality is because $\delta \geq (1-p)^{T-1}$. Combining both cases, we establish that $F(p,\delta,T)$ is pseudo-concave in T.

Next, observe that

$$\frac{\partial}{\partial p}\left(\frac{F(p,\delta,T+1)}{F(p,\delta,T)}\right) = \frac{\partial}{\partial p}\left(\frac{1-(\delta(1-p))^T}{1-(\delta(1-p))^{T-1}} \times \frac{1-\delta^T}{1-\delta^{T+1}}\right) \leq 0 \Leftrightarrow T\delta(1-p)-(\delta(1-p))^T-(T-1) \leq 0$$

since the function $Tx - x^T - (T-1)$ is nondecreasing in x, for any $x \in [0,1]$, which yields that $Tx - x^T - (T-1) \leq T-1-(T-1) = 0$. Hence, for any T, if $F(p,\delta,T) \geq F(p,\delta,T+1)$ for some p, then $F(p',\delta,T) \geq F(p',\delta,T+1)$ for all $p' \geq p$.

Let $T' \doteq \arg\max_{T\in\{2,3,\dots\}} F(p,\delta,T)$. Because $F(p,\delta,T)$ is pseudo-concave, $F(p,\delta,T') \geq F(p,\delta,T'+1) \geq F(p,\delta,T'+2)$ and so on. Therefore, for any $p' \geq p$, $F(p',\delta,T') \geq F(p',\delta,T'+1) \geq F(p',\delta,T'+2)$ and so on. Hence, $\arg\max_{T\in\{2,3,\dots\}} F(p',\delta,T) \leq T'$, i.e., $\arg\max_{T\in\{2,3,\dots\}} F(p,\delta,T)$ is nonincreasing in p. □

LEMMA EC.7. *Suppose that Assumption 1 holds. Let $T^* = \arg\max_{T\in\{2,3,\dots\}} V^{FI}(T)$. Then, $T^* \in \{\lfloor\tilde{T}\rfloor, \lceil\tilde{T}\rceil\}$ if $\tilde{T} \geq 2$ and $T^* = 2$ otherwise, in which $\tilde{T}$ is the unique solution to:*

$$\sum_{i=1}^n v_i \frac{1-(\delta(1-p_i))^{T-2}}{1-\delta^T}\left(\frac{1}{T}\frac{\delta^T\ln(\delta^T)}{1-\delta^T} - \frac{1}{T-1}\frac{(\delta(1-p_i))^{T-1}\ln((\delta(1-p_i))^{T-1})}{1-(\delta(1-p_i))^{T-1}}\right) = 0. \tag{EC.7}$$

Moreover, T^* is nonincreasing in p_i, for any i. When $p_i = p$ for all i, T^* is independent of v_j, for any j.

Proof. The proof uses Lemmas EC.5 and EC.6 in appendix, which respectively derive a closed-form solution for V_i^{FI} and show the monotonicity of T^* with respect to p_i for any i. The proof consist in showing that $V^{\mathrm{FI}}(T)$ is pseudoconcave and then using the first-order optimality conditions to characterize T^*. For any i, define $\alpha_i \doteq v_i/(1-\delta(1-p_i))$. Then, by Lemma EC.5,

$$V^{\mathrm{FI}}(T) = \sum_{i=1}^n \alpha_i \frac{1-\delta^{T-1}(1-p_i)^{T-1}}{1-\delta^T}.$$

We next show that $V^{\mathrm{FI}}(T)$ is pseudoconcave. When $dV^{\mathrm{FI}}(T)/dT = 0$, i.e.,

$$\frac{\delta^{T-1}}{(1-\delta^T)^2}\left(\sum_{i=1}^n \alpha_i\left(\log(\delta)(\delta-(1-p_i)^{T-1}) - \log(1-p_i)(1-p_i)^{T-1}(1-\delta^T)\right)\right) = 0,$$

the second derivative is negative:

$$\begin{aligned}\frac{d^2V^{\mathrm{FI}}(T)}{dT^2}\Big|_{\frac{dV^{FI}(T)}{dT}=0} &= \frac{d}{dT}\left(\sum_{i=1}^n \alpha_i\left(\log(\delta)(\delta-(1-p_i)^{T-1}) - \log(1-p_i)(1-p_i)^{T-1}(1-\delta^T)\right)\right) \\ &= -\sum_{i=1}^n \alpha_i(1-\delta^T)\left(\log(1-p_i)+\log(\delta)\right)\log(1-p_i)(1-p_i)^{T-1}\end{aligned}$$

$<0.$

Hence, every stationary point is a local maximum, i.e., $V^{\text{FI}}(T)$ is pseudoconcave.

By definition, $\tilde{T}$ solves $dV^{\text{FI}}(T)/dT = 0$. Since $V^{\text{FI}}(T)$ is pseudoconcave, $\tilde{T} = \arg\max V^{\text{FI}}(T)$. If $\tilde{T} \le 2$, then $T^* = 2$. If $\tilde{T} > 2$, then T^* equals the integer just below $\tilde{T}$ or above $\tilde{T}$.

By Lemma EC.6, T^* is nonincreasing in p_i, for any i. If $p_i = p$ for all i, T^* maximizes $(1-\delta^{T-1}(1-p)^{T-1})/(1-\delta^T)$ and is thus independent of α_i. □

Lemma EC.8. *For any $p \in (0,1)$ and $\delta \in (0,1)$, the function*

$$F(p,\delta,T) \doteq \frac{1-(\delta(1-p))^{T-1}}{(1-\delta^T)} \times \frac{(1-\delta)(1+p\delta)}{(1-\delta(1-p))}.$$

is nonincreasing in p, for any $T \ge 4$.

Proof. We first show that when $T \ge 4$, $\frac{\partial F(p,\delta,T)}{\partial p} \ge 0 \Rightarrow \frac{\partial^2 F(p,\delta,T)}{\partial p \partial T} < 0$. Fix $T \ge 4$ and p and suppose that $\partial F(p,\delta,T)/\partial p \ge 0$, i.e.,

$$-\delta^3(1-p)^2 + (\delta(1-p))^T\left((T-1)(1-\delta) + \delta^2(1-p)^2 + \delta^2 p^2(T-2) + p\delta(T-1)(2-\delta) + p\delta^2\right) \ge 0.$$

Then, using first the logarithmic inequality, $\ln(x) \le x-1$ for any $x>0$, and then the fact that $T \ge 4$, we obtain

$$\begin{aligned}
&\frac{\partial^2 F(p,\delta,T)}{\partial p \partial T}\Big|_{\frac{\partial F(p,\delta,T)}{\partial p} \ge 0} \\
=&\frac{(1-\delta)}{\delta(1-\delta^T)^2(1-p)^2(1-\delta(1-p))^2}\delta^T \ln(\delta) \\
&\times \left(-\delta^3(1-p)^2 + (\delta-\delta p)^T\left((T-1)(1-\delta) + \delta^2(1-p)^2 + \delta^2 p^2(T-2) + p\delta(T-1)(2-\delta) + p\delta^2\right)\right) \\
&+ \frac{(1-\delta)}{\delta(1-\delta^T)(1-p)^2(1-\delta(1-p))^2} \times (\delta(1-p))^T \\
&\times \Big(1 - \delta^2(1-p)p - \delta(1-2p) \\
&\qquad + \ln(\delta(1-p)) \times \left((T-1)(1-\delta) + \delta^2(1-p)^2 + \delta^2 p^2(T-2) + p\delta(T-1)(2-\delta) + p\delta^2\right)\Big) \\
\le&\frac{(1-\delta)}{\delta(1-\delta^T)(1-p)^2(1-\delta(1-p))^2} \times (\delta(1-p))^T \\
&\times \Big(1 - \delta^2(1-p)p - \delta(1-2p) \\
&\qquad + (\delta(1-p)-1) \times \left((T-1)(1-\delta) + \delta^2(1-p)^2 + \delta^2 p^2(T-2) + p\delta(T-1)(2-\delta) + p\delta^2\right)\Big) \\
\le&\frac{(1-\delta)}{\delta(1-\delta^T)(1-p)^2(1-\delta(1-p))^2} \times (\delta(1-p))^T \\
&\times \Big(1 - \delta^2(1-p)p - \delta(1-2p) \\
&\qquad + (\delta(1-p)-1) \times \left((4-1)(1-\delta) + \delta^2(1-p)^2 + \delta^2 p^2(4-2) + p\delta(4-1)(2-\delta) + p\delta^2\right)\Big) \\
=&-\frac{(1-\delta)}{\delta(1-\delta^T)(1-p)^2} \times (\delta(1-p))^T(2-\delta+3p\delta) \\
<&0.
\end{aligned}$$

Therefore, if there exists some $\hat{p} \in (0,1)$ and $\hat{T} > 4$ such that $\partial F(\hat{p},\delta,\hat{T})/\partial p = 0$, then, $\partial^2 F(\hat{p},\delta,\hat{T})/\partial p \partial T < 0$. Because $\partial^2 F(p,\delta,T)/\partial p \partial T$ is continuous in T, $\partial^2 F(\hat{p},\delta,T')/\partial p \partial T \le 0$ for all $T' \in (\hat{T}-\gamma, \hat{T}+\gamma)$, for some

$\gamma > 0$. By the fundamental theorem of calculus, $\partial F(\hat{p},\delta,T')/\partial p \geq \partial F(\hat{p},\delta,\hat{T})/\partial p = 0$ for all $T' \in (\hat{T}-\gamma,\hat{T})$. Hence, the function $\partial F(\hat{p},\delta,T)/\partial p$ crosses zero from above at $\hat{p}$. Since $\hat{p}$ was chosen arbitrarily, we obtain that $\partial F(p,\delta,T)/\partial p$ crosses zero at most once as T increases, for any $T \geq 4$, and the crossing is from above. Since, for any p, $\partial F(p,\delta,4)/\partial p = -(\delta^2(1-\delta+\delta p(4-3p)))/(1+\delta+\delta^2+\delta^3) < 0$, we conclude that $\partial F(p,\delta,T)/\partial p \leq 0$ for all p when $T \geq 4$, i.e., $F(p,\delta,T)$ is nonincreasing in p when $T \geq 4$. $\square$

LEMMA EC.9. *For any* $\delta \in [0,1]$, *the function*

$$f(\delta) \doteq \frac{\left(2+4\delta+4\delta^2+\delta^3-\sqrt{\delta(4+\delta(2+\delta)^2)}\right)\left(2+2\delta+2\delta^2+\delta^3+\sqrt{\delta(4+\delta(2+\delta)^2)}\right)}{4(1+\delta+\delta^2)^3}$$

is nonincreasing in δ.

Proof. Because

$$f'(\delta) = -\frac{\delta(4+16\delta+20\delta^2+14\delta^3+8\delta^4+\delta^5)}{2(1+\delta+\delta^2)^4} + \frac{\delta^{\frac{3}{2}}(10+24\delta+18\delta^2+6\delta^3-6\delta^4-6\delta^5-\delta^6)}{2(1+\delta+\delta^2)^4\sqrt{4+\delta(2+\delta)^2}},$$

$$\begin{aligned} f'(\delta) \leq 0 &\Leftrightarrow \delta^{\frac{1}{2}}(10+24\delta+18\delta^2+6\delta^3-6\delta^4-6\delta^5-\delta^6) \leq (4+16\delta+20\delta^2+14\delta^3+8\delta^4+\delta^5)\sqrt{4+\delta(2+\delta)^2} \\ &\Leftrightarrow \delta(10+24\delta+18\delta^2+6\delta^3-6\delta^4-6\delta^5-\delta^6)^2 \leq (4+16\delta+20\delta^2+14\delta^3+8\delta^4+\delta^5)^2(4+\delta(2+\delta)^2) \\ &\Leftrightarrow -4(1+\delta+\delta^2)^4(16+55\delta+60\delta^2+20\delta^3+2\delta^4) \leq 0, \end{aligned}$$

and the latter inequality always holds. $\square$

LEMMA EC.10.

$$\min_{0\leq p\leq 1, 0\leq\delta\leq 1}\ \max_{T\in\{2,3,\ldots\}} \frac{1-(\delta(1-p))^{T-1}}{(1-\delta^T)} \times \frac{(1-\delta)(1+p\delta)}{(1-\delta(1-p))} = \frac{16+\sqrt{13}}{27},$$

and the minimum is achieved when $\delta = 1$, $p = (5-\sqrt{13})/16$ *and* $T \in \{3,4\}$.

Proof. The proof uses Lemmas EC.6, EC.8, and EC.9 in appendix. Let $F(p,\delta,T)$ denote the objective function. By Lemma EC.6, for any $T \geq 2$, there exist breakpoints $\{p_1,p_2,p_3,\ldots\}$ with $p_1 = 1$ and $p_T \leq p_{T-1}$ for any $T > 1$, such that, for any $\hat{T} < \infty$, $\hat{T} = \arg\max_{T'\in\{2,3,\ldots\}} F(p,\delta,T')$ for all $p \in [p_{\hat{T}}, p_{\hat{T}-1}]$. In particular, solving $F(p,\delta,2) = F(p,\delta,3)$ for p yields that $p_2 = 1/(1+\delta)$ and solving $F(p,\delta,3) = F(p,\delta,4)$ for p yields that $p_3 = (2+2\delta+\delta^2-\sqrt{\delta(4+4\delta+4\delta^2+\delta^3)})/(2(1+\delta+\delta^2))$.

We first show that for any p, $\max_{T\in\{2,3,\ldots\}} F(p,\delta,T) \geq \min_{p\in\{p_2,p_3\}} \max_{T\in\{2,3,\ldots\}} F(p,\delta,T) = \min_{p\in\{p_2,p_3\}} F(p,\delta,3)$. On the one hand, for any $p \leq p_3$, $\max_{T\in\{2,3,\ldots\}} F(p,\delta,T) \geq F(p,\delta,4) \geq F(p_3,\delta,4) = F(p_3,\delta,3)$, because $F(p,\delta,T)$ is nonincreasing in p for all $T \geq 4$ by Lemma EC.8 and $p \leq p_3$ and because $\arg\max F(p_3,\delta,T) = \{3,4\}$. On the other hand, for any $p \geq p_2$, $\max_{T\in\{2,3,\ldots\}} F(p,\delta,T) = F(p,\delta,2) = (1+\delta p)/(1+\delta) \geq (1+\delta p_2)/(1+\delta) = F(p_2,\delta,2) = F(p_2,\delta,3)$ because $\arg\max F(p_2,\delta,T) = \{2,3\}$.

Therefore,

$$\begin{aligned} &\min_{0\leq p\leq 1, 0\leq\delta\leq 1}\ \max_{T\in\{2,3,\ldots\}} F(p,\delta,T) = \min_{p\in\{p_2,p_3\}, 0\leq\delta\leq 1} F(p,\delta,3) \\ &= \min_{0\leq\delta\leq 1} \min\left\{F\left(\frac{1}{1+\delta},\delta,3\right), F\left(\frac{2+2\delta+\delta^2-\sqrt{\delta(4+4\delta+4\delta^2+\delta^3)}}{2(1+\delta+\delta^2)}\right)\right\} \end{aligned}$$

$$= \min_{0\le\delta\le 1} \min\left\{\frac{1+2\delta}{(1+\delta)^2}, \frac{\left(2+4\delta+4\delta^2+\delta^3-\sqrt{\delta(4+\delta(2+\delta)^2)}\right)\left(2+2\delta+2\delta^2+\delta^3+\sqrt{\delta(4+\delta(2+\delta)^2)}\right)}{4(1+\delta+\delta^2)^3}\right\}.$$

By Lemma EC.9, the last term in brackets is nonincreasing in δ. Similarly, the term $(1+2\delta)/(1+\delta)^2$ is nonincreasing since its derivative equals $-2\delta/(1+\delta)^3$. Hence, both terms are minimized when $\delta=1$, yielding:

$$\min_{0\le p\le 1, 0\le\delta\le 1} \max_{T\in\{2,3,\ldots\}} F(p,\delta,T) = \min\left\{\frac{3}{4}, \frac{16+\sqrt{13}}{27}\right\} = \frac{16+\sqrt{13}}{27},$$

and the bound is attained when $p=p_3$ and $\delta=1$. □

Proposition 6 characterizes the robustness of the FI coordination scheduling rule and its sensitivity around the optimal fixed cycle duration.

Proof of Proposition 6. The proof uses Lemmas EC.5 and EC.10 in appendix, which respectively derive a closed-form solution for V_i^{FI} and a worst-case on an algebraic expression, which will turn out to correspond to $V^{\mathrm{FI}}(T)/V^{\mathrm{FB}}$. The idea of the proof is to derive a worst-case bound on $V^{\mathrm{FB}}-V^{\mathrm{FI}}(T)$ across all problem instances. The worst case turns out to be attained when $\sum_i f(x_i)\in\{0,\sum_i f_i(0)\}$, which can be achieved with only one worker and a binary productivity function. For this type of problem instance, the ratio $V^{\mathrm{FI}}(T)/V^{\mathrm{FB}}$ can be expressed as a function of δ and the likelihood of having at least one question, denoted by p, and this expression can be bounded from below using Lemma EC.10.

Fix any $T'\ge T-1$ and suppose coordination must happen by time $T'+1$ under $\pi\in\{\mathrm{FI},\mathrm{FB}\}$. Since T' is chosen arbitrarily, this is without loss of generality. Let $\sigma=(\boldsymbol{\xi}_1^\sigma,\boldsymbol{\xi}_2^\sigma,\ldots,\boldsymbol{\xi}_{T'-1}^\sigma)$ be a sample path defined as follows: In any period t and state $\mathbf{x}$, if production takes place, transitions occur to state $\mathbf{x}+\boldsymbol{\xi}_{t+1}^\sigma$. We denote by $\xi_{t,i}^\sigma$ the ith component of $\boldsymbol{\xi}_t^\sigma$. Let $\mathbb{P}_\sigma$ be the probability of sample path σ and let $\mathcal{U}_{T'}$ be the set of all possible sample paths. For $\pi\in\{\mathrm{FI},\mathrm{FB}\}$, let $1\le\theta_\sigma^\pi\le T'$ be the number of periods of production under π on σ until coordination happens. Obviously, $\theta_\sigma^{\mathrm{FI}}=T-1$. By Proposition 1, $f_i\left(0+\sum_{k=1}^t \xi_{k,i}^\sigma\right)>\phi$ if and only if $t\le\theta_\sigma^{\mathrm{FB}}$. Accordingly, we have

$$V^{\mathrm{FI}}(T)=\sum_\sigma \mathbb{P}_\sigma \sum_{t=0}^{T-2}\delta^t\sum_{i=1}^n f_i\left(0+\sum_{k=1}^t \xi_{k,i}^\sigma\right)+\delta^T V^{\mathrm{FI}}(T)=\frac{1}{1-\delta^T}\sum_\sigma \mathbb{P}_\sigma \sum_{t=0}^{T-2}\delta^t\sum_{i=1}^n f_i\left(0+\sum_{k=1}^t \xi_{k,i}^\sigma\right),$$

and similarly,

$$V^{\mathrm{FB}}=\frac{1}{1-\left(\sum_\sigma \mathbb{P}_\sigma\delta^{\theta_\sigma^{\mathrm{FB}}+1}\right)}\sum_\sigma \mathbb{P}_\sigma\sum_{t=0}^{\theta_\sigma^{\mathrm{FB}}-1}\delta^t\sum_{i=1}^n f_i\left(0+\sum_{k=1}^t \xi_{k,i}^\sigma\right).$$

For any $\mathbf{x}$, denote the set of sample paths that visit state $\mathbf{x}$ under either FI or FB before coordination happens by $\mathcal{S}(\mathbf{x})=\{\sigma\in\mathcal{U}_{T'}:\exists t\le\max\{T,\theta_\sigma^{\mathrm{FB}}\}:\mathbf{x}=\sum_{k=1}^t\boldsymbol{\xi}_t^\sigma\}$ and the time at which this visit happens on σ by $\tau_\sigma(\mathbf{x})$. (If $\sigma\ne\mathcal{S}(\mathbf{x})$, set $\tau_\sigma(\mathbf{x})>T'$.). Accordingly,

$$V^{\mathrm{FB}}-V^{\mathrm{FI}}(T)=\sum_{\mathbf{x}\ge\mathbf{0}}\sum_{\sigma\in\mathcal{S}(\mathbf{x})}\mathbb{P}_\sigma\delta^{\tau_\sigma(\mathbf{x})}\left(\frac{\mathbb{1}[\tau_\sigma(\mathbf{x})<\theta_\sigma^{\mathrm{FB}}]}{1-\left(\sum_{\sigma'}\mathbb{P}_{\sigma'}\delta^{\theta_{\sigma'}^{\mathrm{FB}}+1}\right)}-\frac{\mathbb{1}[\tau_\sigma(\mathbf{x})<T-1]}{1-\delta^T}\right)\left(\sum_{i=1}^n f_i(x_i)\right).$$

We next propose an iterative algorithm that increases the gap $V^{\mathrm{FB}}-V^{\mathrm{FI}}(T)$. An iteration starts as follows. Suppose that there exists a state $\mathbf{x}$ such that $\sum_i f_i(x_i)>\phi$, i.e., such that $\tau_\sigma(\mathbf{x})<\theta_\sigma^{\mathrm{FB}}$ for all $\sigma\in$

$\mathcal{S}(\mathbf{x})$, and that $\sum_{\sigma\in\mathcal{S}(\mathbf{x})}\mathbb{P}_\sigma\delta^{\tau_\sigma(\mathbf{x})}\left(\frac{\mathbb{1}[\tau_\sigma(\mathbf{x})<\theta_\sigma^{\mathrm{FB}}]}{1-\left(\sum_{\sigma'}\mathbb{P}_{\sigma'}\delta^{\theta_{\sigma'}^{\mathrm{FB}}+1}\right)}-\frac{\mathbb{1}[\tau_\sigma(\mathbf{x})<T-1]}{1-\delta^T}\right)<0$. Since the coefficient of $\sum_i f_i(x_i)$ in $V^{\mathrm{FB}}-V^{\mathrm{FI}}(T)$ is negative, $V^{\mathrm{FB}}-V^{\mathrm{FI}}(T)$ is maximized when $\sum_{i=1}^n f_i(x_i)$ is as small as possible. Replacing $\sum_{i=1}^n f_i(x_i)$ with ϕ thus strictly increases $V^{\mathrm{FB}}-V^{\mathrm{FI}}(T)$. Moreover, if $\sum_{i=1}^n f_i(x_i)=\phi$, it is no longer optimal to produce until reaching state $\mathbf{x}$ by Proposition 1; that is, on all sample paths $\sigma\in\mathcal{S}(\mathbf{x})$, $\theta_\sigma^{\mathrm{FB}}$ should decrease to $\tau_\sigma(\mathbf{x})$. Doing so weakly increases the terms that are associated with all other states $\mathbf{x}'\neq\mathbf{x}$ increase through a decrease in the denominator $1-\left(\sum_{\sigma'}\mathbb{P}_{\sigma'}\delta^{\theta_{\sigma'}^{\mathrm{FB}}+1}\right)$. If there is another such state, we proceed to another iteration. If there is no other such state, the algorithm stops. Let us denote by $\overline{f}$ the revised productivity functions and by $\overline{\theta}_\sigma^{\mathrm{FB}}$ the coordination times on same path σ under the FB policy associated with these revised productivity functions. Hence,

$$V^{\mathrm{FB}}-V^{\mathrm{FI}}(T)\le\sum_{\mathbf{x}\ge\mathbf{0}}\sum_{\sigma\in\mathcal{S}(\mathbf{x})}\mathbb{P}_\sigma\delta^{\tau_\sigma(\mathbf{x})}\left(\frac{\mathbb{1}[\tau_\sigma(\mathbf{x})<\overline{\theta}_\sigma^{\mathrm{FB}}]}{1-\left(\sum_{\sigma'}\mathbb{P}_{\sigma'}\delta^{\overline{\theta}_{\sigma'}^{\mathrm{FB}}+1}\right)}-\frac{\mathbb{1}[\tau_\sigma(\mathbf{x})<T-1]}{1-\delta^T}\right)\left(\sum_{i=1}^n\overline{f}_i(x_i)\right).$$

As a result of the algorithm, for any state $\mathbf{x}$, either $\sum_i\overline{f}_i(x_i)>\phi$, i.e., $\tau_\sigma(\mathbf{x})<\overline{\theta}_\sigma^{\mathrm{FB}}$ for all $\sigma\in\mathcal{S}(\mathbf{x})$ and $\sum_{\sigma\in\mathcal{S}(\mathbf{x})}\mathbb{P}_\sigma\delta^{\tau_\sigma(\mathbf{x})}\left(\frac{\mathbb{1}[\tau_\sigma(\mathbf{x})<\overline{\theta}_\sigma^{\mathrm{FB}}]}{1-\left(\sum_{\sigma'}\mathbb{P}_{\sigma'}\delta^{\overline{\theta}_{\sigma'}^{\mathrm{FB}}+1}\right)}-\frac{\mathbb{1}[\tau_\sigma(\mathbf{x})<T-1]}{1-\delta^T}\right)>0$ or (ii) $\sum_i\overline{f}_i(x_i)\le\phi$, i.e., $\tau_\sigma(\mathbf{x})\ge\overline{\theta}_\sigma^{\mathrm{FB}}$ for all $\sigma\in\mathcal{S}(\mathbf{x})$. In case (i), an upper bound on $V^{\mathrm{FB}}-V^{\mathrm{FI}}(T)$ is attained by replacing $\sum_i f_i(x_i)$ with $\sum_i f_i(0)$. In case (ii), an upper bound on $V^{\mathrm{FB}}-V^{\mathrm{FI}}(T)$ is attained by replacing $\sum_i f_i(x_i)$ with 0. That is,

$$V^{\mathrm{FB}}-V^{\mathrm{FI}}(T)\le\sum_{\mathbf{x}\ge\mathbf{0}:\sum_i\overline{f}(x_i)>\phi}\sum_{\sigma\in\mathcal{S}(\mathbf{x})}\mathbb{P}_\sigma\delta^{\tau_\sigma(\mathbf{x})}\left(\frac{1}{1-\left(\sum_{\sigma'}\mathbb{P}_{\sigma'}\delta^{\overline{\theta}_{\sigma'}^{\mathrm{FB}}+1}\right)}-\frac{\mathbb{1}[\tau_\sigma(\mathbf{x})<T-1]}{1-\delta^T}\right)\left(\sum_{i=1}^n f_i(0)\right).$$

Hence, the worst-case suboptimality gap is achieved when the sum of the individual productivity functions is binary, i.e., $\sum_i f_i(x_i)\in\{0,\sum_i f_i(0)\}$.

Accordingly, we henceforth assume, without loss of generality and for simplicity, that $n=1$, $f(x)=v>0$ if $x=0$ and zero if $x\ge1$, and let $p\doteq\mathbb{P}[\xi>0]$. Under the FB policy, the worker coordinates (with herself) each time she has one question. Thus, $V^{\mathrm{FB}}=v+\delta^2pV^{\mathrm{FB}}+\delta(1-p)V^{\mathrm{FB}}=v/(1-\delta^2p-\delta(1-p))=v/[(1-\delta)(1+p\delta)]$. Under the FI rule, by Lemma EC.5, $V^{\mathrm{FI}}(T)=v(1-(\delta(1-p))^{T-1})/(1-\delta(1-p))\times1/(1-\delta^T)$. Hence,

$$\frac{\max_{T\in\{2,3,\dots\}}V^{\mathrm{FI}}(T)}{V^{\mathrm{FB}}}=\max_{T\in\{2,3,\dots\}}\frac{1-(\delta(1-p))^{T-1}}{(1-\delta^T)}\times\frac{(1-\delta)(1+p\delta)}{(1-\delta(1-p))}.$$

The result then follows from Lemma EC.10.

We finally show the sensitivity result, when T^* is ill-chosen. Similar to above,

$$V^{\mathrm{FI}}(T+1)=\frac{1}{1-\delta^{T+1}}\sum_\sigma\mathbb{P}_\sigma\sum_{t=0}^{T-1}\delta^t\sum_{i=1}^n f_i\left(0+\sum_{k=1}^t\xi_{k,i}^\sigma\right).$$

The ratio $V^{\mathrm{FI}}(T+1)/V^{\mathrm{FI}}(T)$ is minimized if, on every sample path σ, $\sum_{i=1}^n f_i\left(0+\sum_{k=1}^{T-1}\xi_{k,i}^\sigma\right)=0$. This can easily be achieved on every sample path by considering deterministic transitions, i.e., $\mathbb{P}[\mathbf{1}]=1$ with $f_i(x_i)=0$ if $x_i\ge T-1$. Accordingly, we obtain:

$$\frac{V^{\mathrm{FI}}(T+1)}{V^{\mathrm{FI}}(T)}=\frac{1-\delta^T}{1-\delta^{T+1}}\times\frac{\sum_{t=0}^{T-2}\delta^t\sum_{i=1}^n f_i\left(0+\sum_{k=1}^t\xi_{k,i}^\sigma\right)}{\sum_{t=0}^{T-2}\delta^t\sum_{i=1}^n f_i\left(0+\sum_{k=1}^t\xi_{k,i}^\sigma\right)}=\frac{1-\delta^T}{1-\delta^{T+1}}.$$

For any T, $(1-\delta^T)/(1-\delta^{T+1})$ is nonincreasing in T. Therefore,

$$\frac{V^{\mathrm{FI}}(T^*+1)}{V^{\mathrm{FI}}(T^*)}\ge\frac{T^*}{T^*+1}\ge\frac{2}{3},$$

The bound is tight since when $\delta\to1$ and $p\to1$, $T^*=2$. $\square$

Proposition 7 describes how PC, PP, HS, and FI compare to one another as team size n increases.

Proof of Proposition 7. The proof uses Lemmas EC.4, EC.5, and EC.7 in appendix, which respectively derive a closed-form solution for V_i^{HS}, for V_i^{FI}, and provide an expression for the optimal fixed schedule T^*. We first show that the cycle duration under the FI rule is independent of $\sum_{i=1}^n v_i$ when $p_i = p$. As a result, the cycle duration that maximizes the sum of the workers' value functions also maximizes each individual worker's value function. In contrast, the cycle duration under the HS rule is a random variable for all workers $i > 1$ as it depends on worker 1's likelihood of encountering issues. The comparison between the optimal cycle duration and a random one will establish that $V_i^{\text{FI}} > V_i^{\text{HS}}$ for all $i > 1$. Even though $V_1^{\text{HS}} > V_1^{\text{FI}}$, the term $\sum_{i=2}^n (V_i^{\text{FI}} - V_i^{\text{HS}})$ will end up dominating as n becomes large.

We first show that the cycle duration under the FI rule is independent of $\sum_{i=1}^n v_i$ when $p_i = p$. By (EC.7) in Lemma EC.7 and because $p_i = p$ for all $i > 1$, when $n \to \infty$, $T^* \in \{\lfloor \tilde{T} \rfloor, \lceil \tilde{T} \rceil\}$, in which $\tilde{T}$ solves

$$\begin{aligned}0 =& v_1 \frac{1-(\delta(1-p_1))^{T-2}}{1-\delta^T}\left(\frac{1}{T}\frac{\delta^T\ln(\delta^T)}{1-\delta^T} - \frac{1}{T-1}\frac{(\delta(1-p_1))^{T-1}\ln\left((\delta(1-p_1))^{T-1}\right)}{1-(\delta(1-p_1))^{T-1}}\right)\\ &+ \sum_{i=2}^{\infty} v_i \frac{1-(\delta(1-p))^{T-2}}{1-\delta^T}\left(\frac{1}{T}\frac{\delta^T\ln(\delta^T)}{1-\delta^T} - \frac{1}{T-1}\frac{(\delta(1-p))^{T-1}\ln\left((\delta(1-p))^{T-1}\right)}{1-(\delta(1-p))^{T-1}}\right).\end{aligned}$$

This implies that $\tilde{T}$ solves

$$0 = \frac{1}{T}\frac{\delta^T\ln(\delta^T)}{1-\delta^T} - \frac{1}{T-1}\frac{(\delta(1-p))^{T-1}\ln\left((\delta(1-p))^{T-1}\right)}{1-(\delta(1-p))^{T-1}}.$$

Otherwise, $\left(\frac{1}{T}\frac{\delta^T\ln(\delta^T)}{1-\delta^T} - \frac{1}{T-1}\frac{(\delta(1-p_1))^{T-1}\ln\left((\delta(1-p_1))^{T-1}\right)}{1-(\delta(1-p_1))^{T-1}}\right)$ should be infinite since $\sum_{i=2}^{\infty} v_i = \infty$, which cannot be true. Therefore, $T^* = \arg\max_{T\in\{2,3,\dots\}} \frac{v_i}{1-\delta(1-p)}\frac{1-\delta^{T-1}(1-p)^{T-1}}{1-\delta^T}$ for every $i \geq 1$.

We next compare the value functions under the two policies and establish that $V_i^{\text{FI}} > V_i^{\text{HS}}$ for all $i > 1$. By Lemma EC.5,

$$V_i^{\text{FI}} = \frac{v_i}{1-\delta(1-p)}\frac{1-\delta^{T^*-1}(1-p)^{T^*-1}}{1-\delta^{T^*}} = \frac{v_i\left(1-\delta^{T^*-1}(1-p)^{T^*-1}\right)}{1-\delta(1-p)} + \delta^{T^*}\overline{V}_i^{\text{FI}},$$

in which $\overline{V}_i^{\text{FI}}$ denotes the continuation value after the first production cycle. By Lemma EC.4, for any $i > 1$,

$$V_i^{\text{HS}} = \sum_{t=2}^{\infty} p_1(1-p_1)^{t-2}\left(\frac{v_i\,(1-\delta^{t-1}(1-p)^{t-1})}{1-\delta(1-p)} + \delta^t\overline{V}_i^{\text{HS}}\right), \tag{EC.8}$$

in which $\overline{V}_i^{\text{HS}}$ denotes the continuation value after the first production cycle.

The proof of the comparison proceeds by induction. Fix $i > 1$ and suppose that $\overline{V}_i^{\text{FI}} \geq \overline{V}_i^{\text{HS}}$, then

$$\begin{aligned}V_i^{\text{FI}} \geq& \frac{v_i\left(1-\delta^{T^*-1}(1-p)^{T^*-1}\right)}{1-\delta(1-p)} + \delta^{T^*}\overline{V}_i^{\text{HS}}\\ >& \sum_{T=2}^{\infty} p_1(1-p_1)^{T-2}\left(\frac{v_i\,(1-\delta^{T-1}(1-p)^{T-1})}{1-\delta(1-p)} + \delta^T\overline{V}_i^{\text{HS}}\right) = V_i^{\text{HS}};\end{aligned}$$

here, the first inequality is by induction hypothesis and the second inequality follows from the optimality of T^* and the fact that, under HS, the duration of the first production cycle is random.

Accordingly, since V_1^{FI} and V_1^{HS} are both independent of n,

$$\lim_{n\to\infty}\frac{1}{n}\sum_{i=1}^n V_i^{\text{FI}} = \lim_{n\to\infty}\frac{1}{n}\sum_{i=2}^n V_i^{\text{FI}} > \lim_{n\to\infty}\frac{1}{n}\sum_{i=2}^n V_i^{\text{HS}} = \lim_{n\to\infty}\frac{1}{n}\sum_{i=1}^n V_i^{\text{HS}}.$$

By continuity, there exists a threshold $\tilde{n}$ such that for all $n \geq \tilde{n}$, $\frac{1}{n}\sum_{i=1}^{n} V_i^{\mathrm{FI}} \geq \frac{1}{n}\sum_{i=1}^{n} V_i^{\mathrm{HS}}$.

The second inequality follows from Proposition 4, which statement can be generalized so that $p_1 \neq p_i$ for all $i > 1$. □

The proof of Proposition 8 uses several lemmas, namely, Lemmas EC.11, EC.12, and EC.13, all of which are of technical nature, to establish either nonnegativity or monotonicity of algebraic expressions, which emerge in the comparison of the value functions under FI and under PC-C$^{\min}$.

Lemma EC.11. *For any $\delta \in (0,1)$, $p_i \in (0,1)$ and $T \geq 2$,*

$$F(T) \doteq (1-\delta)\delta^T(1-p_i)^{T-1} + \delta p_i(1-\delta^T) + (1-\delta)(1-\delta-\delta^T) > 0.$$

Proof. We first show that $F(T)$ is pseudoconvex and then that $F'(2) > 0$, thereby showing that $F'(T) > 0$ for all $T \geq 2$. Observing that $F(2) > 0$ then establishes the result. The function $F(T)$ is pseudo-convex because

$$F''(T)\Big|_{F'(T)=0} = (1-\delta)(1-p_i)^{T-1}\ln(1-p_i)\left(\ln(\delta)+\ln(1-p_i)\right) > 0.$$

Moreover, because $-p_i\ln(\delta) + (1-\delta)(1-p_i)\ln(1-p_i)$ is decreasing in δ since its derivative with respect to δ equals $-p_i/\delta - (1-p_i)\ln(1-p_i) < p_i(1-1/\delta) < 0$ (here, we used the inequality $-\ln(1-x) < x/(1-x)$ for all $x < 1$ and $x \neq 0$),

$$\begin{aligned} F'(2) =& \delta^2\left(-p_i\ln(\delta) + (1-\delta)(1-p_i)\ln(1-p_i)\right) \\ >& \delta^2\left(-p_i\ln(\delta) + (1-\delta)(1-p_i)\ln(1-p_i)\right)_{\delta=1} = 0. \end{aligned}$$

Combining these two facts, we obtain that $F'(T) > 0$ for all $T \geq 2$. As a result, $F(T) \geq F(2) = (1-\delta)(1-\delta(1-p_i)) > 0$. □

Lemma EC.12. *For any $p \in (0,1)$, $\delta \in (0,1)$ and $T \geq 2$,*

$$f(p,\delta,T) \doteq (1-\delta)^2 + \delta p(1-\delta^T) - \delta^T(1-\delta)(1-(1-p)^{T-1}) > 0.$$

Proof. We first show that $f(p,\delta,T)$ is pseudoconvex and then that $f(p,\delta,2) > 0$, thereby showing that $\partial f(p,\delta,T)/\partial T > 0$ for all $T \geq 2$. Observing that $f(p,\delta,2) > 0$ then establishes the result. The function $f(p,\delta,T)$ is pseudoconvex in T because when $\partial f(p,\delta,T)/\partial T = 0$, i.e., when

$$\begin{aligned} &-\ln(\delta)\delta p - \ln(\delta)(1-\delta) + (1-p)^{T-1}(1-\delta)(\ln(\delta)+\ln(1-p)) = 0, \\ &\frac{\partial^2 f(p,\delta,T)}{\partial T^2}\Big|_{\frac{\partial f(p,\delta,T)}{\partial T}=0} = \ln(1-p)(1-p)^{T-1}(1-\delta)(\ln(\delta)+\ln(1-p)) > 0. \end{aligned}$$

Moreover,

$$\begin{aligned} \frac{\partial f(p,\delta,2)}{\partial T} =& \delta^2\left(-\ln(\delta)\delta p - \ln(\delta)(1-\delta) + (1-p)(1-\delta)(\ln(\delta)+\ln(1-p))\right) \\ =& \delta^2\left(-p\ln(\delta) + (1-p)(1-\delta)\ln(1-p)\right) \\ \geq& \delta^2(1-\delta)\left(-p + (1-p)\ln(1-p)\right) > 0. \end{aligned}$$

Therefore, $\partial f(p,\delta,T)/\partial T > 0$ for all $T \geq 2$. Thus for any $T \geq 2$, $f(p,\delta,T) \geq f(p,\delta,2) = (1-\delta)^2 + \delta p(1-\delta) > 0$. □

LEMMA EC.13. *For any $P \in (0,1)$, $\delta \in (0,1)$ and $T \geq 2$,*

$$f(P,\delta,T) \doteq -(1-\delta^T) + \delta P(1-\delta^T) - P^{T-1}(1-\delta)\delta^{T-1}$$

is nondecreasing in P.

Proof. Because $\partial^2 f(P,\delta,T)/\partial P^2 \leq 0$, $f(P,\delta,T)$ is concave in P. Hence, $f(P,\delta,T)$ is nondecreasing for all $P \in [0,1]$ if and only if $\partial f(1,\delta,T)/\partial P = \delta(1-\delta^T) - (T-1)(1-\delta)\delta^{T-1} \geq 0$. Let $g(\delta,T) \doteq \delta(1-\delta^T) - (T-1)(1-\delta)\delta^{T-1}$. Because

$$\frac{\partial^2 g(\delta,T)}{\partial T^2}\Big|_{\frac{\partial g(\delta,T)}{\partial T}=0} = -\delta^{T-1}\ln(\delta)(1-\delta) > 0$$

and because $g(\delta,2) = \delta^2(1-\delta) > 0$, $g(\delta,T) > 0$ for all $T \geq 2$. Hence, $\partial f(1,\delta,T)/\partial P > 0$ and therefore $\partial f(P,\delta,T)/\partial P \geq 0$ for all $P \in [0,1]$. □

Proposition 8 compares PC-C$^{\min}$, PP-C$^{\max}$, HS-C$^{\min}$, and HS-C$^{\max}$ with FI.

Proof of Proposition 8. The proof uses Lemmas EC.3, EC.5, EC.11, EC.12, and EC.13 in appendix. We prove all bullet points independently of each other.

- To show the first bullet point in the proposition, fix T. By Lemmas EC.3 and EC.5, $V_i^{\text{PC-C}^{\min}}(T) \geq V_i^{\text{FI}}(T)$ if and only if

$$\begin{aligned}
&\frac{v_i\left(\frac{1-(\delta(1-p_i))^{T-1}}{1-\delta(1-p_i)} + \frac{\left(\prod_{j=1}^n(1-p_j)\right)^{T-1}\delta^{T-1}}{1-\delta\prod_{j=1}^n(1-p_j)}\right)}{1-\left(\prod_{j=1}^n(1-p_j)\right)^{T-1}\delta^{T+1}\frac{1-\prod_{j=1}^n(1-p_j)}{1-\delta\prod_{j=1}^n(1-p_j)} - \left(1-\left(\prod_{j=1}^n(1-p_j)\right)^{T-1}\right)\delta^T}\\
&\geq \frac{v_i}{1-\delta(1-p_i)}\cdot\frac{1-\delta^{T-1}(1-p_i)^{T-1}}{1-\delta^T}\\
\Leftrightarrow &\frac{\delta^{T-1}\left(\prod_{j=1}^n(1-p_j)\right)^{T-1}((1-\delta)\delta^T(1-p_i)^{T-1}+\delta p_i(1-\delta^T)+(1-\delta)(1-\delta-\delta^T))}{(1-\delta^T)(1-\delta(1-p_i))\left((1-\delta)\delta^T\left(\prod_{j=1}^n(1-p_j)\right)^{T-1}+(1-\delta^T)\left(1-\delta\prod_{j=1}^n(1-p_j)\right)\right)} \geq 0\\
\Leftrightarrow &(1-\delta)\delta^T(1-p_i)^{T-1}+\delta p_i(1-\delta^T)+(1-\delta)(1-\delta-\delta^T) \geq 0.
\end{aligned}$$

  By Lemma EC.11, the last inequality always holds. Hence, $V_i^{\text{PC-C}^{\min}}(T) \geq V_i^{\text{FI}}(T)$ for any T. Therefore, $\max_{T^{\min}\in\{2,3,\ldots\}} V_i^{\text{PC-C}^{\min}}(T^{\min}) \geq \max_{T\in\{2,3,\ldots\}} V_i^{\text{FI}}(T)$.

  By Lemmas EC.3 and EC.5, $V_i^{\text{PP-C}^{\max}}(T) \geq V_i^{\text{FI}}(T)$ if and only if

$$\begin{aligned}
&\frac{v_i\frac{1-(\delta(1-p_i))^{T-1}}{1-\delta(1-p_i)}}{1-\left(\sum_{t=0}^{T-3}(\delta^{t+2}-\delta^T)\left(\sum_{j=1}^n p_j(1-p_j)^t\left(\prod_{k=1}^{j-1}(1-(1-p_k)^t)\right)\cdot\left(\prod_{k=j+1}^n(1-(1-p_k)^{t+1})\right)\right)+\delta^T\right)}\\
&\geq \frac{v_i}{1-\delta(1-p_i)}\cdot\frac{1-\delta^{T-1}(1-p_i)^{T-1}}{1-\delta^T}\\
\Leftrightarrow &1-\delta^T \geq 1-\left(\sum_{t=0}^{T-3}(\delta^{t+2}-\delta^T)\left(\sum_{j=1}^n p_j(1-p_j)^t\left(\prod_{k=1}^{j-1}(1-(1-p_k)^t)\right)\cdot\left(\prod_{k=j+1}^n(1-(1-p_k)^{t+1})\right)\right)+\delta^T\right),
\end{aligned}$$

  which always holds. Hence, $V_i^{\text{PP-C}^{\max}}(T) \geq V_i^{\text{FI}}(T)$ for any T. Therefore, $\max_{T^{\max}\in\{2,3,\ldots\}} V_i^{\text{PP-C}^{\max}}(T^{\max}) \geq \max_{T\in\{2,3,\ldots\}} V_i^{\text{FI}}(T)$.

- The second bullet point can be established in a similar way to the way the first inequality in Proposition 7 was derived, with the exception that (EC.8) needs to be modified as follows to reflect that HS-C$^{\min}$ has a minimum cycle duration of $T^{\min}$ and that HS-C$^{\max}$ has a maximum cycle duration of $T^{\max}$:

$$V_i^{\text{HS-C}^{\min}} = \sum_{T=1}^{T^{\min}} p_1(1-p_1)^{T-2}\left(\frac{v_i\left(1-\delta^{T^{\min}-1}(1-p)^{T^{\min}-1}\right)}{1-\delta(1-p)} + \delta^{T^{\min}}\overline{V}_i^{\text{HS-C}^{\min}}\right)$$
$$+ \sum_{T=T^{\min}+1}^{\infty} p_1(1-p_1)^{T-2}\left(\frac{v_i\left(1-\delta^{T-1}(1-p)^{T-1}\right)}{1-\delta(1-p)} + \delta^{T}\overline{V}_i^{\text{HS-C}^{\min}}\right)$$
$$V_i^{\text{HS-C}^{\max}} = \sum_{T=2}^{T^{\max}-1} p_1(1-p_1)^{T-2}\left(\frac{v_i\left(1-\delta^{T-1}(1-p)^{T-1}\right)}{1-\delta(1-p)} + \delta^{T}\overline{V}_i^{\text{HS-C}^{\max}}\right)$$
$$+ \sum_{T=T^{\max}}^{\infty} p_1(1-p_1)^{T-2}\left(\frac{v_i\left(1-\delta^{T^{\max}-1}(1-p)^{T^{\max}-1}\right)}{1-\delta(1-p)} + \delta^{T^{\max}}\overline{V}_i^{\text{HS-C}^{\max}}\right)$$

  in which $\overline{V}_i^{\text{HS-C}^{\min}}$ and $\overline{V}_i^{\text{HS-C}^{\max}}$ denote the continuation values after the first production cycle. Despite these modifications, the argument of the proof of Proposition 7 remains essentially identical and is omitted for brevity.

- To show the third result, we separately consider PC-C$^{\min}$ and then PP-C$^{\max}$. In both cases, first note that the enhanced worker-driven coordination scheduling rule yields the same value as the FI policy if someone wants to coordinate before or as soon as the minimum cycle duration has been reached (under PC-C$^{\min}$) or if someone wants to produce up to the maximum cycle duration while another worker wants to coordinate (under PP-C$^{\max}$). Taking these situations as base cases, one can use Taylor's theorem to bound from above the value under the enhanced worker-driven policies.

  —First, we consider PC-C$^{\min}$. Denote $P(n) \doteq \prod_{j=1}^{n}(1-p_j)$ and $K \doteq \frac{\delta^{T^{\min}-1}\left((1-\delta)^2+\delta(1-\delta^{T^{\min}}))\right)}{\left(1-\delta^{T^{\min}}\right)^2}$. By Lemma EC.3,

$$V_i^{\text{PC-C}^{\min}}(P(n)) = \frac{v_i\left(\frac{1-(\delta(1-p_i))^{T^{\min}-1}}{1-\delta(1-p_i)} + \frac{(P(n))^{T^{\min}-1}\delta^{T^{\min}-1}}{1-\delta P(n)}\right)}{1-(P(n))^{T^{\min}-1}\delta^{T^{\min}+1}\frac{1-P(n)}{1-\delta P(n)} - \left(1-(P(n))^{T^{\min}-1}\right)\delta^{T^{\min}}}.$$

  By Lemma EC.5, $V_i^{\text{PC-C}^{\min}}(0) = V_i^{\text{FI}}$. Therefore, by Taylor's theorem,

$$V_i^{\text{PC-C}^{\min}}(P(n)) \leq V_i^{\text{PC-C}^{\min}}(0) + \max_{P\in[0,1]} \frac{dV_i^{\text{PC-C}^{\min}}(P)}{dP}\cdot P(n) = V_i^{\text{FI}} + \max_{P\in[0,1]} \frac{dV_i^{\text{PC-C}^{\min}}(P)}{dP}\cdot P(n).$$

  We next derive a uniform bound on the right-hand side. For any $P(n)\in[0,1]$, the derivative of $V_i^{\text{PC-C}^{\min}}(P)$ is bounded as follows:

$$\begin{aligned}
&\frac{dV_i^{\text{PC-C}^{\min}}(P(n))}{dP}\\
=&v_i\delta^{T^{\min}-1}P^{T^{\min}-2}(1+(T^{\min}-2)(1-\delta P(n)))\\
&\times\frac{(1-\delta)^2+\delta p_i(1-\delta^{T^{\min}})-\delta^{T^{\min}}(1-\delta)(1-(1-p_i)^{T^{\min}-1})}{(1-\delta(1-p_i))\left((1-\delta^{T^{\min}})(1-\delta P(n))+\delta^{T^{\min}}(P(n))^{T^{\min}-1}(1-\delta)\right)^2}\\
\leq&v_i\frac{\delta^{T^{\min}-1}\left((1-\delta)^2+\delta p_i(1-\delta^{T^{\min}})-\delta^{T^{\min}}(1-\delta)(1-(1-p_i)^{T^{\min}-1})\right)}{(1-\delta(1-p_i))\left(1-\delta^{T^{\min}}\right)^2}
\end{aligned}$$

$$\leq v_i K.$$

Here, the first inequality follows from Lemmas EC.12 and EC.13 and the second inequality follows from having $0 \leq p_i \leq 1$. Therefore,

$$V_i^{\text{PC-C}^{\min}}(P(n)) \leq V_i^{\text{FI}} + \max_{P\in[0,1]} \frac{dV_i^{\text{PC-C}^{\min}}(P)}{dP} \cdot P(n) \leq V_i^{\text{FI}} + v_i \cdot K \cdot P(n).$$

Because $p_i \geq \underline{p}$ for all i, $P(n) \leq (1-\underline{p})^n$ and therefore $V^{\text{PC-C}^{\min}} - V^{\text{FI}} \leq n\overline{v}K(1-\underline{p})^n$. For any $\epsilon > 0$, setting $\tilde{n} > W(\ln(1-\underline{p})\epsilon/(\overline{v}K))/\ln(1-\underline{p})$, in which $W(x)$ is the principal solution for z in $x = ze^z$, yields the desired result.

—Next, we consider PP-C$^{\max}$. Denote

$$Q(n) \doteq \sum_{t=0}^{T^{\max}-3} (\delta^{t+2} - \delta^{T^{\max}}) \left(\sum_{j=1}^{n} p_j (1-p_j)^t \left(\prod_{k=1}^{j-1} (1-(1-p_k)^t) \right) \cdot \left(\prod_{k=j+1}^{n} \left(1-(1-p_k)^{t+1}\right) \right) \right)$$

and

$$\overline{Q} \doteq \max_{n\in\{2,3,\dots\}} \max_{p_i\in[\underline{p},\overline{p}]\ \forall i} \sum_{t=0}^{T^{\max}-3} (\delta^{t+2} - \delta^{T^{\max}}) \left(\sum_{j=1}^{n} p_j (1-p_j)^t \left(\prod_{k=1}^{j-1} (1-(1-p_k)^t) \right) \cdot \left(\prod_{k=j+1}^{n} \left(1-(1-p_k)^{t+1}\right) \right) \right),$$

and note that $\overline{Q}$ is a uniform upper bound on $Q(n)$ and that $\overline{Q} < 1 - \delta^{T^{\max}}$. Let also $L \doteq \frac{1}{(1-\delta)(1-\overline{Q}-\delta^{T^{\max}})^2}$.

By Lemma EC.3,

$$V_i^{\text{PP-C}^{\max}}(Q(n)) = \frac{v_i \frac{1-(\delta(1-p_i))^{T^{\max}-1}}{1-\delta(1-p_i)}}{1-Q(n)-\delta^{T^{\max}}}.$$

By Lemma EC.5, $V_i^{\text{PP-C}^{\max}}(0) = V_i^{\text{FI}}$. Therefore, by Taylor's theorem,

$$V_i^{\text{PP-C}^{\max}}(Q(n)) \leq V_i^{\text{PP-C}^{\max}}(0) + \max_{Q\in[0,\overline{Q}]} \frac{dV_i^{\text{PP-C}^{\max}}(Q(n))}{dQ} Q(n).$$

We next derive a uniform bound on the right-hand side. For any $Q(n) \in [0, \overline{Q}]$, the derivative of $V_i^{\text{PP-C}^{\max}}(Q)$ is bounded as follows:

$$\frac{dV_i^{\text{PP-C}^{\max}}(Q(n))}{dQ} = \frac{v_i \frac{1}{1-\delta}}{(1-Q(n)-\delta^{T^{\max}})^2} \leq v_i L.$$

Therefore, by Taylor's theorem,

$$V_i^{\text{PP-C}^{\max}}(Q(n)) \leq V_i^{\text{FI}} + \max_{Q\in[0,\overline{Q}]} \frac{dV_i^{\text{PP-C}^{\max}}(Q(n))}{dQ} Q(n) \leq V_i^{\text{FI}} + v_i \cdot L \cdot Q(n).$$

Because

$$\begin{aligned} nQ(n) \leq& n \sum_{t=0}^{T^{\max}-3} (\delta^{t+2} - \delta^{T^{\max}}) \left(\sum_{j=1}^{n} \frac{1}{t+1} \left(1 - \frac{1}{t+1}\right)^t \left(\prod_{k=1}^{j-1} (1-(1-\overline{p})^t) \right) \cdot \left(\prod_{k=j+1}^{n} \left(1-(1-\overline{p})^{t+1}\right) \right) \right) \\ =& n \sum_{t=0}^{T^{\max}-3} (\delta^{t+2} - \delta^{T^{\max}}) \left(\left(1-(1-\overline{p})^{t+1}\right)^n - \left(1-(1-\overline{p})^{t+1}\right)^n \right) \cdot \left(\frac{t}{(t+1)(1-\overline{p})} \right)^t \frac{1}{\overline{p}(1+t)}, \end{aligned}$$

$\lim_{n\to\infty} nQ(n) = 0$. Because $nQ(n)$ is continuous, for any $\epsilon > 0$, there exists a threshold $\tilde{n}$ such that $nQ(n)\overline{v}L < \epsilon$ for all $n \geq \tilde{n}$, and therefore such that $V^{\text{PP-C}^{\max}} - V^{\text{FI}} < \epsilon$ for all $n \geq \tilde{n}$.

□

EC.2.5. Equilibrium Characterization under General Productivity Functions with No Base Value

To simplify the notations in the proofs, we henceforth assume that $\mathbb{P}[\mathbf{0}]=0$. This is without loss of generality by redefining $f_i(x_i) \leftarrow f_i(x_i)/(1-\delta\mathbb{P}[\mathbf{0}])$ and $\mathbb{P}[\boldsymbol{\xi}] \leftarrow \mathbb{P}[\boldsymbol{\xi}]/(1-\delta\mathbb{P}[\mathbf{0}])$ for all $\boldsymbol{\xi} \geq \mathbf{0}$ and $\boldsymbol{\xi} \neq \mathbf{0}$. Moreover, we omit the redundant time index in the value and policy functions since we restrict our attention to worker-driven, time-independent rules.

To characterize the equilibrium policy under PC for general productivity functions (Proposition B-1), we first establish that worker i's value function is monotonic in the number of accumulated issues (Lemma EC.14). We will then use this property to characterize a local property on worker i's stated preferences, which we will then use in the proof of Proposition B-1 to initialize an induction argument.

LEMMA EC.14. *Suppose that Assumption 1(i) holds. $V_i^{PC}(x_i, x_{-i})$ is nonincreasing in (x_i, x_{-i}) for $i=1,2$.*

Proof. Throughout the proof, we omit the superscript PC. Fix i. The proof proceeds by induction. By Lemma EC.1, there exists a state $\hat{\mathbf{x}} \leq \overline{\mathbf{x}}$ such that $a_i(\mathbf{x}) = C$ for all $x_i \geq \hat{x}_i$ and $a_{-i}(\mathbf{x}) = C$ for all $x_{-i} \geq \hat{x}_{-i}$. Accordingly, $A(\mathbf{x}) = C$ for all $\mathbf{x}$ such that either $x_i \geq \hat{x}_i$ or $x_{-i} \geq \hat{x}_{-i}$. Therefore, $V_i(\mathbf{x})$ is constant and equal to $\delta V_i(\mathbf{0})$ for all $\mathbf{x}$ such that either $x_i \geq \hat{x}_i$ or $x_{-i} \geq \hat{x}_{-i}$.

Fix $\mathbf{x} \leq \hat{\mathbf{x}}$ with $\mathbf{x} \neq \hat{\mathbf{x}}$ and suppose that $V_i(\mathbf{x}+\boldsymbol{\xi})$ is nonincreasing in $\boldsymbol{\xi}$ for all $\boldsymbol{\xi} \geq \mathbf{0}$, $\boldsymbol{\xi} \neq \mathbf{0}$. For any $\mathbf{y} \geq \mathbf{x}$, we obtain

$$f_i(x_i) + \delta\mathbb{E}\left[V_i(\mathbf{x}+\boldsymbol{\xi})\right] \geq f_i(y_i) + \delta\mathbb{E}\left[V_i(\mathbf{y}+\boldsymbol{\xi})\right] \tag{EC.9}$$

$$f_{-i}(x_{-i}) + \delta\mathbb{E}\left[V_{-i}(\mathbf{x}+\boldsymbol{\xi})\right] \geq f_{-i}(y_{-i}) + \delta\mathbb{E}\left[V_{-i}(\mathbf{y}+\boldsymbol{\xi})\right], \tag{EC.10}$$

because $f_j'(x_j) \leq 0$ for $j=1,2$ and by induction hypothesis.

We next consider three exhaustive cases, depending on the equilibrium outcome and who exerts the coordination preemption and show that the induction hypothesis is preserved, i.e., $V_j(\mathbf{y}) \leq V_j(\mathbf{x})$ for $j=1,2$.

Case 1: $a_1(\mathbf{x}) = a_2(\mathbf{x}) = P$. In this case, $A(\mathbf{x}) = P$ by (1), i.e., $V_j(\mathbf{x}) = f_j(x_j) + \delta\mathbb{E}\left[V_j(\mathbf{x}+\boldsymbol{\xi})\right]$. We consider two cases, depending on $A(\mathbf{y})$.

- If $A(\mathbf{y}) = C$, $V_j(\mathbf{y}) = \delta V_j(\mathbf{0})$ for $j=1,2$. By assumption, $\delta V_j(\mathbf{0}) < f_j(x_j) + \delta\mathbb{E}\left[V_j(\mathbf{x}+\boldsymbol{\xi})\right]$ for $j=1,2$. Therefore $V_j(\mathbf{y}) < V_j(\mathbf{x})$ for $j=1,2$.
- If $A(\mathbf{y}) = P$, $V_j(\mathbf{y}) = f_j(y_j) + \delta\mathbb{E}\left[V_j(\mathbf{y}+\boldsymbol{\xi})\right]$ for $j=1,2$. Then, by (EC.9) and (EC.10), $V_j(\mathbf{y}) \leq f_j(x_j) + \delta\mathbb{E}\left[V_j(\mathbf{x}+\boldsymbol{\xi})\right] = V_j(\mathbf{x})$ for $j=1,2$.

Case 2: $a_i(\mathbf{x}) = P$ and $a_{-i}(\mathbf{x}) = C$. In this case, $A(\mathbf{x}) = C$ by (1), i.e., $V_j(\mathbf{x}) = \delta V_j(\mathbf{0})$ for $j=1,2$. By (EC.10), $f_{-i}(y_{-i}) + \delta\mathbb{E}\left[V_{-i}(\mathbf{y}+\boldsymbol{\xi})\right] \leq f_{-i}(x_{-i}) + \delta\mathbb{E}\left[V_{-i}(\mathbf{x}+\boldsymbol{\xi})\right] \leq \delta V_{-i}(\mathbf{0})$; therefore, $a_{-i}(\mathbf{y}) = C$ and by (1), $A(\mathbf{y}) = C$. Hence, $V_j(\mathbf{y}) = \delta V_j(\mathbf{0}) = V_j(\mathbf{x})$ for $j=1,2$.

Case 3: $a_1(\mathbf{x}) = a_2(\mathbf{x}) = C$. In this case, $A(\mathbf{x}) = C$ by (1), i.e., $V_j(\mathbf{x}) = \delta V_j(\mathbf{0})$ for $j=1,2$. By (EC.9), $f_j(y_i) + \delta\mathbb{E}\left[V_j(\mathbf{y}+\boldsymbol{\xi})\right] \leq f_j(x_j) + \delta\mathbb{E}\left[V_j(\mathbf{x}+\boldsymbol{\xi})\right] \leq \delta V_j(\mathbf{0})$ for $j=1,2$. Therefore, $a_i(\mathbf{y}) = C$ for $j=1,2$ and by (1), $A(\mathbf{y}) = C$. Hence, $V_j(\mathbf{y}) = \delta V_j(\mathbf{0}) = V_j(\mathbf{x})$ for $j=1,2$.

Combining all three cases shows that $V_j(\mathbf{x}) \leq V_j(\mathbf{y})$ for $j=1,2$, completing the induction step. $\square$

LEMMA EC.15. *Suppose that Assumption 1(i) holds. If, for some $i=1,2$, $a_i^{PC}(\mathbf{x})=P$, $a_i^{PC}(x_i+1,x_{-i})=C$, then $a_i^{PC}(x_i,x_{-i}+1)=P$.*

Proof. The proof uses Lemma EC.14 in appendix. Throughout the proof, we omit the superscript PC. The proof proceeds by contradiction. Suppose that $a_i(\mathbf{x})=P$, $a_i(x_i+1,x_{-i})=C$, but also that $a_i(x_i,x_{-i}+1)=C$.

By Lemma EC.14, $V_i(x_i+\xi_i,x_{-i}+1+\xi_{-i})\geq V_i(\mathbf{y}+\boldsymbol{\xi})$ for any $\mathbf{y}\geq(x_i,x_{-i}+1)$ and any $\boldsymbol{\xi}\geq\mathbf{0}$. Hence for any $\mathbf{y}\geq(x_i,x_{-i}+1)$, $f_i(x_i)+\delta\mathbb{E}\left[V_i(x_i+\xi_i,x_{-i}+\xi_{-i}+1)\right]\geq f_i(y_i)+\delta\mathbb{E}\left[V_i(\mathbf{y}+\boldsymbol{\xi})\right]$ because $f_j'(x_j)\leq 0$ for $j=1,2$ and because $\mathbb{P}[\boldsymbol{\xi}]$ is independent of the current stock of questions. By (5), since $a_i(x_i+1,x_{-i})=C$, we obtain that $a_i(\mathbf{y})=C$. Hence, by (1), $A(\mathbf{y})=C$, and therefore, $V_i(\mathbf{y})=\delta V_i(\mathbf{0})$ for all $\mathbf{y}\geq(x_i,x_{-i}+1)$.

Using a similar logic with $a_i(x_i,x_{-i}+1)=C$, we obtain: $V_i(\mathbf{y})=\delta V_i(\mathbf{0})$ for all $\mathbf{y}\geq(x_i+1,x_{-i})$.

Combining these two results with the assumption that $\mathbb{P}[\mathbf{0}]=0$, shows that $\mathbb{E}\left[V_i(\mathbf{x}+\boldsymbol{\xi})\right]=\delta V_i(\mathbf{0})$. Hence, by (5), $a_i(\mathbf{x})=P$ can be equivalently expressed as follows: $f_i(x_i)+\delta^2V_i(\mathbf{0})>\delta V_i(\mathbf{0})$. Similarly, $a_i(x_i,x_{-i}+1)=C$ can be equivalently expressed as: $f_i(x_i)+\delta^2V_i(\mathbf{0})\leq\delta V_i(\mathbf{0})$. Clearly, both inequalities cannot hold at the same time, and we therefore obtain a contradiction. □

Proposition B-1 characterizes the equilibrium policy under PC for general productivity function and establishes, in particular, the existence of a "coordination trigger point".

Proof of Proposition B-1. The proof uses Lemmas EC.14 and EC.15 in appendix. Lemma EC.14 establishes monotonicity of $V_i^{\mathrm{PC}}(\mathbf{x})$, which will be useful to establish that worker i's dominant strategy is a threshold policy. In particular, Lemma EC.15 initiates an induction argument by showing that worker i's decision to produce in $\mathbf{x}$, if she wants to coordinate with one more issue, is independent of x_{-i}. Throughout the proof, we omit the superscript PC. Fix i.

First, consider the "upper right corner," where coordination arises. By Lemma EC.1, there exists a state $\hat{\mathbf{x}}\leq\overline{\mathbf{x}}$ such that $a_i(\mathbf{x})=C$ for all $x_i\geq\hat{x}_i$ and $a_{-i}(\mathbf{x})=C$ for all $x_{-i}\geq\hat{x}_{-i}$. Accordingly, $A(\mathbf{x})=C$ for all $\mathbf{x}$ such that either $x_i\geq\hat{x}_i$ or $x_{-i}\geq\hat{x}_{-i}$.

Second, consider all other regions where production arises. Specifically, we show by induction that $a_i(\mathbf{y})=P$ for any $\mathbf{y}$ such that $y_i\leq\hat{x}_i-1$ for $i=1,2$. By (1), this will establish that $A(\mathbf{x})=P$ for all $\mathbf{x}<\hat{\mathbf{x}}$.

Fix i. We first consider the states at the boundary of the region of interest by showing that $a_i(\hat{x}_i-1,y_{-i})=P$ for all y_{-i}. The argument relies on two parts.

- By construction of $\hat{\mathbf{x}}$, there exists a value x_{-i} such that $a_i(\hat{x}_i-1,x_{-i})=P$. By (5), $f_i(\hat{x}_i-1)+\delta\mathbb{E}[V_i(\hat{x}_i-1+\xi_i,x_{-i}+\xi_{-i})]>\delta V_i(\mathbf{0})$. By Lemma EC.14, $V_i(y_i,y_{-i})$ is nonincreasing in y_{-i}. Hence, for all $y_{-i}\leq x_{-i}$, $f_i(\hat{x}_i-1)+\delta\mathbb{E}[V_i(\hat{x}_i-1+\xi_i,y_{-i}+\xi_{-i})]\geq f_i(\hat{x}_i-1)+\delta\mathbb{E}[V_i(\hat{x}_i-1+\xi_i,x_{-i}+\xi_{-i})]>\delta V_i(\mathbf{0})$. Therefore by (5), $a_i(\hat{x}_i-1,y_{-i})=P$ for all $y_{-i}\leq x_{-i}$.
- Because $a_i(\hat{x}_i,y_{-i})=C$ for all y_i, applying Lemma EC.15 iteratively shows that, for all $y_{-i}>x_{-i}$, $f_i(\hat{x}_i-1)+\delta\mathbb{E}[V_i(\hat{x}_i-1+\xi_i,y_{-i}+\xi_{-i})]>\delta V_i(\mathbf{0})$. Therefore by (5), $a_i(\hat{x}_i-1,y_{-i})=P$ for all $y_{-i}>x_{-i}$.

Combining these two results shows that $a_i(\hat{x}_i-1,y_{-i})=P$ for all y_{-i}.

We next consider the states that lie in the interior of the region of interest. By Lemma EC.14, $V_i(\mathbf{y})$ is nonincreasing in y_i. Moreover, $f_i(y_i)$ is nonincreasing. Hence, for any $\mathbf{y}$ such that $y_i\leq\hat{x}_i-1$, $f_i(y_i)+$

$\delta\mathbb{E}[V_i(y_i+\xi_i-1, y_{-i}+\xi_{-i})] \geq f_i(\hat{x}_i-1)+\delta\mathbb{E}[V_i(\hat{x}_i-1+\xi_i, y_{-i}+\xi_{-i})] > \delta V_i(\mathbf{0})$. Therefore by (5), $a_i(\mathbf{y}) = P$ for any $\mathbf{y}$ such that $y_i \leq \hat{x}_i - 1$.

We next characterize the value of $\hat{\mathbf{x}}$. Since $\mathbb{P}[\mathbf{0}] = 0$ and $A(\mathbf{x}) = C$ for all $\mathbf{x} \geq \hat{\mathbf{x}}$, $\mathbf{x} \neq \hat{\mathbf{x}}$, $V_i(\hat{x}_i - 1, \hat{x}_{-i} - 1) = f_{-i}(\hat{x}_i - 1) + \delta^2 V_i(\mathbf{0})$. Therefore, $a_i(\hat{x}_i - 1, \hat{x}_i - 1) = P \Leftrightarrow f_i(\hat{x}_i - 1) + \delta^2 V_i(\mathbf{0}) > \delta V_i(\mathbf{0}) \Leftrightarrow f_i(\hat{x}_i - 1) > \delta(1-\delta)V_i(\mathbf{0})$ and $a_i(\hat{x}_i, \hat{x}_{-i}) = C \Leftrightarrow f_i(\hat{x}_i) + \delta^2 V_i(\mathbf{0}) \leq \delta V_i(\mathbf{0}) \Leftrightarrow f_i(\hat{x}_i) \leq \delta(1-\delta)V_i(\mathbf{0})$. Combining these inequalities and applying the same logic to worker $-i$ yield the desired characterization of $\hat{\mathbf{x}}$. □

To characterize the equilibrium policy under PP for general productivity functions (Proposition B-2), we first establish that worker i's value function is monotonic in the number of accumulated issues, specifically, increasing in x_i and decreasing in x_{-i} (Lemma EC.17). The proof of this monotonocity property is by induction and, for the induction to be initialized, relies on a characterization of the value function in the upper right area of the orthant $\mathbf{x} \geq \mathbf{0}$, namely, when coordination happens in equilibrium under PP (Lemma EC.16).

Lemma EC.16. *Suppose that Assumption 1(i) holds. There exists a threshold $\hat{x}_i \leq \overline{x}_i$ such that, for all $x_i \geq \hat{x}_i$ and for all x_{-i}, $a_i^{PP}(\mathbf{x}) = C$ and $V_{-i}^{PP}(\mathbf{x})$ is constant in x_i and nonincreasing in x_{-i}. Moreover, for all $x_i \geq \overline{x}_i$ and for all x_{-i}, $V_i^{PP}(\mathbf{x})$ is constant in x_i and nondecreasing in x_{-i}.*

Proof. In the proof, we omit the 'PP' superscript. By Lemma EC.1, there exists a state $\hat{\mathbf{x}} \leq \overline{\mathbf{x}}$ such that $a_i(\mathbf{x}) = C$ for all $x_i \geq \hat{x}_i$.

We next characterize the value functions by induction. The initialization step is common to both $V_{-i}(\mathbf{x})$ and $V_i(\mathbf{x})$. For any $\mathbf{x} \geq \overline{x}$, $a_j(\mathbf{x}) = C$ for $j = 1, 2$ by Lemma EC.1. Hence, by (2), $A(\mathbf{x}) = C$. Therefore, $V_j(\mathbf{x}) = \delta V_j(\mathbf{0})$ for $j = 1, 2$, which are both constant.

Building on this initialization step, we next show the result regarding $V_{-i}(\mathbf{x})$, namely that $V_{-i}(\mathbf{x})$ is constant in x_i and nonincreasing in x_{-i} for any $\mathbf{x}$ such that $x_i \geq \hat{x}_i$. Consider some $\mathbf{x}$ such that $x_i \geq \hat{x}_i$ and $\mathbf{x} \not\geq \overline{\mathbf{x}}$ and suppose that $V_{-i}(\mathbf{x}+\boldsymbol{\xi})$ is constant in ξ_i and nonincreasing in ξ_{-i}, for all $\boldsymbol{\xi} \geq \mathbf{0}$ and $\boldsymbol{\xi} \neq \mathbf{0}$. For any $\mathbf{y} \geq \mathbf{x}$, we thus obtain that $f_{-i}(x_{-i}) + \delta\mathbb{E}\left[V_{-i}(\mathbf{x}+\boldsymbol{\xi})\right] \geq f_{-i}(y_{-i}) + \delta\mathbb{E}\left[V_{-i}(\mathbf{y}+\boldsymbol{\xi})\right]$ because $f'_{-i}(x) \leq 0$ and by the induction hypothesis, and the inequality is tight when $x_{-i} = y_{-i}$. Moreover, since $x_i \geq \overline{x}_i$, $a_i(\mathbf{x}) = C$. Accordingly, by (2), $V_{-i}(\mathbf{x}) = \max\{\delta V_{-i}(\mathbf{0}), f_{-i}(x_{-i}) + \delta\mathbb{E}[V_{-i}(\mathbf{x}+\boldsymbol{\xi})]\}$. Therefore, for any $\mathbf{y} \geq \mathbf{x}$

$$
\begin{aligned}
V_{-i}(\mathbf{x}) &= \max\{\delta V_{-i}(\mathbf{0}), f_{-i}(x_{-i}) + \delta\mathbb{E}\left[V_{-i}(\mathbf{x}+\boldsymbol{\xi})\right]\} \\
&\geq \max\{\delta V_{-i}(\mathbf{0}), f_{-i}(y_{-i}) + \delta\mathbb{E}\left[V_{-i}(\mathbf{y}+\boldsymbol{\xi})\right]\} = V_{-i}(\mathbf{y}), \qquad \text{(EC.11)}
\end{aligned}
$$

with equality when $x_{-i} = y_{-i}$.

We finally show the result regarding $V_i(\mathbf{x})$, building on the initialization step above, namely, that $V_i(\mathbf{x})$ is constant in x_i and nondecreasing in x_{-i} for any $\mathbf{x}$ such that $x_i \geq \overline{x}_i$. Consider some $\mathbf{x}$ such that $x_i \geq \overline{x}_i$ and $x_{-i} < \overline{x}_{-i}$ and suppose that $V_i(\mathbf{x}+\boldsymbol{\xi})$ is constant in ξ_i and nondecreasing in ξ_{-i} for all $\boldsymbol{\xi} \geq \mathbf{0}$ and $\boldsymbol{\xi} \neq \mathbf{0}$. For any $\mathbf{y} \geq \mathbf{x}$, we obtain that

$$
\delta\mathbb{E}\left[V_i(\mathbf{x}+\boldsymbol{\xi})\right] \leq \delta\mathbb{E}\left[V_i(\mathbf{y}+\boldsymbol{\xi})\right] \qquad \text{(EC.12)}
$$

by the induction hypothesis, and the inequality is tight when $x_{-i} = y_{-i}$. To finalize the induction step, we consider two cases.

- Suppose that $a_{-i}(\mathbf{y}) = C$. Since $a_i(\mathbf{y}) = C$ given that $y_i \geq x_i \geq \overline{x}_i$, $A(\mathbf{y}) = C$ by (2). Therefore, $V_i(\mathbf{y}) = \delta V_i(\mathbf{0})$. Since $a_i(\mathbf{x}) = C$ given that $x_i \geq \overline{x}_i$, $\delta V_i(\mathbf{0}) \geq V_i(\mathbf{x})$ by (5), and therefore, $V_i(\mathbf{y}) \geq V_i(\mathbf{x})$. Moreover, this holds at equality if $x_{-i} = y_{-i}$ since in that case $\delta V_{-i}(\mathbf{0}) \geq f_{-i}(y_{-i}) + \delta \mathbb{E}[V_{-i}(\mathbf{y}+\boldsymbol{\xi})] = f_{-i}(x_{-i}) + \delta \mathbb{E}[V_{-i}(\mathbf{x}+\boldsymbol{\xi})]$ given that (EC.12) is tight when $x_{-i} = y_{-i}$ as shown above; hence, $a_{-i}(\mathbf{x}) = C$ by (5), and therefore $A(\mathbf{x}) = C$, yielding that $V_i(\mathbf{x}) = \delta V_i(\mathbf{0})$.
- Suppose that $a_{-i}(\mathbf{y}) = P$. Since $a_i(\mathbf{y}) = C$ given that $y_i \geq x_i \geq \overline{x}_i$, $A(\mathbf{y}) = P$ by (2). Therefore, $V_i(\mathbf{y}) = f_i(y_i) + \delta \mathbb{E}[V_i(\mathbf{y}+\boldsymbol{\xi})] = \delta \mathbb{E}[V_i(\mathbf{y}+\boldsymbol{\xi})]$ since $f(y_i) = 0$ given that $y_i \geq \overline{x}_i$. Moreover $a_{-i}(\mathbf{x}) = P$ since, given that $a_{-i}(\mathbf{x}) = P$, (5), and (EC.11), we have $\delta V_{-i}(\mathbf{0}) < V_{-i}(\mathbf{y}) \leq V_{-i}(\mathbf{x})$. Thus, by (2), $A(\mathbf{x}) = P$. Hence, $V_i(\mathbf{x}) = f_i(x_i) + \delta \mathbb{E}[V_i(\mathbf{x}+\boldsymbol{\xi})] = \delta \mathbb{E}[V_i(\mathbf{x}+\boldsymbol{\xi})]$ since $f_i(x_i) = 0$ for all $x_i \geq \overline{x}_i$. As a result, using (EC.12), we obtain $V_i(\mathbf{x}) = \delta \mathbb{E}[V_i(\mathbf{x}+\boldsymbol{\xi})] \leq \delta \mathbb{E}[V_i(\mathbf{y}+\boldsymbol{\xi})] = V_i(\mathbf{y})$, with equality when $x_{-i} = y_{-i}$.

Combining both cases shows that $V_i(\mathbf{x}) \leq V_i(\mathbf{y})$, with equality when $x_{-i} = y_{-i}$. □

Lemma EC.17. *Suppose that Assumption 1(i) holds. $V_i^{PP}(x_i, x_{-i})$ is decreasing in x_i and increasing in x_{-i} for $i = 1, 2$.*

Proof. The proof uses Lemma EC.16 in appendix. In the proof, we omit the 'PP' superscript. Fix i. The proof proceeds by induction. By Lemma EC.16, for all $x_i \geq \overline{x}_i$, $V_i(x_i, x_{-i})$ is constant in x_i and nondecreasing in x_{-i}, whereas $V_{-i}(x_{-i}, x_i)$ is constant in x_i and nonincreasing in x_{-i}.

We next proceed to the induction step. Fix $\mathbf{x}$ such that $x_i < \overline{x}_i$, and suppose, as induction hypothesis, that $V_j(\mathbf{x}+\boldsymbol{\xi})$ is nonincreasing in ξ_j and nondecreasing in ξ_{-j} for $j = 1, 2$, for all $\xi_{-j} < 0$ if $\xi_j = 0$ and all ξ_{-j} if $\xi_j > 0$. For any $\mathbf{y}$ such that $y_i \geq x_i$ and $y_{-i} \leq x_{-i}$, we obtain

$$f_i(x_i) + \delta \mathbb{E}[V_i(\mathbf{x}+\boldsymbol{\xi})] \geq f_i(y_i) + \delta \mathbb{E}[V_i(\mathbf{y}+\boldsymbol{\xi})] \tag{EC.13}$$

$$f_{-i}(x_{-i}) + \delta \mathbb{E}[V_{-i}(\mathbf{x}+\boldsymbol{\xi})] \leq f_{-i}(y_{-i}) + \delta \mathbb{E}[V_{-i}(\mathbf{y}+\boldsymbol{\xi})], \tag{EC.14}$$

because $f'_j(x_j) \leq 0$ for $j = 1, 2$ and by induction hypothesis.

We next consider three exhaustive cases, depending on the equilibrium outcome and who exerts the production preemption to establish that $V_i(\mathbf{x}) \leq V_i(\mathbf{y})$ and $V_{-i}(\mathbf{y}) \geq V_{-i}(\mathbf{x})$ for any $\mathbf{y}$ such that $y_i \geq x_i$ and $y_{-i} \leq x_{-i}$.

Case 1: $a_j(\mathbf{x}) = C$ for $j = 1, 2$. In this case, $A(\mathbf{x}) = C$ by (2), i.e., $V_j(\mathbf{x}) = \delta V_j(\mathbf{0})$ for $j = 1, 2$. We consider two subcases, depending on $A(\mathbf{y})$.

- If $A(\mathbf{y}) = C$, $V_j(\mathbf{y}) = \delta V_j(\mathbf{0})$ for $j = 1, 2$, and therefore $V_j(\mathbf{y}) = V_j(\mathbf{x})$ for $j = 1, 2$.
- If $A(\mathbf{y}) = P$, then $V_i(\mathbf{y}) = f_i(y_i) + \delta \mathbb{E}[V_i(\mathbf{y}+\boldsymbol{\xi})]$. Then, by (EC.13), $V_i(\mathbf{y}) \leq f_i(x_i) + \delta \mathbb{E}[V_i(\mathbf{x}+\boldsymbol{\xi})]$. Because $f_i(x_i) + \delta \mathbb{E}[V_i(\mathbf{x}+\boldsymbol{\xi})] \leq \delta V_i(\mathbf{0})$ by (5) since $a_i(\mathbf{x}) = C$, we obtain that $V_i(\mathbf{y}) \leq \delta V_i(\mathbf{0}) = V_i(\mathbf{x})$. In particular, $a_i(\mathbf{y}) = C$. Hence we must have that $a_{-i}(\mathbf{y}) = P$ for $A(\mathbf{y}) = P$ to be true by (2), which implies, by (5), that $f_{-i}(y_{-i}) + \delta \mathbb{E}[V_{-i}(\mathbf{y}+\boldsymbol{\xi})] > \delta V_{-i}(\mathbf{0})$. Therefore, $V_{-i}(\mathbf{y}) > V_{-i}(\mathbf{x})$.

Case 2: $a_i(\mathbf{x}) = C$ and $a_{-i}(\mathbf{x}) = P$. By (2), $A(\mathbf{x}) = P$; hence, $V_j(\mathbf{x}) = f_j(x_j) + \delta \mathbb{E}[V_j(\mathbf{x}+\boldsymbol{\xi})]$ for $j = 1, 2$. By (EC.14), $f_{-i}(y_{-i}) + \delta \mathbb{E}[V_{-i}(\mathbf{y}+\boldsymbol{\xi})] \geq f_{-i}(x_{-i}) + \delta \mathbb{E}[V_{-i}(\mathbf{x}+\boldsymbol{\xi})]$ and because $a_i(\mathbf{x}) = P$, by (5), $f_{-i}(x_{-i}) + \delta \mathbb{E}[V_{-i}(\mathbf{x}+\boldsymbol{\xi})] > \delta V_{-i}(\mathbf{0})$; therefore $a_{-i}(\mathbf{y}) = P$, which implies, by (2), that $A(\mathbf{y}) = P$. Hence, $V_j(\mathbf{y}) = f_j(y_j) + \delta \mathbb{E}[V_j(\mathbf{y}+\boldsymbol{\xi})]$ for $j = 1, 2$. By (EC.13) and (EC.14), $V_i(\mathbf{y}) \leq V_i(\mathbf{x})$ and $V_{-i}(\mathbf{y}) \geq V_{-i}(\mathbf{x})$.

Case 3: $a_j(\mathbf{x}) = P$ for $j = 1,2$. In this case, $A(\mathbf{x}) = P$ by (2), i.e., $V_j(\mathbf{x}) = f_j(x_j) + \delta\mathbb{E}[V_j(\mathbf{x}+\boldsymbol{\xi})]$ for $j = 1,2$. We consider two subcases, depending on $A(\mathbf{y})$.

- If $A(\mathbf{y}) = C$, then $V_j(\mathbf{y}) = \delta V_j(\mathbf{0})$ for $j = 1,2$. Since $a_i(\mathbf{x}) = P$, $\delta V_i(\mathbf{0}) < f_i(x_i) + \delta\mathbb{E}[V_i(\mathbf{x}+\boldsymbol{\xi})]$ by (5). Hence, $V_i(\mathbf{y}) < V_i(\mathbf{x})$. Moreover, $V_{-i}(\mathbf{y}) = \delta V_{-i}(\mathbf{0}) \geq f_{-i}(y_{-i}) + \delta\mathbb{E}[V_{-i}(\mathbf{y}+\boldsymbol{\xi})]$ given that one must have $a_{-i}(\mathbf{y}) = C$ for having $A(\mathbf{y}) = C$ by (2). By (EC.14), $f_{-i}(y_{-i}) + \delta\mathbb{E}[V_{-i}(\mathbf{y}+\boldsymbol{\xi})] \geq f_{-i}(x_{-i}) + \delta\mathbb{E}[V_{-i}(\mathbf{x}+\boldsymbol{\xi})]$. Therefore, $V_{-i}(\mathbf{y}) \geq V_{-i}(\mathbf{x})$. (In fact, this case is infeasible since it implies that $\delta V_{-i}(\mathbf{0}) = V_{-i}(\mathbf{y}) \geq V_{-i}(\mathbf{x}) > \delta V_{-i}(\mathbf{0})$.)
- If $A(\mathbf{y}) = P$, then $V_j(\mathbf{y}) = f_j(y_j) + \delta\mathbb{E}[V_j(\mathbf{y}+\boldsymbol{\xi})]$ for $j = 1,2$. Then, by (EC.13) and (EC.14), $V_i(\mathbf{y}) \leq V_i(\mathbf{x})$ and $V_{-i}(\mathbf{y}) \geq V_{-i}(\mathbf{x})$.

Combining all three cases shows that $V_i(\mathbf{x}) \leq V_i(\mathbf{y})$ and $V_{-i}(\mathbf{y}) \geq V_{-i}(\mathbf{x})$, completing the induction step. □

Proposition B-2 characterizes the structure of the PP equilibrium outcome.

Proof of Proposition B-2. The proof uses Lemmas EC.16 and EC.17 in appendix. Lemma EC.17 establishes monotonicity of $V_i^{\mathrm{PP}}(\mathbf{x})$; unlike PC, worker i's value is increasing in his co-workers needs x_{-i}. Lemma EC.16 characterizes the equilibrium in the "upper right" corner when both workers want to coordinate; the characterization of the value functions in this region will be useful to initiate the induction in the other regions. In the proof, we omit the 'PP' superscript. As in Lemma EC.16, define $\hat{\mathbf{x}}$ such that $a_i(\mathbf{x}) = C$ for any $\mathbf{x}$ such that $x_i \geq \hat{x}_i$, for $i = 1,2$. Hence by construction, $A(\mathbf{x}) = C$ in all states $\mathbf{x} \geq \hat{\mathbf{x}}$.

First, consider the "upper left" (or "lower right") region $\{\mathbf{x} | x_i \geq \hat{x}_i, x_{-i} \leq \hat{x}_{-i} - 1\}$. To initialize the characterization, we first consider its boundary of the region of interest. By definition of $\hat{\mathbf{x}}$, $a_{-i}(\hat{x}_{-i} - 1, x_i) = P$ for some x_i; by (5), $V_{-i}(\hat{x}_{-i} - 1, x_i) > \delta V_{-i}(\mathbf{0})$. Fix this particular x_i. If $x_i \geq \hat{x}_i$, $V_{-i}(\hat{x}_{-i} - 1, x_i) = V_{-i}(\hat{x}_{-i} - 1, \hat{x}_i)$ by Lemma EC.16. If $x_i < \hat{x}_i$, $V_{-i}(\hat{x}_{-i} - 1, x_i) \leq V_{-i}(\hat{x}_{-i} - 1, \hat{x}_i)$ by Lemma EC.17. Combining both cases, we obtain that $V_{-i}(\hat{x}_{-i} - 1, \hat{x}_i + \xi_i) \geq V_{-i}(\hat{x}_{-i} - 1, x_i) > \delta V_{-i}(\mathbf{0})$ for all $\xi_i \geq 0$; thus, $a_{-i}(\hat{x}_{-i} - 1, \hat{x}_i + \xi_i) = P$ for all $\xi_i \geq 0$.

We next consider the interior of the region of interest. Consider any $\mathbf{x}$ such that $x_i \geq \hat{x}_i$ and $x_{-i} < \hat{x}_{-i}$. By Lemma EC.16, $V_{-i}(\mathbf{y})$ is nonincreasing in y_{-i} for all $y_i \geq \hat{x}_i$. Hence, $V_{-i}(\mathbf{x}) \geq V_{-i}(\hat{x}_{-i} - 1, x_i) > \delta V_{-i}(\mathbf{0})$; thus, $a_{-i}(\mathbf{x}) = P$. By (2), $A(\mathbf{x}) = P$.

In the lower right region, i.e., states $\mathbf{x} < \hat{\mathbf{x}}$, the equilibrium characterization comes from Lemma EC.17 and (2).

We next characterize the value of $\hat{\mathbf{x}}$. For any $i = 1,2$, denote by p_i the probability that worker i has at least one issue. Then, because $V_{-i}(\mathbf{y})$ is constant in y_i when $y_i \geq \hat{x}_i$ by Lemma EC.16,

$$\begin{aligned} V_{-i}(\hat{x}_{-i} - 1, \hat{x}_i) =& f_{-i}(\hat{x}_{-i} - 1) + (1 - p_{-i})\delta V_{-i}(\hat{x}_{-i} - 1, \hat{x}_i) + p_{-i}\delta^2 V_{-i}(\mathbf{0}) \\ =& \frac{1}{1 - \delta(1 - p_{-i})} f_{-i}(\hat{x}_{-i} - 1) + \frac{p_{-i}\delta^2}{1 - \delta(1 - p_{-i})} V_{-i}(\mathbf{0}), \end{aligned}$$

Therefore,

$$\begin{aligned} a_{-i}(\hat{x}_{-i} - 1, \hat{x}_i) = P \Leftrightarrow & \frac{1}{1 - \delta(1 - p_{-i})} f_{-i}(\hat{x}_{-i} - 1) + \frac{p_{-i}\delta^2}{1 - \delta(1 - p_{-i})} V_{-i}(\mathbf{0}) > \delta V_{-i}(\mathbf{0}) \\ \Leftrightarrow & f_{-i}(\hat{x}_{-i} - 1) > \delta(1 - \delta) V_{-i}(\mathbf{0}), \end{aligned}$$

and

$$a_{-i}(\hat{\mathbf{x}}) = C \Leftrightarrow f_{-i}(\hat{x}_{-i}) + \delta^2 V_{-i}(\mathbf{0}) \leq V_{-i}(\mathbf{0}) \Leftrightarrow f_{-i}(\hat{x}_{-i}) \leq \delta(1-\delta) V_{-i}(\mathbf{0}).$$

Combining these inequalities and applying the same logic to worker i yield the desired characterization of $\hat{\mathbf{x}}$. □

Proposition B-3 characterizes the structure of the HS equilibrium outcome.

Proof of Proposition B-3. This is a corollary of Proposition 1 with $n = 1$. □

EC.2.6. Equilibrium Characterization under Binary Productivity Functions with Base Value and Team Incentives

In this appendix, we derive necessary and sufficient conditions such that the equilibrium characterizations of PC, PP, and HS established under Assumption 1 using (1), (2), and (3) together with (9), remain valid in the presence of a base value when workers have accumulated some issues (Appendix B.2) and in the presence of team incentives (Appendix C). Lemmas EC.18, EC.19, and EC.20 respectively do so for PC, PP, and HS.

LEMMA EC.18. *Suppose that $n = 2$, that Assumption 1(i) holds, that $f_i(x) = b_i + (v_i - b_i)\mathbb{1}[x = 0]$ for $i = 1, 2$, and that $w_i(\mathbf{f}(\mathbf{x})) = \gamma f_i(x_i) + (1-\gamma) f_{-i}(x_{-i})$ for $i = 1, 2$. The policy $A^{PC}(\mathbf{x}) = P \Leftrightarrow \mathbf{x} = \mathbf{0}$ arises in equilibrium under PC if and only if, for $i = 1, 2$,*

$$\delta \frac{\gamma v_i + (1-\gamma) v_{-i}}{1 + \delta - \delta(1-p_i)(1-p_{-i})} \geq \gamma b_i + (1-\gamma) v_{-i}. \tag{EC.15}$$

Proof. Denote by π the policy such that $A^\pi(\mathbf{x}) = P$ if and only if $\mathbf{x} = \mathbf{0}$, and let $V_i^\pi(\mathbf{x})$ be the associated value-to-go functions. Let also $V_i^\pi(\mathbf{x}|P)$ be worker i's value-to-go if action P is chosen in state $\mathbf{x}$ and policy π is then followed. For policy π to arise in equilibrium under PC, we need to have $V_i^\pi(\mathbf{x}|P) \leq \delta V_i^\pi(\mathbf{0})$ if and only if $x_i \geq 1$. Similar to (EC.3), we obtain:

$$(1 - \delta(1-p_1)(1-p_2)) V_i^\pi(\mathbf{0}) = \gamma v_i + (1-\gamma) v_{-i} + \delta^2 (1 - (1-p_1)(1-p_2)) V_i^\pi(\mathbf{0}),$$

or equivalently,

$$V_i^\pi(\mathbf{0}) = \frac{\gamma v_i + (1-\gamma) v_{-i}}{(1-\delta)(1 + \delta - \delta(1-p_1)(1-p_2))}. \tag{EC.16}$$

Without loss of generality, we can restrict our attention to the following four states: $\{(0,0), (1,0), (0,1), (1,1)\}$. Out of these four possible values of $\mathbf{x}$, we only need to consider two cases so that worker i prefers to coordinate whenever $x_i \geq 1$, namely, $\mathbf{x} = (1,1)$ and $\mathbf{x} = (1,0)$. If so, coordination is guaranteed to happen under PC when $\min\{x_1, x_2\} \geq 1$. When $x_1 = x_2 = 0$, production always happens since $V_i((0,0)|P) = V_i^\pi(\mathbf{0}) > \delta V_i^\pi(\mathbf{0})$ given that $\delta < 1$.

First, $V_i((1,1)|P) \leq \delta V_i^\pi(\mathbf{0})$ if and only if

$$\gamma b_i + (1-\gamma) b_{-i} + \delta^2 (1 - (1-p_1)(1-p_2)) V_i^\pi(\mathbf{0}) \leq \delta V_i^\pi(\mathbf{0}) (1 - \delta(1-p_1)(1-p_2)),$$

or equivalently, $\gamma b_i + (1-\gamma) b_{-i} \leq \delta(1-\delta) V_i^\pi(\mathbf{0})$, which holds by (EC.15) after replacing $V_i^\pi(\mathbf{0})$ with (EC.16) since $b_{-i} \leq v_{-i}$.

Second, $V_i((1,0)|P) \le \delta V_i^\pi(\mathbf{0})$ if and only if

$$\gamma b_i + (1-\gamma)v_{-i} + \delta^2 p_{-i} V_i^\pi(\mathbf{0}) \le \delta V_i^\pi(\mathbf{0})\left(1-\delta(1-p_{-i})\right),$$

or equivalently, $\gamma b_i + (1-\gamma)v_{-i} \le \delta(1-\delta)V_i^\pi(\mathbf{0})$, which holds by (EC.15) after replacing $V_i^\pi(\mathbf{0})$ with (EC.16). $\square$

Lemma EC.19. *Suppose that $n=2$, that Assumption 1(i) holds, that $f_i(x) = b_i + (v_i - b_i)\mathbb{1}[x=0]$ for $i=1,2$, and that $w_i(\mathbf{f}(\mathbf{x})) = \gamma f_i(x_i) + (1-\gamma) f_{-i}(x_{-i})$ for $i=1,2$. The policy $A^{PP}(\mathbf{x}) = C \Leftrightarrow \mathbf{x} \ge \mathbf{1}$ arises in equilibrium under PP if and only if, for $i=1,2$,*

$$\gamma v_i + (1-\gamma)b_{-i} > \delta(1-\delta)V_i(\mathbf{0}) \ge \gamma b_i + (1-\gamma)b_{-i}, \tag{EC.17}$$

in which

$$\begin{aligned} V_i(\mathbf{0}) = &\left(\gamma v_i \frac{1-\delta(1-p_i)(1-p_{-i})}{1-\delta(1-p_i)} + (1-\gamma)v_{-i}\frac{1-\delta(1-p_i)(1-p_{-i})}{1-\delta(1-p_{-i})}\right.\\ &\left.+\gamma b_i \frac{\delta p_i(1-p_{-i})}{1-\delta(1-p_{-i})} + (1-\gamma)b_{-i}\frac{\delta p_{-i}(1-p_i)}{1-\delta(1-p_i)}\right) \\ &\times \frac{(1-\delta(1-p_i))(1-\delta(1-p_{-i}))}{(1-\delta)\left(\delta^2(p_1^2+p_2^2+p_1p_2(1-\delta p_1 p_2)) + \delta(1-\delta)(2-\delta p_1 p_2)(p_1+p_2) + (1-\delta)^2(1-\delta p_1 p_2)\right)}. \end{aligned} \tag{EC.18}$$

Proof. Denote by π the policy such that $A(\mathbf{x}) = C$ if and only if $\mathbf{x} \ge \mathbf{1}$, and let $V_i^\pi(\mathbf{x})$ be the associated value functions, omitting the time argument. Let also $V_i^\pi(\mathbf{x}|P)$ be worker i's value if action P were chosen in state $\mathbf{x}$ and policy π were followed in the other states. For policy π to arise in equilibrium under PP, we need to have $V_i^\pi(\mathbf{x}|P) > \delta V_i^\pi(\mathbf{0})$ if and only if $x_i = 0$. We have:

$$\begin{aligned} \left(1-\delta(1-p_1)(1-p_2)\right)V_i^\pi(\mathbf{0}) = &\ \gamma v_i + (1-\gamma)v_{-i} + \delta^2 p_1 p_2 V_i^\pi(\mathbf{0}) \\ &+\delta p_i(1-p_{-i})\frac{\gamma b_i + (1-\gamma)v_{-i} + p_{-i}\delta^2 V_i^\pi(\mathbf{0})}{1-\delta(1-p_{-i})} \\ &+\delta p_{-i}(1-p_i)\frac{\gamma v_i + (1-\gamma)b_{-i} + p_i\delta^2 V_i^\pi(\mathbf{0})}{1-\delta(1-p_i)}, \end{aligned}$$

or equivalently, $V_i^\pi(\mathbf{0})$ is as defined by (EC.18).

Without loss of generality, we can restrict our attention to the following four states: $\{(0,0),(1,0),(0,1),(1,1)\}$. Out of the four possible values of $\mathbf{x}$, we only need to consider two cases so that, when $x_{-i}=1$, worker i prefers to produce whenever if and only if $x_i = 0$, namely, $\mathbf{x} = (1,1)$ and $\mathbf{x} = (0,1)$. When $\mathbf{x} = \mathbf{0}$, production always happens since $V_i((0,0)|P) = V_i^\pi(\mathbf{0}) > \delta V_i^\pi(\mathbf{0})$ given that $\delta < 1$. If so, production is guaranteed to happen under PP when $\min\{x_1, x_2\} = 0$.

First, $V_i((1,1)|P) \le \delta V_i^\pi(\mathbf{0})$ if and only if

$$\gamma b_i + (1-\gamma)b_{-i} + \delta^2\left(1-(1-p_1)(1-p_2)\right)V_i^\pi(\mathbf{0}) \le \delta V_i^\pi(\mathbf{0})\left(1-\delta(1-p_1)(1-p_2)\right),$$

or equivalently, $\gamma b_i + (1-\gamma)b_{-i} \le \delta(1-\delta)V_i^\pi(\mathbf{0})$, which holds by (EC.17).

Second, $V_i((0,1)|P) > \delta V_i^\pi(\mathbf{0})$ if and only if

$$\gamma v_i + (1-\gamma)b_{-i} + \delta^2\left(1-(1-p_i)\right)V_i^\pi(\mathbf{0}) > \delta V_i^\pi(\mathbf{0})\left(1-\delta(1-p_i)\right),$$

or equivalently, $\gamma v_i + (1-\gamma)b_{-i} > \delta(1-\delta)V_i^\pi(\mathbf{0})$, which holds by (EC.17). $\square$

LEMMA EC.20. *Suppose that $n=2$, that Assumption 1(i) holds, that $f_i(x) = b_i + (v_i - b_i)\mathbb{1}[x=0]$ for $i=1,2$, and that $w_i(\mathbf{f}(\mathbf{x})) = \gamma f_i(x_i) + (1-\gamma) f_{-i}(x_{-i})$ for $i=1,2$. The policy $A^{HS_1}(\mathbf{x}) = C \Leftrightarrow x_1 \geq 1$ arises in equilibrium under HS_1 if and only if*

$$\gamma v_1 + (1-\gamma)b_2 > \delta(1-\delta)V_1(\mathbf{0}) \geq \gamma b_1 + (1-\gamma)v_2, \tag{EC.19}$$

in which

$$V_1(\mathbf{0}) = \left(\gamma v_1 \frac{1-\delta(1-p_1)(1-p_2)}{1-\delta(1-p_1)} + (1-\gamma)v_2 + (1-\gamma)b_2 \frac{\delta p_2(1-p_1)}{1-\delta(1-p_1)}\right) \times \frac{(1-\delta(1-p_1))}{(1-\delta)(1-\delta(1-p_1)(1-p_2))(1+\delta p_1)}. \tag{EC.20}$$

Proof. Denote by π the policy such that $A(\mathbf{x}) = C$ if and only if $x_1 \geq 1$, and let $V_i^\pi(\mathbf{x})$ be the associated value functions, omitting the time argument. Let also $V_i^\pi(\mathbf{x}|P)$ be worker i's value if action P were chosen in state $\mathbf{x}$ and policy π were followed in the other states. For policy π to arise in equilibrium under HS, we need to have $V_1^\pi(\mathbf{x}|P) > \delta V_1^\pi(\mathbf{0})$ if and only if $x_1 = 0$. Similar to the proof of Lemma EC.4 or (EC.5), we obtain:

$$(1-\delta(1-p_1)(1-p_2))\, V_1^\pi(\mathbf{0}) = \gamma v_1 + (1-\gamma)v_2 + \delta^2 p_1 V_1^\pi(\mathbf{0}) + \delta p_2(1-p_1)\frac{\gamma v_1 + (1-\gamma)b_2 + p_1\delta^2 V_1^\pi(\mathbf{0})}{1-\delta(1-p_1)},$$

or equivalently, $V_1^\pi(\mathbf{0})$ is as defined by (EC.20).

Without loss of generality, we can restrict our attention to the following four states: $\{(0,0),(1,0),(0,1),(1,1)\}$. When $\mathbf{x} = \mathbf{0}$, production always happens since $V_1((0,0)|P) = V_1^\pi(\mathbf{0}) > \delta V_1^\pi(\mathbf{0})$ given that $\delta < 1$. Let us consider the other three cases.

First, $V_1((1,0)|P) \leq \delta V_1^\pi(\mathbf{0})$ if and only if

$$\gamma b_1 + (1-\gamma)v_2 + \delta^2 p_2 V_1^\pi(\mathbf{0}) \leq \delta V_1^\pi(\mathbf{0})\,(1-\delta(1-p_2)),$$

or equivalently, $\gamma b_1 + (1-\gamma)v_2 \leq \delta(1-\delta)V_1^\pi(\mathbf{0})$, which holds by (EC.19).

Second, $V_1((1,1)|P) \leq \delta V_1^\pi(\mathbf{0})$ if and only if

$$\gamma b_1 + (1-\gamma)b_2 + \delta^2\,(1-(1-p_1)(1-p_2))\, V_1^\pi(\mathbf{0}) \leq \delta V_1^\pi(\mathbf{0})\,(1-\delta(1-p_1)(1-p_2)),$$

or equivalently, $\gamma b_1 + (1-\gamma)b_2 \leq \delta(1-\delta)V_1^\pi(\mathbf{0})$, which holds by (EC.19) since $b_2 < v_2$.

Third, $V_1((0,1)|P) > \delta V_1^\pi(\mathbf{0})$ if and only if

$$\gamma v_1 + (1-\gamma)b_2 + \delta^2\,(1-(1-p_1))\, V_1^\pi(\mathbf{0}) > \delta V_1^\pi(\mathbf{0})\,(1-\delta(1-p_1)),$$

or equivalently, $\gamma v_1 + (1-\gamma)b_2 > \delta(1-\delta)V_1^\pi(\mathbf{0})$, which holds by (EC.19). □

Proposition B-4 shows which equilibrium outcome under Assumption 1(ii), among the coordination scheduling rules PC, PP, and HS, might be the most affected by the introduction of a base productivity value.

Proof of Proposition B-4. The proof uses Lemmas EC.18-EC.20 in the electronic companion, which characterize the conditions under which the equilibrium policies considered in the main body of the paper under PC, PP, and HS apply when productivity has a base value and in the presence of team incentives. When $\gamma = 1$, condition (EC.15) simplifies to:

$$\frac{\delta}{1+\delta-\delta(1-p_i)(1-p_{-i})} \geq \frac{b_i}{v_i}. \tag{EC.21}$$

When $\gamma = 1$, condition (EC.17) simplifies to:

$$\frac{1-\delta(1-p_{-i})(1-\delta+(1-p_i)(1+\delta p_{-i}))}{\delta^2 p_i(1-p_{-i})} > \frac{b_i}{v_i} \quad \text{and} \quad \frac{\delta}{1+\delta p_i} \geq \frac{b_i}{v_i}.$$

It can be verified that $1-\delta(1-p_{-i})(1-\delta+(1-p_i)(1+\delta p_{-i})) > 0$ so the first inequality is well defined. Since $\frac{1-\delta(1-p_{-i})(1-\delta+(1-p_i)(1+\delta p_{-i}))}{\delta^2 p_i(1-p_{-i})} > \frac{\delta}{1+\delta p_i}$, the first inequality is redundant. Hence, the condition simplifies to

$$\frac{\delta}{1+\delta p_i} \geq \frac{b_i}{v_i}. \tag{EC.22}$$

When $\gamma = 1$, condition (EC.19) simplifies to:

$$\frac{\delta}{1+\delta p_1} \geq \frac{b_1}{v_1}. \tag{EC.23}$$

Set $i = 2$. The result follows from comparing (EC.21), (EC.22), and (EC.23) and noting that

$$\infty > \frac{\delta}{1+\delta p_2} \geq \frac{\delta}{1+\delta-\delta(1-p_1)(1-p_2)}.$$

□

Proposition C-1 shows which equilibrium outcome in our base model, among the coordination scheduling rules PC, PP, and HS, might be the most affected by the introduction of team incentives.

Proof of Proposition C-1. The proof uses Lemmas EC.18-EC.20 in the electronic companion, which characterize the conditions under which the equilibrium policies considered in the main body of the paper under PC, PP, and HS apply when productivity has a base value and in the presence of team incentives. Recall also the definitions of $\alpha(p_1, p_2, \delta)$ and $\beta(p_1, p_2, \delta)$ in (10).

When $\mathbf{b} = \mathbf{0}$, condition (EC.15) simplifies to:

$$\frac{\delta}{1-\delta(1-p_i)(1-p_{-i})} \frac{v_i}{v_{-i}} \geq \frac{1-\gamma}{\gamma}. \tag{EC.24}$$

or equivalently, $\alpha(p_1, p_2, \delta)\frac{v_i}{v_{-i}} \geq \frac{1-\gamma}{\gamma}$, which generalizes Proposition 2.

When $\mathbf{b} = \mathbf{0}$, condition (EC.17) simplifies to:

$$\frac{v_i}{v_{-i}} \frac{1-\delta(1-p_{-i})(1-\delta+(1-p_i)(1+\delta p_{-i}))}{\delta\,(1-\delta(1-p_i)(1-p_{-i}))} > \frac{1-\gamma}{\gamma}, \tag{EC.25}$$

or equivalently, when $\beta(p_{-i}, p_i, \delta)\frac{v_i}{v_{-i}} \geq \frac{1-\gamma}{\gamma}$, which generalizes Proposition 2.

When $\mathbf{b} = \mathbf{0}$, condition (EC.19) simplifies to:

$$\frac{v_1}{v_2} \min\left\{\frac{1-\delta(1-p_1)(1-p_2)}{\delta}, \frac{\delta(1-\delta(1-p_1)(1-p_2))}{(1+\delta p_1)(1-\delta(1-p_1)(1-p_2)) - \delta(1-\delta(1-p_1))}\right\} \geq \frac{1-\gamma}{\gamma}, \tag{EC.26}$$

or equivalently, when $\frac{v_1}{v_2}\min\left\{\frac{1}{\alpha(p_1,p_2,\delta)}, \frac{1}{\beta(p_1,p_2,\delta)}\right\} \geq \frac{1-\gamma}{\gamma}$, which generalizes Proposition 2.

The rest of the proof consists in comparing conditions (EC.24), (EC.25), and (EC.26), both when $\delta \geq 1-\delta(1-p_1)(1-p_2)$ and when the opposite holds. It can easily be shown that $\alpha(p_1,p_2,\delta) \geq 1$ if and only if $\beta(p_1,p_2,\delta) \leq 1$ if and only if $\beta(p_2,p_1,\delta) \leq 1$; and that $\alpha(p_1,p_2,\delta) \geq 1$ if and only if $\alpha(p_1,p_2,\delta)\beta(p_1,p_2,\delta) \geq 1$ if and only if $\alpha(p_1,p_2,\delta)\beta(p_2,p_1,\delta) \geq 1$.

First, suppose that $\delta \geq 1-\delta(1-p_1)(1-p_2)$, i.e., $\alpha(p_1,p_2,\delta) \geq 1$. Accordingly,

$$\alpha(p_1,p_2,\delta)\min\left\{\frac{v_1}{v_2},\frac{v_2}{v_1}\right\} \geq \min\left\{\frac{v_1}{v_2}\beta(p_2,p_1,\delta),\frac{v_2}{v_1}\beta(p_1,p_2,\delta)\right\},$$

i.e., Condition (EC.24) is looser than (EC.25). Moreover, since $\alpha(p_1,p_2,\delta) \geq 1$,

$$\begin{aligned}\frac{v_1}{v_2}\beta(p_2,p_1,\delta) &\geq \frac{v_1}{v_2}\min\left\{\frac{1}{\alpha(p_1,p_2,\delta)},\frac{1}{\beta(p_1,p_2,\delta)}\right\},\\ \frac{v_2}{v_1}\beta(p_1,p_2,\delta) &\geq \frac{v_2}{v_1}\min\left\{\frac{1}{\alpha(p_1,p_2,\delta)},\frac{1}{\beta(p_2,p_1,\delta)}\right\}.\end{aligned}$$

Therefore,

$$\begin{aligned}&\min\left\{\frac{v_1}{v_2}\min\left\{\frac{1}{\alpha(p_1,p_2,\delta)},\frac{1}{\beta(p_1,p_2,\delta)}\right\},\frac{v_2}{v_1}\min\left\{\frac{1}{\alpha(p_1,p_2,\delta)},\frac{1}{\beta(p_2,p_1,\delta)}\right\}\right\}\\ &\leq \min\left\{\frac{v_1}{v_2}\beta(p_2,p_1,\delta),\frac{v_2}{v_1}\beta(p_1,p_2,\delta)\right\},\end{aligned}$$

i.e., Condition (EC.25) is looser than (EC.26), for at least one $i \in \{1,2\}$. Finally, since $\alpha(p_1,p_2,\delta) \geq 1$,

$$\frac{v_1}{v_2}\min\left\{\frac{1}{\alpha(p_1,p_2,\delta)},\frac{1}{\beta(p_1,p_2,\delta)}\right\} \leq \frac{v_2}{v_1}\min\left\{\frac{1}{\alpha(p_1,p_2,\delta)},\frac{1}{\beta(p_2,p_1,\delta)}\right\}$$

if and only if $v_1 \leq v_2$.

Second, suppose that $\delta \leq 1-\delta(1-p_1)(1-p_2)$, i.e., $\alpha(p_1,p_2,\delta) \leq 1$. Accordingly,

$$\alpha(p_1,p_2,\delta)\min\left\{\frac{v_1}{v_2},\frac{v_2}{v_1}\right\} \leq \left\{\begin{array}{l}\alpha(p_1,p_2,\delta)\frac{v_1}{v_2} \leq \frac{v_1}{v_2}\min\left\{\frac{1}{\alpha(p_1,p_2,\delta)},\frac{1}{\beta(p_1,p_2,\delta)}\right\}\\ \alpha(p_1,p_2,\delta)\frac{v_2}{v_1} \leq \frac{v_2}{v_1}\min\left\{\frac{1}{\alpha(p_1,p_2,\delta)},\frac{1}{\beta(p_2,p_1,\delta)}\right\}\end{array}\right.,$$

i.e., Condition (EC.24) is stricter than (EC.26) for $i=1,2$. Moreover, since $\alpha(p_1,p_2,\delta) \leq 1$,

$$\begin{aligned}\frac{v_1}{v_2}\min\left\{\frac{1}{\alpha(p_1,p_2,\delta)},\frac{1}{\beta(p_1,p_2,\delta)}\right\} &= \frac{v_1}{v_2}\frac{1}{\beta(p_1,p_2,\delta)} \leq \frac{v_1}{v_2}\beta(p_2,p_1,\delta)\\ \frac{v_2}{v_1}\min\left\{\frac{1}{\alpha(p_1,p_2,\delta)},\frac{1}{\beta(p_2,p_1,\delta)}\right\} &= \frac{v_2}{v_1}\frac{1}{\beta(p_2,p_1,\delta)} \leq \frac{v_2}{v_1}\beta(p_1,p_2,\delta).\end{aligned}$$

Therefore,

$$\begin{aligned}&\min\left\{\frac{v_1}{v_2}\min\left\{\frac{1}{\alpha(p_1,p_2,\delta)},\frac{1}{\beta(p_1,p_2,\delta)}\right\},\frac{v_2}{v_1}\min\left\{\frac{1}{\alpha(p_1,p_2,\delta)},\frac{1}{\beta(p_2,p_1,\delta)}\right\}\right\}\\ &\leq \min\left\{\frac{v_1}{v_2}\beta(p_2,p_1,\delta),\frac{v_2}{v_1}\beta(p_1,p_2,\delta)\right\},\end{aligned}$$

i.e., Condition (EC.25) is looser than (EC.26), for at least one $i \in \{1,2\}$. □